Pure Mathematical Sciences, Vol. 9, 2020, no. 1, 53 - 59
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https://doi.org/10.12988/pms.2020.91220

Super Fuzzy n^k – Based Graceful Labeling of Some Classes of Trees

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Abstract

Fuzzy sets were introduced by Lotfi. A. Zadeh in 1965, which had a greater improvement in mathematical modeling. In this paper we define the super fuzzy n^k– based graceful labeling for path graph, star graph and the product of path and star graph.

Keywords: Star graph, super fuzzy, graceful labeling

1. Introduction

Fuzzy sets were introduced by Lotfi. A. Zadeh in 1965 [1], which had a greater improvement in mathematical modeling. Fuzzy sets and fuzzy relations on it had been leading to make a fuzzy graph model when there is an ambiguity in vertices and edges. Narsingh [2] extended the concepts of graph theory. A. Nagoorgani, Muhammed Akram and D. Rajalakshmi [3, 4] introduced the concepts of labeling of fuzzy graph. Some results of fuzzy bi-magic and anti-magic labeling on star graph were proved by K.Thirusangu and D. Jeevitha [5], also fuzzy vertex graceful labeling are obtained by K. Ameenal and M. Devi on bi star graph [6]. A. Solairaju and T. Narppasalai [7] introduced supper fuzzy $10^k -$ based graceful labeling for some classes of fuzzy trees. N.Sujatha, C. Dharuman and K. Thirusangu [8] obtained graceful and magic labeling for special fuzzy graphs.

2. Preliminaries

Definition 2.1: let Y be a space of points with a generic element of Y, y. A fuzzy set A in Y is characterized by a membership function $f_A(y)$ which associates with each point in Y a real number in the interval $[0, 1]$ with the value of $f_A(y)$ at y, representing the grade of membership of y in A.

Definition 2.2: A graph $G = (V, E)$ is characteristic by a set of vertices $V = \{v_1, v_2, \dots\dots, v_n\}$ and a set of edges $E = \{e_1, e_2, \dots\dots, e_m\}$ which coupled a pair of vertices of V.

Definition 2.3: The path graph P_n is a sequence of vertices and edges with no repetition of its vertices, a path with n vertices has n – 1 as a number of edges.

Definition 2.4: The star graph S_n is a tree with n vertices and n – 1 edges, every vertex has degree one except the center of star which has degree n – 1, also a star graph is a complete bipartite graph $K_{1,n}$.

Definition 2.5: Let U and X be two sets, a relation ρ is said to be a fuzzy relation from U into X if ρ is a fuzzy set of U × X. A fuzzy graph $G = (\mu, \sigma)$ is a pair of functions defined by $\sigma: V \to [0, 1]$ and $\mu: V \times V \to [0, 1]$, for all $u, v \in V$ we have $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$.

Definition 2.6: A graph $G = (\mu, \sigma)$ is said to be a fuzzy labeling graph if $\sigma: V \to [0, 1]$ and $\mu: V \times V \to [0, 1]$ are bijectives, such that the membership value of edges and vertices are distinct and $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$.

Definition 2.7: A fuzzy labeling graph $G = (\mu, \sigma)$ is said to be a fuzzy graceful graph if $\sigma: V \to [0, 1]$ and $\mu: V \times V \to [0, 1]$ such that $\mu(u, v) = |\sigma(u) - \sigma(v)|$ and $\mu(u, v)$ are distinct for all $u, v \in V$.

3. Main results

Theorem 3.1: Every path P_n is a super fuzzy n^k- based graceful.

Proof: Consider a graph $G = P_n$ with vertex set $V = \{v_1, v_2, \dots \dots, v_n\}$ and edge set $E = \{v_i v_{i+1}, i = 1, 2, 3, \dots \dots, n-1\}$. Assume $w = n, q = n - 1$, where w and q represent number of vertices and edges of the path graph, respectively.
Define $\sigma: V \to [0, 1]$ by the rules:
(a) $\sigma(v_1) = \frac{w+q}{n^j} - 1$, where $j = 1, 2, \dots \dots, n$ (for this case, j =*1*).
(b) $\sigma(v_2) = \left|\sigma(v_1) - \frac{q^2}{n^2}\right|$
(b) $\sigma(v_i) = \left|\sigma(v_{i-2}) - \frac{q^i}{n^j}\right|, i, j = 3, 4, \dots \dots, n.$
Also, define $\mu: V \times V \to [0, 1]$, for all $u, v \in V$ and uv is an edge in E, by:
$\mu(uv) = |\sigma(u) - \sigma(v)|$, then σ, μ both satisfy the supper fuzzy n^k- based graceful labeling.

Example 3.1:
The super fuzzy n^k- based graceful for P_3 is obtained in the following figure. We have $w = 3, q = 2$

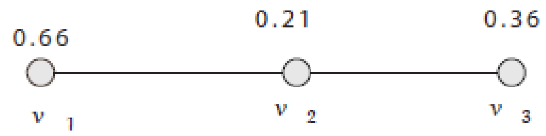

Figure 1: Super fuzzy n^k- based graceful labeling for P_3.

In this case we have:

$$\sigma(v_1) = \frac{5}{3} - 1 = 0.66$$

$$\sigma(v_2) = \left|0.66 - \frac{2^2}{3^2}\right| = 0.21$$

$$\sigma(v_3) = \left|0.66 - \frac{2^3}{3^3}\right| = 0.36$$

And:

$$\mu(v_1 v_2) = |\sigma(v_1) - \sigma(v_2)| = 0.45$$

$$\mu(v_2 v_3) = |\sigma(v_2) - \sigma(v_3)| = 0.15$$

Theorem 3.2: Every star graph S_n is a super fuzzy n^k- based graceful labeling.

Proof:
Consider a star graph S_n whose vertex set $\{v_1, v_2, \dots \dots \dots, v_n\}$ and edge set $\{v_1 v_2, v_1 v_3, \dots \dots \dots, v_1 v_n\}$ with $w = n$ and $q = n - 1$, where *w* and *q* are

number of vertices and edges of star graph with index n, respectively.
Define a function $\sigma: V \to [0,1]$ by
$(a)\ \sigma(v_1) = \frac{w+q}{n^j} - 1$, where $j = 1$
$(b)\ \sigma(v_i) = \left|\sigma(v_{i-1}) - \frac{q}{n^j}\right|, i, j = 2, 3, \dots \dots, n-1$
And $\mu: V \times V \to [0,1]$, for all $u, v \in V$ and uv is an edge in E, by:
$\mu(uv) = |\sigma(u) - \sigma(v)|$, then σ, μ both satisfy the supper fuzzy n^k- based graceful labeling.

Example 3.2: Consider a star graph S_n with n = 7, then S_7 is a super fuzzy $n^k -$ based graceful labeling. Consider the following star graph with 7 vertices and 6 edges. By applying the set of rules, we have:

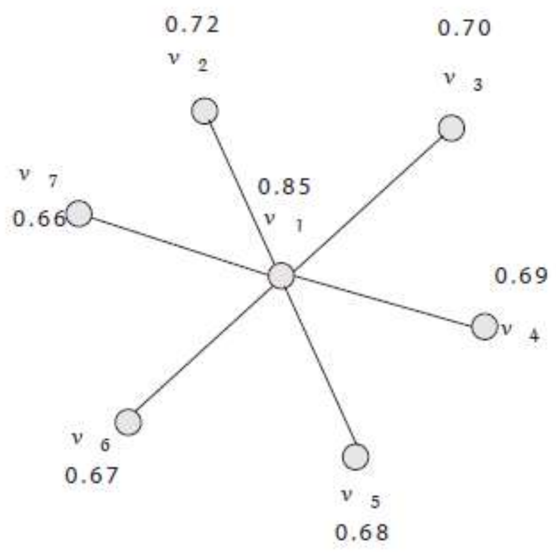

Figure 2: Super fuzzy $n^k -$ based graceful labeling for S_7

$\sigma(v_1) = \frac{13}{7} - 1 = 0.85$
$\sigma(v_2) = \left|0.85 - \frac{6}{7^2}\right| = 0.72$
$\sigma(v_3) = 0.70, \sigma(v_4) = 0.69, \sigma(v_5) = 0.68, \sigma(v_6) = 0.67$ and $\sigma(v_7) = 0.66$
$\mu(v_1v_2) = 0.13,\quad \mu(v_1v_3) = 0.15,\quad \mu(v_1v_4) = 0.16,\quad \mu(v_1v_5) = 0.17,$
$\mu(v_1v_6) = 0.18,\quad \mu(v_1v_7) = 0.19$

Theorem 3.3: The graph $(P_n * S_m)$ is a super fuzzy $n^k -$ based graceful.

Proof: The graph $(P_n * S_m)$ has two cases for which P_n is merged with S_m.

Case1: One of the two vertices of degree one of P_n is merged with the center vertex of S_m.
Case 2: One of the two vertices of degree one of P_n is merged with a non-centered vertex of S_m.
In case 1, define $\sigma: V \to [0, 1]$ and $\mu: V \times V \to [0, 1]$ by:

$$\sigma(v_1) = \frac{w+q}{n} - 1$$

$$\sigma(v_2) = \left|\sigma(v_1) - \frac{q^2}{n^2}\right|$$

$\sigma(v_i) = \left|\sigma(v_{i-2}) - \frac{q^i}{n^j}\right|, i, j = 3, 4, \dots \dots, n$, with v_i a vertex of P_n.

$\sigma(v_i) = \left|\sigma(v_{i-1}) - \frac{q}{n^j}\right|, j = 2, 3, \dots \dots, m-1$ and v_i is a vertex of S_m.
In second case, define $\sigma: V \to [0, 1]$ and $\mu: V \times V \to [0, 1]$ by:
$(a)\ \sigma(v_1) = \frac{w+q}{n^j} - 1$, where $j = 1, 2, \dots, n$ (for this case, $j = 1$).
(b) $\sigma(v_2) = \left|\sigma(v_1) - \frac{q^2}{n^2}\right|$

$$(b)\ \sigma(v_i) = \left|\sigma(v_{i-2}) - \frac{q^i}{n^j}\right|$$

$\mu(v_i v_j) = |\sigma(v_i) - \sigma(v_j)|$, for $v_i v_j$ is an edge of G, so σ, μ both satisfy the supper fuzzy n^k- based graceful labeling for $G = (P_n * S_m)$.

Example 3.3: Consider path P_3 is merged with S_3, the result graph has 5vertices and 4 edges, in either one of the two cases, we have the following label:

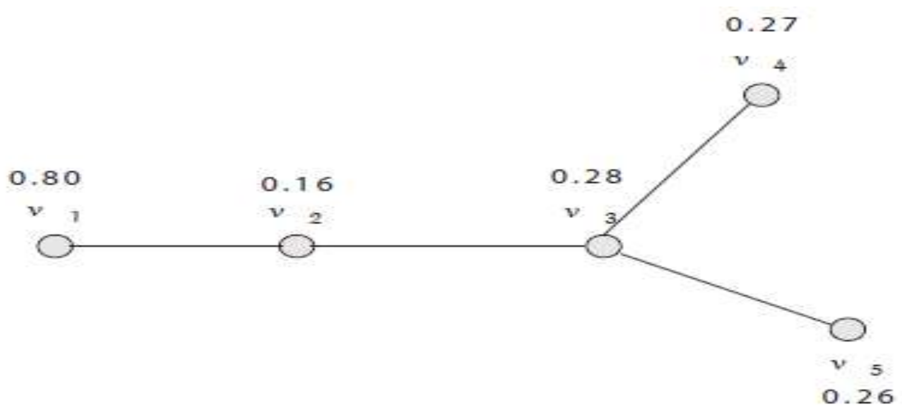

Figure 3: Super fuzzy $n^k -$ based graceful labeling for $(P_3 * S_3)$ of case 1.

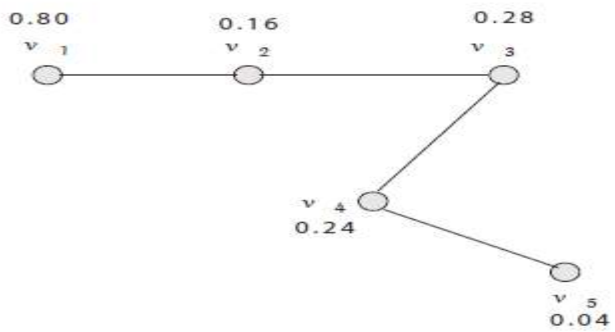

Figure 4: Super fuzzy $n^k -$ based graceful labeling for $(P_3 * S_3)$ of case 2.

For case 1, the labeling of vertices and edges are:
$\sigma(v_1) = 0.80, \sigma(v_2) = 0.16, \sigma(v_3) = 0.28, \sigma(v_4) = 0.27$ and $\sigma(v_5) = 0.26$
$\mu(v_1v_2) = 0.64,\quad \mu(v_2v_3) = 0.12,\quad \mu(v_3v_4) = 0.01,\quad \mu(v_3v_5) = 0.02$
For case 2, the labeling of vertices and edges are:
$\sigma(v_1) = 0.80, \sigma(v_2) = 0.16, \sigma(v_3) = 0.28, \sigma(v_4) = 0.24$ and $\sigma(v_5) = 0.04$
$\mu(v_1v_2) = 0.64,\quad \mu(v_2v_3) = 0.12,\quad \mu(v_3v_4) = 0.04,\quad \mu(v_4v_5) = 0.2$

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Received: October 25, 2020; Published: December 8, 2020