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# Parallelism of Weil Bundles

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**Abstract**

Let $M$ be a smooth manifold, $A$ a Weil algebra and $M^A$ the associed Weil bundle. We use the structure of $C^\infty(M^A, A)$-module on the set $\mathfrak{X}(M^A)$ of vector fields on $M^A$ for to give the equivalence of parallelism of the $A$-manifold $M^A$.

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## 1 Preliminaries

A Weil algebra or a local algebra (in the sense of André Weil) is a unitary commutative real algebra of finite dimension with unique maximal ideal of codimension 1.

Let $A$ be a Weil algebra and let $\mathfrak{m}$ be its unique maximal ideal. We have

$$A = \mathbb{R} \oplus \mathfrak{m}.$$

The first projection

$$A = \mathbb{R} \oplus \mathfrak{m} \longrightarrow \mathbb{R}$$

is a surjective homomorphism of algebras, called augmention and the unique integer $k \in \mathbb{N}$ such that $\mathfrak{m}^k \neq (0)$ and $\mathfrak{m}^{k+1} = (0)$ is the order of $A$.

We have, as examples of Weil algebras:

**Example 1** *:*

1. *$\mathbb{R} = \mathbb{R} \oplus (0)$ is a Weil algebra of order* 0*.*

2. *The algebra of dual numbers, $\mathbb{D} = \mathbb{R}[T]/(T^2)$, is a Weil algebra of order* 1*.*

3. *$\mathbb{A} = \mathbb{R}[T]/(T^3)$ is a Weil algebra of order* 2*. More generally, the algebra of truncated polynomials*

$$A = \mathbb{R}[X_1, ..., X_n]/(X_1, ..., X_n)^{k+1}$$

   *is a Weil algebra of order $k$.*

4. *If $M$ is a smooth manifold of dimension $n$, the space, $J_x^k(M, \mathbb{R})$, of jets at $x \in M$ of order $k$, is a Weil algebra of dimension $\complement_{n+k}^k$ and of order $k$.*

5. *If $A$ is a Weil algebra with maximal ideal $\mathfrak{m}_A$ and of order $h$ and $B$ a Weil algebra with maximal ideal $\mathfrak{m}_B$ and of order $l$, then the tensor product $A \otimes B$ is a Weil algebra of maximal ideal $\mathfrak{m}_{A\otimes B} = \mathfrak{m}_A \otimes B + A \otimes \mathfrak{m}_B$ and of order $h + l$. Thus, $\mathbb{D} \otimes \mathbb{D} = \mathbb{R}[T_1, T_2]/(T_1^2, T_2^2)$ is a Weil algebra of order* 2*.*

**Remark 1** *The tensor product of two algebras of truncated polynomials is not a truncated polynomials algebra. This is the case for $\mathbb{D} \otimes \mathbb{D}$.*

If $M$ is a smooth manifold, $C^\infty(M)$ the algebra of real functions on $M$ and $A$ a Weil algebra with maximal ideal $\mathfrak{m}$, a near point of $x \in M$ of kind $A$ is a homomorphism of algebras

$$\xi : C^\infty(M) \longrightarrow A$$

such that $[\xi(f) - f(x)] \in \mathfrak{m}$ for every $f \in C^\infty(M)$[10].

Let $M_x^A$ denote the set of near points of $x$ of kind $A$ and

$$M^A = \bigcup_{x \in M} M_x^A.$$

The set $M^A$ is a smooth manifold of dimension $\dim M \times \dim A$: its the manifold of near points of $M$ of kind $A$ or simply the Weil bundle over $M$ of kind $A$.

**Example 2** *: For any smooth manifold $M$, $M^{\mathbb{R}} = M$.*

1. *For any smooth manifold $M$, the map*
$$TM \longrightarrow Hom_{A\lg}(C^\infty(M),\mathbb{D}), v \longmapsto \xi_v,$$
*defined by*
$$\xi_v(f) = f(p) + v(f)\cdot\varepsilon$$
*if $v \in T_pM$, identifies $TM = J_0^1(\mathbb{R}, M)$ at $M^{\mathbb{D}}$. We verify that $v$ is a tangent vector at $p \in M$ if only if $\xi_v$ is a near point of $p \in M$ of kind $\mathbb{D}$.*

2. *If $A = \mathbb{R}[X]/(X^3)$, $M^A = J_0^2(\mathbb{R}, M)$. More generally, if $A$ is the algebra of truncated polynomials*
$$\mathbb{R}[X_1, ..., X_s]/(X_1, ..., X_s)^{k+1},$$
*then $M^A = J_0^k(\mathbb{R}^s, M)$ is the set of jets at $0$ of order $k$ of differentiable applications from $\mathbb{R}^s$ in $M$.*

3. *The application $\xi \longmapsto \xi(id_{\mathbb{R}})$ identifies $\mathbb{R}^A$ to $A$.*

4. *If $V$ is a real vector space of finite dimension, if $(e_i)_{i=1,\dots,r}$ is a basis of $V$ and if $(e_i^*)_{i=1,\dots,r}$ is a dual basis of the basis $(e_i)_{i=1,\dots,r}$, then*
$$V^A \longrightarrow V \otimes A, \xi \longmapsto \sum_{i=1}^{r} e_i \otimes \xi(e_i^*)$$
*is a canonical isomorphism of $A$-modules.*

When $M$ and $N$ are two manifolds, and when
$$h : M \longrightarrow N$$
is a differentiable application of class $C^\infty$, then the application
$$h^A : M^A \longrightarrow N^A, \xi \longmapsto h^A(\xi),$$
such that for all $g \in C^\infty(N)$,
$$\left[h^A(\xi)\right](g) = \xi(g \circ h),$$
is differentiable of class $C^\infty$. When $h$ is a diffeomorphism, it is even $h^A$.

Furthermore, if $\varphi : A \longrightarrow B$ is a homomorphism of Weil algebras, for any smooth manifold $M$, the application
$$\varphi_M : M^A \longrightarrow M^B, \xi \longmapsto \varphi \circ \xi$$
is differentiable. In particular, the augmention
$$A \longrightarrow \mathbb{R}$$
defined for any smooth manifold $M$, the projection
$$M^A \longrightarrow M,$$
which associates to each near point of $x \in M$, its origin $x$.

# 2 Parallelism of Weil bundles

In what follows $M$ is a smooth manifold of dimension $n$, $A$ a Weil algebra with unit $1_A$, $C^\infty(M)$ the algebra of real functions of class $C^\infty$ on $M$, $\mathfrak{X}(M)$ the $C^\infty(M)$-module of vector fields on $M$, $TM$ the tangent bundle of $M$ and

$$\pi_M : TM \longrightarrow M$$

the canonical projection.

If $(U, \varphi)$ is a local chart of $M$ with local coordinates $(x_1, x_2, ..., x_n)$, the application,

$$U^A \longrightarrow A^n, \xi \longmapsto (\xi(x_1), \xi(x_2), ..., \xi(x_n)),$$

is a bijection from $U^A$ to a open of $A^n$. The manifold $M^A$ is modeled on $A^n$, i.e $M^A$ is a $A$-manifold of dimension $n$.

The set, $C^\infty(M^A, A)$ of the functions of class $C^\infty$ on $M^A$ with values in $A$, is a commutative $A$-algebra with unit. By identifying $\mathbb{R}^A$ at $A$, for $f \in C^\infty(M)$, the application

$$f^A : M^A \longrightarrow A, \xi \longmapsto \xi(f),$$

is of class $C^\infty$. Futher the application

$$C^\infty(M) \longrightarrow C^\infty(M^A, A), f \longmapsto f^A,$$

is a injective homomorphism of algebras and we have:

$$(f + g)^A = f^A + g^A; \quad (\lambda \cdot f)^A = \lambda \cdot f^A; \quad (f \cdot g)^A = f^A \cdot g^A$$

with $\lambda \in \mathbb{R}$, $f$ and $g$ belonging to $C^\infty(M)$.

When $(a_\alpha)_{\alpha=1,2,...,\dim(A)}$ is a basis of $A$ and when $(a_\alpha^*)_{\alpha=1,2,...,\dim(A)}$ is a dual basis of the basis $(a_\alpha)_{\alpha=1,2,...,\dim(A)}$, the application

$$\sigma : C^\infty(M^A, A) \longrightarrow A \otimes C^\infty(M^A), \varphi \longmapsto \sum_{\alpha=1}^{\dim(A)} a_\alpha \otimes (a_\alpha^* \circ \varphi),$$

is an isomorphism of $A$-algebras. This isomorphism does not depend on the chosen basis and the application

$$\gamma : C^\infty(M) \longrightarrow A \otimes C^\infty(M^A), f \longmapsto \sigma(f^A),$$

is a morphism of algebras.

We note $\mathfrak{X}(M^A)$, the set of all vector fields on $M^A$. The following assertions are then equivalent [4]:

1. $X : C^\infty(M^A) \longrightarrow C^\infty(M^A)$ is a vector field on $M^A$;
2. $X : C^\infty(M) \longrightarrow C^\infty(M^A, A)$ is a linear application satisfying

$$X(fg) = X(f) \cdot g^A + f^A \cdot X(g)$$

for all $f$ and $g$ in $C^\infty(M)$.

Then, when

$$\theta : C^\infty(M) \longrightarrow C^\infty(M)$$

is a vector field over $M$, the application

$$\theta^A : C^\infty(M) \longrightarrow C^\infty(M^A, A), f \longmapsto [\theta(f)]^A,$$

is a vector field on $M^A$: the vector field $\theta^A$ is the prolongation to $M^A$ of the vector field $\theta$.

When $X$ is a vector field on $M^A$, considered as derivation from $C^\infty(M)$ to $C^\infty(M^A, A)$, then there exists a unique derivation [4],

$$\widetilde{X} : C^\infty(M^A, A) \longrightarrow C^\infty(M^A, A)$$

such that:

1. $\widetilde{X}$ is $A$-linear;
2. $\widetilde{X}\left[C^\infty(M^A)\right] \subset C^\infty(M^A)$;
3. $\widetilde{X}(f^A) = X(f)$ for any $f \in C^\infty(M)$.

The set $\mathfrak{X}(M^A)$ of vector fields on $M^A$ is in these conditions, a $C^\infty(M^A, A)$-module and a Lie algebra over $A$ [4].

**Theorem 1** *[10]. If $M$ is a smooth manifold and if $A$ and $B$ are two Weil algebras, then the application*

$$(M^A)^B \longrightarrow M^{A\otimes B}, \eta \longmapsto (id_A \otimes \eta) \circ \gamma$$

*is a diffeomorphism.*

In particular, we have an isomorphism of manifolds between $TM^A$ and $(TM)^A$.

For $x \in M$, $T_xM$ denotes the tangent space at $x$ to $M$.

We recall that the manifold $M$ is parallelizable if its tangent bundle $TM$ is trivial i.e there is a diffeomorphism

$$\sigma : TM \longrightarrow M \times \mathbb{R}^n$$

such that the following diagramm

$$\begin{array}{ccc} TM & \stackrel{\sigma}{\longrightarrow} & M \times \mathbb{R}^n \\ & \searrow_{\pi_M} & \downarrow pr_1 \\ & & M \end{array}$$

commutes and that for all $x \in M$ the restriction

$$\sigma_{|T_xM} : T_xM \longrightarrow \{x\} \times \mathbb{R}^n$$

is a isomorphism of vector spaces.

When $(U, \varphi)$ is a local chart of $M$ with the local coordinates $(x_1, x_2, ..., x_n)$, the application

$$\psi : TU^A \longrightarrow U^A \ \times A^n, \sum_{i=1}^{n} \lambda_i \cdot \left(\frac{\partial}{\partial x_i}\right)^A |_{\xi} \longmapsto (\xi, \lambda_1, ..., \lambda_n)$$

is a diffeomorphism of $A$-manifolds satisfiying $pr_1 \circ \psi = \pi_{M^A}$.

Thus the local parallelism of $M^A$ expressed in terms of existence of a diffeomorphism of $A$-manifolds whose restriction in each tangent space is an isomorphism of $A$-modules.

The aim of this work is to give the equivalence of parallelism of $M^A$ in terms of $A$-manifolds. We recall that when $M$ is a smooth manifold, the basic algebra of $M$ is $C^\infty(M)$. Since $\mathfrak{X}(M^A)$ is a $C^\infty(M^A, A)$-module, considered as the set of derivations from $C^\infty(M)$ to $C^\infty(M^A, A)$, and is a Lie algebra over $A$, and as $M^A$ is a $A$-manifold, this means that the basic algebra of $M^A$ is $C^\infty(M^A, A)$ and not $C^\infty(M^A)$.

**Proposition 2** *The manifold $M^A$ is parallelizable if only if there is a diffeomorphism of $A$-manifolds*

$$H : TM^A \longrightarrow M^A \times A^n$$

*such that the following diagram*

$$\begin{array}{ccc} TM^A & \stackrel{H}{\longrightarrow} & M^A \times A^n \\ & \searrow_{\pi_{M^A}} & \downarrow pr_1 \\ & & M^A \end{array}$$

*commutes and that for every $\xi \in M^A$, the restriction*

$$H_{|T_\xi M^A} : T_\xi M^A \longrightarrow \{\xi\} \times A^n$$

*is an isomorphism of $A$-modules.*

**Proof.** / $\Longrightarrow$ As the manifold is parallelizable, there exists a diffeomorphism

$$TM^A \quad \xrightarrow{\sigma} \quad M^A \times \mathbb{R}^{n \cdot \dim A}$$

such that

$$\begin{array}{ccc} TM^A & \xrightarrow{\sigma} & M^A \times \mathbb{R}^{n\cdot\dim A} \\ & \searrow_{\pi_{M^A}} & \downarrow pr_1 \\ & & M^A \end{array}$$

commutes i.e $pr_1 \circ \sigma = \pi_{M^A}$, and that for every $\xi \in M^A$, the restriction

$$\sigma_{|T_\xi M^A} : T_\xi M^A \longrightarrow \{\xi\} \times \mathbb{R}^{n \cdot \dim A}$$

is an isomorphism of vector spaces.

Let

$$h : A^n \longrightarrow \mathbb{R}^{n \cdot \dim A}$$

be an isomorphism of vector espaces. By transport of structure, we equip $\mathbb{R}^{n \cdot \dim A}$ of a structure of $A$-module defined on $A^n$. Thus $h$ becomes an isomorphism of $A$-modules. In the same way,

$$\sigma_{|T_\xi M^A} : T_\xi M^A \longrightarrow \{\xi\} \times \mathbb{R}^{n \cdot \dim A}$$

becomes an isomorphism of $A$-modules. Asking $H = (id_{M^A} \times h^{-1}) \circ \sigma$. For every $\xi \in M^A$, we deduce that the restriction

$$H|_{T_\xi M^A} : T_\xi M^A \longrightarrow \{\xi\} \times A^n$$

is an isomorphism of $A$-modules.

Since $\sigma$ is differentiable on a open of $\mathbb{R}^{2n \cdot \dim A}$, it is the same for $H$: thus the differentiability of $H$ carried on open of $A^n$.

$\Longleftarrow$ / The sufficient condition is oubvious. ■

The description of the parallelism of $M^A$ is the following:

**Theorem 3** *If $M$ is a smooth manifold of dimension $n$ and if $M^A$ is the Weil bundle of $M$ of kind $A$, then the following assertions are equivalent:*

1. *The manifold $M^A$ is parallelizable;*

2. *There are $n$ vector fields $X_1, ..., X_n$ on $M^A$ such that for every $\xi \in M^A$, the vectors $X_1(\xi), ..., X_n(\xi)$ provide a basis of $T_\xi M^A$;*

3. *The $C^\infty(M^A, A)$-module, $\mathfrak{X}(M^A)$, of vector fields on $M^A$ is a free $C^\infty(M^A, A)$-module of rank $n$.*

**Proof.** Show $1/ \Longleftrightarrow 2/$

$1/ \Longrightarrow 2/$ As the manifold $M^A$ est parallelizable, then there exists a difféomorphism of $A$-manifolds

$$H : TM^A \longrightarrow M^A \times A^n$$

such that the following diagram

$$\begin{array}{ccc} TM^A & \xrightarrow{H} & M^A \times A^n \\ & \searrow_{\pi_{M^A}} & \downarrow pr_1 \\ & & M^A \end{array}$$

commutes and for every $\xi \in M^A$, the restriction

$$H|_{T_\xi M^A} : T_\xi M^A \longrightarrow \{\xi\} \times A^n$$

is an isomorphism of $A$-modules.

For all $i = 1, 2, ..., n$, let $a_i = (0, 0, ..., 1_A, 0, ..., 0)$ where $1_A$ is to $i$-th place. Obviously $(a_1, a_2, ..., a_n)$ is a basis of the $A$-module $A^n$. For all $i = 1, 2, ..., n$, the applications

$$\sigma_i : M^A \longrightarrow M^A \times A^n, \xi \longmapsto (\xi, a_i)$$

and

$$X_i = H^{-1} \circ \sigma_i : M^A \longrightarrow TM^A$$

are diferentiables. Moreover

$$X_i : M^A \longrightarrow TM^A$$

is a section of the tangent bundle, since for $\xi \in M^A$ we have

$$\begin{aligned} (\pi_{M^A} \circ X_i)(\xi) &= (pr_1 \circ H)\left[(H^{-1} \circ \sigma_i)(\xi)\right] \\ &= (pr_1 \circ H)\left[H^{-1}(\xi, a_i)\right] \\ &= pr_1(\xi, a_i) \\ &= \xi. \end{aligned}$$

Thus

$$\pi_{M^A} \circ X_i = id_{M^A}.$$

We conclude that $X_i$ is a vector field on $M^A$.

For any $\xi \in M^A$, as $(\xi, a_i)_{i=1,2,...,n}$ is a basis of the $A$-module $\{\xi\} \times A^n$, then $[H^{-1}(\xi, a_i)]_{i=1,2,...,n}$ is a basis of the $A$-module $T_\xi M^A$. We conclude that the vectors $X_1(\xi), ..., X_n(\xi)$ form a basis of the $A$-module $T_\xi M^A$.

2/ $\Longrightarrow$ 1/ Assume that there are $n$ vector fiels $X_1, ..., X_n$ on $M^A$ such that for $\xi \in M^A$, $(X_1(\xi), ..., X_n(\xi))$ is a basis of $T_\xi M^A$.

The application

$$\varphi : M^A \ \times A^n \ \longrightarrow \ TM^A, (\xi, \lambda_1, ..., \lambda_n) \longmapsto \sum_{i=1}^{n} \lambda_i \cdot X_i(\xi)$$

is an diffeomorphism of $A$-manifolds and

$$\varphi_{|_{\{\xi\} \times A^n}} : \{\xi\} \times A^n \ \longrightarrow \ T_\xi M^A, (\xi, \lambda_1, ..., \lambda_n) \longmapsto \sum_{i=1}^{n} \lambda_i X_i(\xi)$$

is an isomorphism of $A$-modules and its reciprocal

$$\varphi^{-1} : \ TM^A \longrightarrow M^A \ \times A^n, \sum_{i=1}^{n} \lambda_i X_i(\xi) \longmapsto (\xi, \lambda_1, ..., \lambda_n)$$

is such that

$$pr_1 \circ \ \varphi^{-1} = pr_1 \circ H = \pi_{M^A}.$$

Then concludes that the manifold $M^A$ is parallelizable.

Show 2/ $\Longleftrightarrow$ 3/

2/ $\Longrightarrow$ 3/ Assume that there are $n$ vector fields $X_1, ..., X_n$ on $M^A$ such that for $\xi \in M^A$, $(X_1(\xi), ..., X_n(\xi))$ is a basis of $T_\xi M^A$.

The vector fields $X_1, ..., X_n$ are linearly independent. Indeed, if, $g_1, ..., g_n \in C^\infty(M^A, A)$ are such that

$$\sum_{i=1}^{n} g_i \cdot X_i = 0,$$

then for all $\xi \in M^A$, we have

$$\sum_{i=1}^{n} g_i(\xi) \cdot X_i(\xi) = 0.$$

As $(X_1(\xi), ..., X_n(\xi))$ is a basis of $T_\xi M^A$, then $g_i(\xi) = 0$ for all $i = 1, 2, ..., n$. As $\xi$ is arbitrary, we concludes that $g_i = 0$ for all $i = 1, 2, ..., n$.

The familly $X_1, ..., X_n$ generates $\mathfrak{X}(M^A)$, in fact, if $Y \in \mathfrak{X}(M^A)$ and $\xi \in M^A$, we have:

$$Y(\xi) = \sum_{i=1}^{n} \lambda_i \cdot X_i(\xi)$$

with the $\lambda_i \in A$.

The application

$$M^A \xrightarrow{Y} TM^A \xrightarrow{H} M^A \times A^n \xrightarrow{pr_2} A^n \xrightarrow{pr_i} A, \xi \longmapsto \lambda_i,$$

is differentiable. Asking $f_i = pr_i \circ pr_2 \circ H \circ Y$, we have $f_i(\xi) = \lambda_i$ and

$$\begin{aligned} Y(\xi) &= \sum_{i=1}^{n} f_i(\xi) \cdot X_i(\xi) \\ &= \left( \sum_{i=1}^{n} f_i \cdot X_i \right)(\xi). \end{aligned}$$

As $\xi$ is arbitrary, then

$$Y = \sum_{i=1}^{n} f_i \cdot X_i.$$

Thus $X_1, ..., X_n$ is a basis of the $C^\infty(M^A, A)$-module $\mathfrak{X}(M^A)$. We concludes that $\mathfrak{X}(M^A)$ is a free $C^\infty(M^A, A)$-module of rank $n$.

3/ $\Longrightarrow$ 2/ Assume that $\mathfrak{X}(M^A)$ is a free $C^\infty(M^A, A)$-module of rank $n$. Let $(X_1, ..., X_n)$ a basis of the $C^\infty(M^A, A)$-module $\mathfrak{X}(M^A)$.

If $\alpha_1(\xi), ..., \alpha_n(\xi)$ are elements of $A$ such that

$$\sum_{i=1}^{n} \alpha_i(\xi) \cdot X_i(\xi) = 0$$

for all $\xi \in M^A$, and for all $i = 1, 2, ..., n$, let

$$f_i : M^A \longrightarrow A, \xi \longmapsto \alpha_i(\xi).$$

For $\eta \in M^A$, there exists $Y \in \mathfrak{X}(M^A)$ such that $Y(\eta) = \sum_{i=1}^{n} f_i(\eta) \cdot X_i(\eta)$. As $Y$ is differentiable in a neighborhood of $\eta$, it is even $f_i$ in a neighborhood of $\eta$.

Since $\eta$ is arbitrary, we deduce that the $f_i$ are differentiable. Thus, we have

$$\begin{aligned} 0 &= \sum_{i=1}^{n} \alpha_i(\xi) \cdot X_i(\xi) \\ &= \sum_{i=1}^{n} f_i(\xi) \cdot X_i(\xi) \\ &= \left( \sum_{i=1}^{n} f_i \cdot X_i \right)(\xi) \end{aligned}$$

four any $\xi \in M^A$. As the vector fields $X_1, ..., X_n$ form a basis of the $C^\infty(M^A, A)$-module $\mathfrak{X}(M^A)$, then

$$f_1 = ... = f_n = 0$$

i.e that for all $\xi \in M^A$, the $\alpha_i(\xi) = 0$. It is concluded that the family $(X_1(\xi), ..., X_n(\xi))$ is free for all $\xi \in M^A$.

Moreover, for $v \in T_\xi M^A$, there exists a vector field $Y \in \mathfrak{X}(M^A)$ such that $Y(\xi) = v$. Since

$$Y = \sum_{i=1}^{n} f_i \cdot X_i$$

where each $f_i \in C^\infty(M^A, A)$, then,

$$v = \sum_{i=1}^{n} f_i(\xi) \cdot X_i(\xi).$$

Thus, the family $(X_1(\xi), ..., X_n(\xi))$ generates the $A$-module $T_\xi M^A$.

Then concludes that at every $\xi \in M^A$, the vectors $X_1(\xi), ..., X_n(\xi)$ form a basis of the $A$-module $T_\xi M^A$. ■

**Corollary 4** *If $M$ is a parallelizable manifold, then the manifold $M^A$ is parallelizable.*

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