Journal of Innovative Technology and Education, Vol. 10, 2023, no. 1, 1 - 5
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/jite.2023.1030

Teaching Consideration and Practice on the Connection of Mathematics at Senior High School and College

Zhi-jie Jiang [*], Yong Yang, Xin-hua Zhang, Hong-bin Bai, Zuo-an Li

School of Mathematics and Statistics, Sichuan University of Science and Engineering, Zigong, Sichuan, 643000, China
[*] Corresponding author

Abstract

We have been engaged in the teaching of college mathematical analysis courses. In teaching, we find that there exists some difficulties for students to understand and grasp in learning mathematical analysis. So, we have been thinking about how to help students overcome such difficulties. However, it is not difficult to find that these difficulties are caused to some extent by the disconnection between middle school mathematics teaching and college mathematics teaching. In this paper, we consider how to realize the connection between middle school mathematics teaching and university mathematics contents.

Keywords: Binomial theorem, mathematical analysis, mathematical thought, mathematical method, mathematical technique

1 Introduction

We have been engaged in the teaching of college mathematical analysis courses. In teaching, students generally reflect that mathematical analysis is difficult to understand, difficult to grasp, and difficult to do. Facing with these difficulties, we have been thinking about how to help students overcome such difficulties. In Literature [1], we analyzed how to overcome the difficulties of learning mathematical analysis in the first grade of college, especially freshmen, and proposed solutions and practiced them in classroom teaching. However, it is not difficult to find that these difficulties are caused to some extent by the disconnection between middle school mathematics teaching and college mathematics teaching. In this regard, this paper puts forward the connection between middle school mathematics teaching and university mathematics teaching from three aspects.

2 Practice and teaching activities

2.1 The methods and techniques in middle school mathematics and college mathematics textbooks can be connected. For example, in middle school mathematics, binomial theorem is an important content. In middle school mathematics teaching, the applications of the theorem are focused on the finding binomial expansion, finding the coefficients of binomial expansion, proving inequalities, finding approximations, and finding combination problems. Although the sequence limit is introduced into the teaching of middle school mathematics, the application of binomial theorem in sequence limit is neglected. Therefore, it is difficult for students to master the methods and skills related to binomial theorem in the limit of university mathematics sequence. The application of binomial theorem in limit in university mathematics. For example, one can refer to the following examples.

Example 1. Find the limit of the following number sequences.

（1） $\lim\limits_{n\to\infty}\sqrt[n]{n^3+3^n}$.

Solution: First, we need to prove that there is a positive integer N ， so that when $n>N$, the inequality $n^3\le 3^n$ holds. To do this, we only need to prove that

$$\lim_{n\to\infty}\frac{n^3}{3^n}=0.$$

Since $3^n=(1+2)^n\ge C_n^4 2^4$ ， we have

$$0\le\frac{n^3}{3^n}\le\frac{n^3}{C_n^4 2^4}=\frac{4!}{2^4}\frac{n^3}{n(n-1)(n-2)(n-3)}.$$

Second, from the sign preserving property of the sequence limit，it is obvious that when $n>N$ ， we have $3\le\sqrt[n]{n^3+3^n}\le 3\sqrt[n]{2}$. Then we have

$$\lim_{n\to\infty}\sqrt[n]{n^3+3^n}=3.$$

（2） $\lim\limits_{n\to\infty}\dfrac{n^5}{e^n}$.

Solution： Let $e=1+h$, then $h>0$ and $e^n=(1+h)^n\geq C_n^6h^6$. Hence, we have

$$0\leq\frac{n^5}{e^n}\leq\frac{n^5}{C_n^6h^6}=\frac{6!}{h^6}\frac{n^5}{n(n-1)\cdots(n-5)}.$$

Then $\lim\limits_{n\to\infty}\dfrac{n^5}{e^n}=0$.

Example 2. Prove $\lim\limits_{n\to\infty}n^2q^n=0$， where $|q|<1$.

Proof： Let $|q|=\dfrac{1}{1+h}$， where $h>0$. From this, we have

$$0\leq n^2|q|^n=\frac{n^2}{(1+h)^n}\leq\frac{n^2}{C_n^3}=\frac{3!n^2}{n(n-1)(n-2)},$$

Then $\lim\limits_{n\to\infty}n^2|q|^n=0$. Hence $\lim\limits_{n\to\infty}n^2q^n=0$.

2.2 The content of middle school mathematics and university mathematics can be connected. With the continuous advancement of the new curriculum reform, many college mathematics teaching contents have entered middle school mathematics. For example, special functions in mathematical analysis (such as Dirichlet function and Riemann function), the convexity of functions, the limits of two important functions, monotone bounded theorem, Hosptial's rule, Taylor formula and Maclaurin formula, differential mean value theorem, series theory, etc. These contents are very prominent in the college entrance examination papers. The focus of the college entrance examination is to examine students' innovation and ability to accept new knowledge through these contents. In teaching, we can realize the connection between these contents in middle school and college mathematics. Examples related to this knowledge can be found as follows.

Example 3. Let $\{x_n\}=1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}-\ln n$. Then $\lim_{n\to\infty} x_n$ exists.

Proof: First of all, we have

$$x_{n+1}-x_n=\frac{1}{n+1}-(\ln(n+1)-\ln n)=\frac{1}{n+1}-\ln\frac{n+1}{n}.$$

By the inequality

$$\left(1+\frac{1}{n}\right)^n<e<\left(1+\frac{1}{n}\right)^{n+1},$$

we have

$$\frac{1}{n+1}<\ln\frac{n+1}{n}<\frac{1}{n},$$

that is, $x_{n+1}-x_n<0$. Then we obtain that $\{x_n\}$ is a monotonically decreasing sequence. Since

$$x_n=1+\frac{1}{2}+\cdots+\frac{1}{n}-\ln n>\ln\frac{2}{1}+\ln\frac{3}{2}+\cdots+\ln\frac{n+1}{n}-\ln n=\ln(n+1)-\ln n>0,$$

the sequence $\{x_n\}$ has a lower bound. From the monotone boundedness theorem, it follows that the sequence $\{x_n\}$ is convergent.

Example 4. Find monotone interval of the function $f(x)=1+x-\mathrm{e}^x$, and compare the size of $\left(1+\frac{1}{n}\right)^n$ and e.

Solution: Since $f'(x)=1-e^x$, the monotonic increasing interval is $(-\infty,0]$, and the monotonic decreasing interval is $[0,+\infty)$. So, when $x>0$, we have $\mathrm{e}^x>1+x$, that is, $\mathrm{e}^{\frac{1}{x}}>1+\frac{1}{x}$. Then $\mathrm{e}^{\frac{1}{n}}>1+\frac{1}{n}$. By taking n power at the same time, we have that

$$\mathrm{e}>\left(1+\frac{1}{n}\right)^n.$$

2.3 Important mathematical ideas can be connected. Middle school mathematics has involved important mathematical ideas such as limit thought and integral thought. These mathematical ideas are also important ideas in university mathematical analysis and important learning content. Therefore, these important ideas of middle school mathematics can be connected with university mathematics. The example of these mathematical ideas can be seen as follows.

Example 5. Let $a>b>0$ and $ab=1$. Which does the following inequality hold ?

A. $a+\frac{1}{b}<\frac{b}{2^a}<\log_2(a+b)$ B. $\frac{b}{2^a}<\log_2(a+b)<a+\frac{1}{b}$

C. $\frac{b}{2^a}<a+\frac{1}{b}<\log_2(a+b)$ D. $\log_2(a+b)<a+\frac{1}{b}<\frac{b}{2^a}$

Solution: The limit method can still be used at this time. Let $a\to+\infty$, then $b\to 0$. So, we have $a+\frac{1}{b}\to+\infty$ and $\frac{b}{2^a}\to 0$. From this, A and D can be excluded. The exclusion of C can be used to conclude that"when $a>b>0$ and $ab=1$, then the growth of logarithm $\log_2(a+b)$ is slower than that of $a+\frac{1}{b}$" (Note: The conclusion is used here: when the base number is greater than 1, the logarithm is a slow growth). Our result $\log_2(a+b)<a+\frac{1}{b}$ shows that we should exclude A, C and D. So we choose B.

Acknowledgments. This work was supported by the Teaching Reform Project of Sichuan University of Science and Engineering (JG-2266).

References

[1] Z. J. Jiang, H. B. Bai, L. Guan, Z. A. Li, Teaching reflection and practice of mathematical analysis for freshman of mathematics and applied mathematics, *International Journal of Mathematical Analysis*, **16** (3) (2022) , 141-148. https://doi.org/10.12988/ijma.2022.912430

[2] Department of Mathematics, East China Normal University, 4th Edition of Mathematical Analysis, Beijing, Higher Education Press, 2010.

Received: January 15, 2023; Published: March 1, 2023