International Mathematical Forum, Vol. 16, 2021, no. 1, 29 - 33
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https://doi.org/10.12988/imf.2021.912195

# Tents of Israel Revisited: Audio Privacy

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**Abstract**

In one of the Biblical stories, prophet Balaam blesses the tents of Israel for being good. But what can be so good about the tents? The traditional Rabbinical interpretation is that the placement of the tents provided full privacy. In our previous paper, we considered the consequences of *visual* privacy: from each entrance, one cannot see what is happening at any other entrance. In this paper, we analyze the possible consequences of *audio* privacy: from each tent, you cannot hear what is going on in other tents.

**Mathematics Subject Classification:** 51M04, 91D25, 68P27

**Keywords:** geometry, privacy, tents of Israel

## 1 Tents of Israel: Reminder and Formulation of the Geometric Problem

**Tents of Israel: a Biblical story.** In one of the Biblical stories, prophet Balaam, when asked to curse the Jewish people, instead blesses them: “How good are your tents, o Israel” (Numbers 24:6).

**How this is related to privacy.** A traditional Rabbinical interpretation of what Balaam meant by goodness of tents is described in the Talmud [1]: what was good about the tents was that they guaranteed visual privacy – from each entrance one could not see what was happening at any other entrance. In our previous paper [3], we explained what geometric arrangements can guarantee this privacy.

**A natural question.** A natural question is: what about *audio* privacy? How would the tents be placed so as from one tent, one cannot hear what is happening at other tents?

## 2 Formulating the Problem in Exact Terms

**How to describe audio privacy in precise terms.** To make sure that no one can overhear what is happening in other tents, we need to make sure that these tents are sufficiently separated, i.e., that the distance between every two tents is larger than or equal to a certain threshold value $D$.

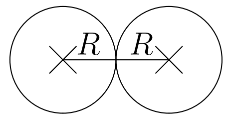
**A natural additional assumption.** In the old days, camps needed to be protected against possible enemies. Such enemies did exist: after all, Balaam was originally sent by the enemies, to curse the Jewish people and thus, to help their enemies defeat them.

The smaller the perimeter, the easier it is to protect the camp. From this viewpoint, it is reasonable to require that the tents locations were selected in such a way that – within the audio privacy constraint – the whole camp would occupy as small an area as possible. Equivalently, we want the density – the number of tents per unit area – to be as large as possible.

**Resulting formulation of the problem.** If we have a large number of tents, how can we place them so that this placement has the largest possible tent density – under the condition that the distance between every two tents would be at least $D$?

## 3 Analysis of the Problem and the Resulting Solution

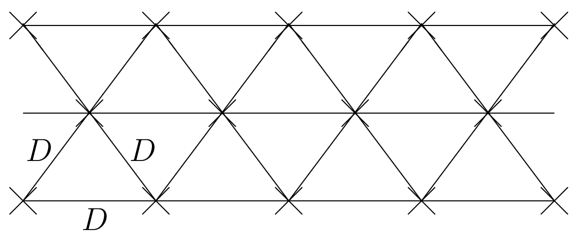
**Analysis of the problem.** One can easily see that the requirement that the distance between every two tents be larger than or equal to $D$ is equivalent to requiring that circles of radius $R = D/2$ with centers at these tends do not overlap:




Thus, the tents-related problem takes the following form: what is the densest arrangement of circles of equal size on a plane so that no overlapping occurs? The desire for maximal compactness means also that no circle can be enlarged without creating an overlap.

**Resulting solution.** The above problem is known as the *circle packing* problem. The solution to this problem is well known: in this solution, centers of the circles (i.e., in our case, tent locations) are arranged in staggered rows.

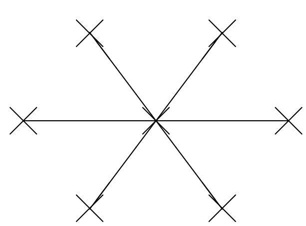
In each row, the next tent is at distance $D$ from the previous one. The next row is shifted by $R = D/2$ in comparison to the previous one – and the distance from each tent to the nearest tents in the next row is also equal to $D$:



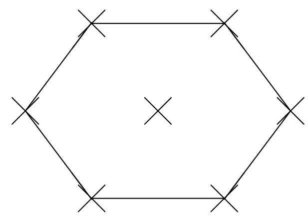

**A brief history.** This arrangement was first formally described in 1773 by Joseph Louis Lagrange, who proved that this is the most dense arrangement among all lattice-shaped arrangements. This result was extended in 1831 by Karl Friedrich Gauss to all possible periodic arrangements. The main ideas of the general-case proof were provided in 1890 by Alex Thue, but the final optimality proof appeared only in 1940 [2], in a paper by László Fejes Tóth.

# 4 Interesting Geometric Consequence: Star of David Naturally Appears

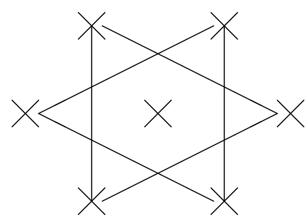
Suppose something good happened in one of the tents. The family living in this tent would then inform their neighbors:

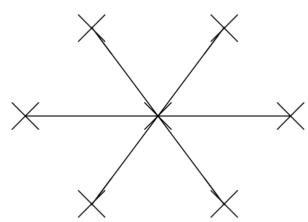


Reasonably, neighbors would exchange opinions about this event, first with their direct neighbors among those who were informed:



and then with next-to-next and next-to-next-to-next neighbors:





Interesting, what we see is the Biblical shape of the Star of David; maybe this is also what Balaam saw?

**Acknowledgments.** This work was supported in part by the National Science Foundation grants 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science), and HRD-1834620 and HRD-2034030 (CAHSI Includes). It was also supported by the program of the development of the Scientific-Educational Mathematical Center of Volga Federal District No. 075-02-2020-1478.

The authors are thankful to Professor Larry Lesser from the University of Texas at El Paso for formulating the problem, and to Rabbi Ben Zeidman from Temple Mount Sinai (El Paso, Texas), and Professor Jason Kalman from The Hebrew Union College–Jewish Institute of Religion for valuable advice.

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**Received: January 15, 2021; Published: February 6, 2021**