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Traveling-Wave Solutions to the Nonlinear Double Degenerate Parabolic Equation of Turbulent Filtration with Absorption

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Abstract

In this paper we use nonlinear scaling methods to prove the existence of finite traveling-wave type solutions to the nonlinear double degenerate parabolic equation of turbulent filtration with absorption and slow diffusion.

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1 Introduction

We consider the nonlinear double degenerate parabolic equation of turbulent filtration with absorption

$$u_t = \left(|(u^m)_x|^{p-1}(u^m)_x\right)_x - bu^\beta,\ x \in \mathbb{R},\ t > 0, \tag{1}$$

with $mp > 1$, $(m, p > 0)$, $0 < \beta < 1$, and $b > 0$. The condition that $mp > 1$ implies that the solutions of equation (1) travel with a finite speed of propagation (slow diffusion case). Equation (1) naturally arises when analyzing the turbulent filtration of a gas through a porous media (see [9]).

In this work we are interested in finding finite traveling-wave solutions to equation (1), that is: $u(x,t) = \varphi(kt - x)$. The function φ is such that: $\varphi(z) \geq 0, \varphi \not\equiv 0$, $\varphi(z) \to 0^+$ as $z \to -\infty$, and $\varphi(0) = 0$.

By substituting $\varphi(kt-x)$ for $u(x,t)$ in equation (1), we have that equation (1) admits a finite traveling-wave solution, if there exists $\varphi \in \mathbb{R}^+$ that satisfies the following initial-value-problem (IVP)

$$\begin{cases} \left(|(\varphi^m)'|^{p-1}(\varphi^m)'\right)' - k\varphi' - b\varphi^\beta = 0, \\ \varphi(0) = (\varphi^m)'(0) = 0, \end{cases} \tag{2}$$

where $\varphi(z) \equiv 0$ for all $z < 0$. All derivatives are understood in the weak sense.

It is of note that currently there is a well-established general theory of nonlinear degenerate parabolic equations (see [1, 2, 4, 5, 10]). Boundary value problems for equation (1) been have been investigated in [11, 15, 13].

Consider the Cauchy problem (CP) for equation (1) with initial function

$$u(x,0) = u_0(x) \sim C(-x)_+^\alpha, \text{ as } x \to 0^-; C, \alpha > 0. \tag{3}$$

The solution of the CP (1), (3) is understood in the weak sense (see Definition 1 from [6]).

The full classification of the interface function: $\eta(t) := \sup\{x : u(x,t) > 0\}$, $\eta(0) = 0$; and local solutions near the interface function for the CP (1), (3) is established in [6] in the slow diffusion case ($mp > 1$). This classification is done for the nonlinear reaction-diffusion equation (equation (1) with $p = 1$) in [8] for the slow diffusion case ($m > 1$) and in [3] for the fast diffusion case ($0 < m < 1$); and for the parabolic p-Laplacian type reaction-diffusion equation (equation (1) with $m = 1$) in [7] for the slow diffusion case ($p > 1$).

For the slow diffusion cases specifically, the use of finite traveling-wave solutions was essential to prove asymptotic results for the interface function and the local solution near the interface function for the CP (1), (3) in the case where diffusion and reaction forces are in balance (see Lemma 5 and Lemma 6 (proof) from [6]).

The existence of traveling-wave solutions with interfaces for the nonlinear reaction-diffusion equation is pursued in [12]. The existence of traveling-wave type solutions for the parabolic p-Laplacian equation with absorption is considered in [14].

2 The Main Result

The following is the main result of this paper.

Theorem 1. *There exists a finite traveling-wave solution to equation* (1)*:* $u(x,t) = \varphi(kt-x)$*, with* $\varphi(0) = 0$ *if* $k \neq 0$*. Further, we have the following limit formulas for* φ

$$(1)\ \lim_{z\to 0^+} z^{-\frac{1+p}{mp-\beta}}\varphi(z) = \left[\frac{b(mp-\beta)^{1+p}}{(m(1+p))^p p(m+\beta)}\right]^{\frac{1}{mp-\beta}} := C_*, \qquad \text{if } p(m+\beta) < 1+p,$$

$$(2)\ \lim_{z\to +\infty} z^{-\frac{1+p}{mp-\beta}}\varphi(z) = C_*, \qquad \text{if } p(m+\beta) > 1+p,$$

$$(3)\ \lim_{z\to +\infty} z^{-\frac{p}{mp-1}}\varphi(z) = \left(\frac{mp-1}{mp}\right)^{\frac{p}{mp-1}} k^{\frac{1}{mp-1}}, \qquad \text{if } k > 0,\ p(m+\beta) < 1+p,$$

(4) $\lim_{z\to 0^+} z^{-\frac{p}{mp-1}}\varphi(z) = \left(\frac{mp-1}{mp}\right)^{\frac{p}{mp-1}} k^{\frac{1}{mp-1}}$, if $k>0$, $p(m+\beta) > 1+p$,

(5) $\lim_{z\to +\infty} z^{-\frac{1}{1-\beta}}\varphi(z) = \left((1-\beta)\left(-\frac{b}{k}\right)\right)^{\frac{1}{1-\beta}}$, if $k<0$, $p(m+\beta) < 1+p$,

(6) $\lim_{z\to 0^+} z^{-\frac{1}{1-\beta}}\varphi(z) = \left((1-\beta)\left(-\frac{b}{k}\right)\right)^{\frac{1}{1-\beta}}$, if $k<0$, $p(m+\beta) > 1+p$.

Our ultimate aim is to prove Theorem 1 – establishing the existence of finite-traveling wave solutions to equation 1. In the next section we will establish some necessary preliminary results to that end.

3 Preliminary Results

In order to prove Theorem 1, some preliminaries must first be established. As done for the analogous problem for the p-Laplacian reaction-diffusion equation in [14], it can be shown that if $\varphi(z) > 0$ exists, then $\varphi(z)$ is increasing for all $z > 0$.

We want to show that there exists such a $\varphi(z) > 0$. We introduce the following change of variable

$$\Theta = \varphi \text{ and } \Upsilon = ((\varphi^m)')^p,$$

it follows that

$$\Theta' = \frac{1}{m}\Theta^{1-m}\Upsilon^{\frac{1}{p}} \text{ and } \Upsilon' = b\Theta^{\beta} + \frac{k}{m}\Theta^{1-m}\Upsilon^{\frac{1}{p}},$$

where (Θ,Υ) starts from (0,0) at $z=0$, exists for any $z>0$, and are contained in the first quadrant. Consider the IVP

$$\begin{cases} \frac{d\Upsilon}{d\Theta} = f(\Theta,\Upsilon) = k + bm\Theta^{m+\beta-1}\Upsilon^{-\frac{1}{p}}, \\ \Upsilon(0) = 0. \end{cases} \tag{4}$$

As done for the analogous IVP for the p-Laplacian reaction-diffusion equation in [14], it can be shown that problem (4) has a unique solution, $\Upsilon(\Theta)$. The proof follows in the same way, with a slight modification required when $k<0$ and $p(m+\beta) < 1+p$: in this case, the borderline curve is decreasing, and by using a similar argument used to prove the other cases, leads to a contradiction if there is no global solution.

Let $\Upsilon = ((\varphi^m)')^p$ be a solution of the problem (4). There exists a unique maximal solution defined on $(-\infty,\varrho)$ to the problem

$$\frac{d\varphi}{dz} = \frac{1}{m}(\varphi(z))^{1-m}\Upsilon^{\frac{1}{p}}(\varphi(z)),\ \varphi(0) = 0, \tag{5}$$

where

$$\lim_{z\to\varrho^-}\varphi(z) = +\infty.$$

By (5) we have that $(\varphi^m)'(0) = \Upsilon^{\frac{1}{p}}(0) = 0$: φ can be continued by zero on $(-\infty,0)$. Moreover, φ is strictly increasing, and the above limit holds if ϱ is finite. By (5) and the boundedness of $\Upsilon^{-\frac{1}{p}}$, the above limit also holds if $\varrho = +\infty$.

The solution of equation (5), $\varphi = \varphi(z)$, defined on $(-\infty, \varrho)$ satisfies the following IVP

$$\begin{cases} (|(\varphi^m)'|^{p-1}(\varphi^m)')' - k\varphi' - b\varphi^\beta = 0, \, z \in (-\infty, \varrho), \\ \varphi(0) = (\varphi^m)'(0) = 0. \end{cases} \tag{6}$$

The solution to IVP (6) is global. To prove it, we establish the following lemma.

Lemma 1. *Let Υ be a solution of IVP* (4)*, then we have the following asymptotic formulas*

(1) $\Upsilon(\Theta) \sim \left[\frac{bm(1+p)}{p(m+\beta)}\right]^{\frac{p}{1+p}} \Theta^{\frac{p(m+\beta)}{1+p}}$, as $\Theta \to 0^+$, if $p(m+\beta) < 1+p$,

(2) $\Upsilon(\Theta) \sim \left[\frac{bm(1+p)}{p(m+\beta)}\right]^{\frac{p}{1+p}} \Theta^{\frac{p(m+\beta)}{1+p}}$, as $\Theta \to +\infty$, if $p(m+\beta) > 1+p$,

(3) $\Upsilon(\Theta) \sim k\Theta$, as $\Theta \to +\infty$, if $k > 0$, $p(m+\beta) < 1+p$,

(4) $\Upsilon(\Theta) \sim k\Theta$, as $\Theta \to 0^+$, if $k > 0$, $p(m+\beta) > 1+p$,

(5) $\Upsilon(\Theta) \sim (-\frac{k}{bm})^{-p}\Theta^{p(m+\beta-1)}$, as $\Theta \to +\infty$, if $k < 0$, $p(m+\beta) < 1+p$,

(6) $\Upsilon(\Theta) \sim (-\frac{k}{bm})^{-p}\Theta^{p(m+\beta-1)}$, as $\Theta \to 0^+$, if $k < 0$, $p(m+\beta) > 1+p$.

Proof of Lemma 1. We will prove formula (5). The proof of formulas (1)-(4) and (6) follow from similar arguments.

We apply nonlinear scaling as follows: we choose $\Upsilon_\ell(\Theta) = \ell\Upsilon(\ell^\gamma\Theta)$, with $\ell > 0$ and γ to be determined. It follows that we have

$$\Upsilon_\ell(\Theta) = \ell\Upsilon(\ell^\gamma\Theta) \iff \Upsilon(\Theta) = \ell^{-1}\Upsilon_\ell(\ell^{-\gamma}\Theta).$$

We set $Z = \ell^\gamma\Theta$. It follows from IVP (4) that

$$\begin{aligned} \frac{d\Upsilon_\ell}{d\Theta} &= \ell^{1+\gamma}\frac{d\Upsilon}{dZ} = \ell^{1+\gamma}\Big(k + bmZ^{m+\beta-1}\Upsilon^{-\frac{1}{p}}\Big) \\ &= k\ell^{1+\gamma} + bm\ell^{1+\gamma}\ell^{\gamma(m+\beta-1)}\ell^{\frac{1}{p}}\Theta^{m+\beta-1}\Upsilon_\ell^{-\frac{1}{p}}. \end{aligned} \tag{7}$$

Now, we choose γ such that

$$1 + \gamma = 1 + \gamma + \gamma(m+\beta-1) + \frac{1}{p} \implies \gamma = -\frac{1}{p(m+\beta-1)}.$$

So, we have that

$$\ell^{\frac{1+p-p(m+\beta)}{p(m+\beta-1)}}\frac{d\Upsilon_\ell}{d\Theta} = k + bm\Theta^{m+\beta-1}\Upsilon_\ell^{-\frac{1}{p}}. \tag{8}$$

From our previous results, we know that there exists a unique solution to equation (8). To prove formula (5), since $p(m+\beta) < 1+p$, we set

$$\lim_{\ell \to 0^+} \Upsilon_\ell(\Theta) = \widetilde{\Upsilon}(\Theta).$$

To show that this limit exists, in this case, it's enough to prove that $\{\Upsilon_\ell\}$ is uniformly bounded on any compact interval, $[\Gamma, \Delta]$. From the equation (4) we have that

$$k + bm\Theta^{m+\beta}\Upsilon_\ell^{-\frac{1}{p}} \geq 0 \implies 0 \leq \Upsilon_\ell(\Theta) \leq \left(-\frac{k}{bm}\right)^{-p} \Theta^{p(m+\beta-1)}, \Theta > 0.$$

It remains to show that $\frac{d\Upsilon_\ell}{d\Theta}$ is uniformly bounded on $[\Gamma, \Delta]$. Consider

$$\ell^{\frac{1+p-p(m+\beta)}{p(m+\beta-1)}} \frac{d\Upsilon_\ell}{d\Theta} = k + bm\Theta^{m+\beta-1}\Upsilon_\ell^{-\frac{1}{p}} \implies \frac{d\Upsilon_\ell}{d\Theta} = \ell^{\frac{p(m+\beta)-(1+p)}{p(m+\beta-1)}}\left(k + bm\Theta^{m+\beta-1}\Upsilon_\ell^{-\frac{1}{p}}\right),$$

$$(\ell+1)^{\frac{1+p-p(m+\beta)}{p(m+\beta-1)}} \frac{d\Upsilon_{\ell+1}}{d\Theta} = k + bm\Theta^{m+\beta-1}\Upsilon_{\ell+1}^{-\frac{1}{p}}.$$

So, it follows that

$$\frac{d\Upsilon_{\ell+1}}{d\Theta} = (\ell+1)^{\frac{p(m+\beta)-(1+p)}{p(m+\beta-1)}}\left(k + bm\Theta^{m+\beta-1}\Upsilon_{\ell+1}^{-\frac{1}{p}}\right) \leq \ell^{\frac{p(m+\beta)-(1+p)}{p(m+\beta-1)}}\left(k + bm\Theta^{m+\beta-1}\Upsilon_{\ell+1}^{-\frac{1}{p}}\right).$$

Define $Z(\Theta) := \Upsilon_{\ell+1}(\Theta) - \Upsilon_l(\Theta)$. By mean value theorem, for all $\theta \in [0, 1]$, we have

$$\frac{dZ}{d\Theta} \leq \ell^{\frac{p(m+\beta)-(1+p)}{p(m+\beta-1)}} bm\Theta^{m+\beta-1}\left(\Upsilon_{\ell+1}^{-\frac{1}{p}} - \Upsilon_\ell^{-\frac{1}{p}}\right) =$$

$$= -\ell^{\frac{p(m+\beta)-(1+p)}{p(m+\beta-1)}} \frac{bm}{p}\Theta^{m+\beta-1}(\Upsilon_\ell + \theta(\Upsilon_{\ell+1} - \Upsilon_\ell))^{-\frac{1+p}{p}} Z.$$

So, we have that

$$\ell^{\frac{1+p-p(m+\beta}{p(m+\beta-1)}} \frac{dZ}{d\Theta} \leq -\frac{bm}{p}\Theta^{m+\beta-1}(\Upsilon_\ell + \theta(\Upsilon_{\ell+1} - \Upsilon_\ell))^{-\frac{1+p}{p}} Z.$$

Since $Z(0) = 0$, it follows from the comparison theorem that $\Upsilon_{\ell+1}(\Theta) \leq \Upsilon_\ell(\Theta)$, $\Theta \in [\Gamma, \Delta]$; see [1] for the general statement of the comparison theorem for degenerate parabolic equations. Hence $\{\Upsilon_\ell\}$ is a monotonically decreasing sequence as $\ell \to 0^+$, and since $\Upsilon_\ell(\Theta) > 0$, for all $\Theta > 0$, there exists $\widetilde{\Upsilon}(\Theta)$ such that

$$\lim_{\ell \to 0^+} \Upsilon_\ell(\Theta) = \widetilde{\Upsilon}(\Theta).$$

Now, for any $\nu \in C_0^\infty(\Gamma, \Delta)$, we appeal to the following integral identity

$$\int_\Gamma^\Delta \ell^{\frac{1+p-p(m+\beta)}{p(m+\beta-1)}} \Upsilon_\ell \nu' + (k + bm\Theta^{m+\beta-1}\Upsilon_\ell^{-\frac{1}{p}})\nu d\Theta = 0.$$

Letting $\ell \to 0^+$ we have

$$\int_\Gamma^\Delta (k + bm\Theta^{m+\beta-1}\widetilde{\Upsilon}^{-\frac{1}{p}})\nu d\Theta = 0.$$

Since ν is arbitrary, we necessarily have that

$$k + bm\Theta^{m+\beta-1}\widetilde{\Upsilon}^{-\frac{1}{p}} = 0.$$

Solving for $\widetilde{\Upsilon}$ we have that

$$\widetilde{\Upsilon} = \widetilde{\Upsilon}(\Theta) = \Big(-\frac{k}{bm}\Big)^{-p}\Theta^{p(m+\beta-1)}. \tag{9}$$

Note that from our choice of scale, $Z = \ell^{\gamma}\Theta$, so we have that: $\Theta = \ell^{-\gamma}Z$. It follows that we have

$$\lim_{\ell\to 0^+}\Upsilon_\ell(\Theta) = \lim_{\ell\to 0^+}\ell\Upsilon(Z) = \widetilde{\Upsilon}(Z) = \Big(-\frac{k}{bm}\Big)^{-p}\ell Z^{p(m+\beta-1)} = \Big(-\frac{k}{bm}\Big)^{-p}\Theta^{p(m+\beta-1)} = \widetilde{\Upsilon}(\Theta).$$

Writing $\widetilde{\Upsilon}(\Theta) = \Upsilon(\Theta)$, it follows that

$$\lim_{\Theta\to+\infty}\frac{\Upsilon(\Theta)}{\Theta^{p(m+\beta-1)}} = \Big(-\frac{k}{bm}\Big)^{-p}.$$

Hence, we have the desired result

$$\Upsilon(\Theta) \sim (-\tfrac{k}{bm})^{-p}\Theta^{p(m+\beta-1)}, \text{ as } \Theta\to+\infty;$$

formula (5) is proved.

As mentioned above, the proof of formulas (1)-(4) and formula (6) follow from similar arguments. Lemma 1 is proved. □

Using the results above, we are now equipped to prove Theorem 1.

4 Proof of the Main Result

Proof of Theorem 1. For $\varphi(z) \neq 0$ ($\Upsilon(\varphi(z)) \neq 0$), we can rewrite equation (5) in the following way

$$m\varphi^{m-1}\Upsilon^{-\frac{1}{p}}(\varphi(z))d\varphi(z) = dz. \tag{10}$$

We will prove formula (2), the proof of formula (1) and formulas (3)-(6) follows in a similar way by choosing the appropriate asymptotic formula for $\Upsilon(\Theta)$ from Lemma 1.

Since $p(m+\beta) > 1+p$, from Lemma 1 we know that

$$\Upsilon(\Theta) \sim \Big[\frac{bm(1+p)}{p(m+\beta)}\Big]^{\frac{p}{1+p}}\Theta^{\frac{p(m+\beta)}{1+p}}, \text{ as } \Theta\to+\infty.$$

By equation (10) we have

$$m\int_0^{\varphi(z)}\Theta^{m-1}\Upsilon^{-\frac{1}{p}}(\Theta)d\Theta = z. \tag{11}$$

Using this fact and the asymptotic estimate above, $\forall\,\varepsilon > 0$, we have that

$$\Big(\tfrac{m(1+p)}{mp-\beta}\Big(\Big[\tfrac{bm(1+p)}{p(m+\beta)}\Big]^{\frac{p}{1+p}}-\varepsilon\Big)^{-\frac{1}{p}}\Big)^{-\frac{1+p}{mp-\beta}} \leq z^{-\frac{1+p}{mp-\beta}}\varphi(z) \leq \Big(\tfrac{m(1+p)}{mp-\beta}\Big(\Big[\tfrac{bm(1+p)}{p(m+\beta)}\Big]^{\frac{p}{1+p}}+\varepsilon\Big)^{-\frac{1}{p}}\Big)^{-\frac{1+p}{mp-\beta}}.$$

Passing $z\to+\infty$, we have

$$\Big(\tfrac{m(1+p)}{mp-\beta}\Big(\Big[\tfrac{bm(1+p)}{p(m+\beta)}\Big]^{\frac{p}{1+p}}-\varepsilon\Big)^{-\frac{1}{p}}\Big)^{-\frac{1+p}{mp-\beta}} \leq \liminf_{z\to+\infty} z^{-\frac{1+p}{mp-\beta}}\varphi(z) \leq$$

$$\leq \limsup_{z\to+\infty} z^{-\frac{1+p}{mp-\beta}}\varphi(z) \leq \Big(\frac{m(1+p)}{mp-\beta}\Big(\Big[\frac{bm(1+p)}{p(m+\beta)}\Big]^{\frac{p}{1+p}}+\varepsilon\Big)^{-\frac{1}{p}}\Big)^{-\frac{1+p}{mp-\beta}}.$$

Now, passing $\varepsilon \to 0^+$, we have our desired result

$$\lim_{z\to+\infty} z^{-\frac{1+p}{mp-\beta}}\varphi(z) = \left(\frac{m(1+p)}{mp-\beta}\left(\left[\frac{bm(1+p)}{p(m+\beta)}\right]^{\frac{p}{1+p}}\right)^{-\frac{1}{p}}\right)^{-\frac{1+p}{mp-\beta}} = C_*;$$

formula (2) is proved.

As mentioned above, the proof of formula (1) and formulas (3)-(6) follow from similar arguments. Theorem 1 is proved. □

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