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# On KS-Semigroups $[0,1]$ and $F_Y^X$

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**Abstract**

In this paper, we define the operations " $*$ " and " $\cdot$ " on the set $[0,1]$ and show that $[0,1]$ is a KS-semigroup. Also, we define the operations " $\otimes$ " and " $\odot$ " on $F_Y^X$, where $Y$ is any KS-semigroup and show that and $F_Y^X$ is also KS-semigroup. We also investigate the structure of these KS-semigroups including the KS-semigroup $F_{[0,1]}^X$.

**Keywords:** BCK-algebra, semigroup, KS-semigroup, ideal of a KS-semigroup, stable KS-semigroup

## 1 Introduction

A class of algebra known as the BCK-algebra was introduced by Y. Imai and K. Iseki in [2]. Since then, a great deal of studies on BCK-algebra has been produced and one of it was that of K.H. Kim in [3] where KS-semigroup was introduced. In this paper, we introduce operations on the sets $[0,1]$ and $F_Y^X$, where $Y$ is any KS-semigroup, and with these operations, the sets $[0,1]$ and $F_Y^X$ are made into KS-semigroups. We also investigate some structure of these KS-semigroups.

# 2 Preliminary

We now review some definitions and results that will be used in this paper.

**Definition 2.1** [2] Let $X$ be a nonempty set, " $*$ " a binary operation in $X$ and $0 \in X$. An algebraic system $(X, *, 0)$ is called a *BCK-algebra* if " $*$ " satisfies the following conditions: For all $x, y, z \in X$,

(1) $((x * y) * (x * z)) * (z * y) = 0$,

(2) $(x * (x * y)) * y = 0$,

(3) $x * x = 0$,

(4) $0 * x = 0$,

(5) $x * y = 0$ and $y * x = 0$ implies $x = y$.

**Definition 2.2** [3] A BCK-algebra $X$ is said to be *commutative* if for all $x, y \in X$, $x * (x * y) = y * (y * x)$.

**Definition 2.3** [3] Let $X$ be a nonempty set. The system $(X, \cdot)$ is called a *semigroup* if " $\cdot$ " is an associative binary operation.

**Definition 2.4** [3] An algebraic system $(X, *, \cdot, 0)$ is called a *KS-semigroup* if it satisfies the following conditions:

(1) $(X, *, 0)$ is a BCK algebra,

(2) $(X, \cdot)$ is a semigroup,

(3) The operation $\cdot$ is distributive (on both sides) over $*$, that is, for all $x, y, z \in X$

  (a) $x \cdot (y * z) = (x \cdot y) * (x \cdot z)$

  (b) $(x * y) \cdot z = (x \cdot z) * (y \cdot z)$

For convenience, we write $x \cdot y$ by $xy$.

**Definition 2.5** [3] A nonempty subset $Y$ of a KS-semigroup $X$ with binary operations " $*$ " and " $\cdot$ " is called a *sub KS-semigroup* if $Y$ is closed under " $*$ " and " $\cdot$ ", that is,

(1) $x * y \in Y$ for all $x, y \in Y$, and

(2) $xy \in Y$ for all $x, y \in Y$.

**Definition 2.6** [3] A nonempty subset $Y$ of a KS-semigroup $X$ is said to be a $left$(respectively $right$) *ideal* of $X$ if

1. $xy \in Y$ (respectively $yx \in Y$) whenever $x \in X$ and $y \in Y$; and
2. $x * y \in Y$ and $y \in Y$ imply that $x \in Y$ for all $x, y \in X$.

If $Y$ is both a left and a right ideal, then $Y$ is called a *two-sided ideal* or simply an *ideal*.

**Remark 2.7** An ideal is a sub KS-semigroup and if $Y$ is an ideal, then $0 \in Y$.

**Definition 2.8** An ideal $Y \neq X$ of a KS-semigroup $X$ is said to be a *maximal ideal* if for every ideal $Z$ of $X$ with $Y \subseteq Z \subseteq X$, then either $Y = Z$ or $Z = X$.

**Definition 2.9** [3] Let $X$ and $Y$ be KS-semigroups and $f : X \longrightarrow Y$ be a function. Then $f$ is a *KS-semigroup homomorphism* or simply a *homomorphism* if $f(xy) = f(x)f(y)$ and $f(x * y) = f(x) * f(y)$ for all $x, y \in X$.

**Definition 2.10** [3] Let $X$ and $Y$ be KS-semigroups and $f : X \longrightarrow Y$ be a homomorphism. Then $f$ is an *epimorphism* if $f$ is onto. If, in addition, $f$ is one-to-one, $f$ is called an *isomorphism*.

**Definition 2.11** [3] Let $f : X \longrightarrow Y$ be a KS-semigroup homomorphism. The *kernel* of $f$ is the set *ker* $f = \{x \in X : f(x) = 0\}$.

**Theorem 2.12** [3] *Let* $f : X \longrightarrow Y$ *be a KS-semigroup homomorphism. If* $B$ *is an ideal of* $Y$, *then* $f^{-1}(B) = \{x \in X : f(x) \in B\}$ *is an ideal of* $X$ *containing* $Ker\ f$.

**Theorem 2.13** [3] *Let* $f : X \longrightarrow Y$ *be a KS-semigroup homomorphism. If* $X$ *is commutative, then* $f(X)$ *is commutative.*

**Definition 2.14** [3] A KS-semigroup $X$ is said to have a *unit element* 1 if for all $x \in X$, $x1 = 1x = x$.

**Definition 2.15** [3] A KS-semigroup $X$ is said to be *strong* if $\forall x, y \in X$, $x * xy = x * y$.

**Definition 2.16** [3] A nonempty subset $A$ of a KS-semigroup $X$ is said to be *stable* if $xa, ax \in A$ whenever $x \in X$ and $a \in A$.

Let $Y$ be an ideal of a KS-semigroup $X$ and define the relation $\widetilde{Y}$ on $X$ by $x\widetilde{Y}y$ if and only if $x * y, y * x \in Y$.

**Remark 2.17** [3] The relation $\widetilde{Y}$ on $X$ is an equivalence relation.

Since an equivalence relation partitions a set into equivalence classes, denote by $Y_x = \left\{y \in X : x\widetilde{Y}y\right\}$ as the equivalence class containing $x$ and the quotient $X/Y = \{Y_x : x \in X\}$ as the set of all equivalence classes in $X$. On $X/Y$, define the operations $\otimes$ and $\odot$ by $Y_x \otimes Y_y = Y_{x*y}$ and $Y_x \odot Y_y = Y_{xy}$. Then the following result holds.

**Remark 2.18** [3] The system $(X/Y, \otimes, \odot)$ is a KS-semigroup with zero element $Y_0 = Y$ and is called the *quotient KS-semigroup*.

# 3 Results and Discussion

On the set $[0,1]$, define the operations " $*$ " and " $\cdot$ " respectively as follows: $x*y = x-y$ if $x \geq y$ and $x*y = 0$ if $x < y$, and $x \cdot y$ as the usual multiplication. Then the following hold.

**Lemma 3.1** *For all $x, y, z \in [0,1]$, $[(x*y)*(x*z)]*(z*y) = 0$.*

*Proof*: Consider the following cases:
Case 1. $x \geq y$
If $y \leq z \leq x$, then $x - y \geq x - z$ so that

$$\begin{aligned}
[(x*y)*(x*z)]*(z*y) &= [(x-y)*(x-z)]*(z-y) \\
&= [(x-y)-(x-z)]*(z-y) \\
&= (z-y)*(z-y) \\
&= (z-y)-(z-y) \\
&= 0.
\end{aligned}$$

If $z \leq y \leq x$, then $x - y \leq x - z$ so that

$$\begin{aligned}
[(x*y)*(x*z)]*(z*y) &= [(x-y)*(x-z)]*0 \\
&= 0*0 \\
&= 0-0 \\
&= 0.
\end{aligned}$$

If $y \leq x \leq z$, then $x - y \leq z - y$ so that

$$\begin{aligned}
[(x*y)*(x*z)]*(z*y) &= [(x-y)*0]*(z-y) \\
&= [(x-y)-0]*(z-y) \\
&= (x-y)*(z-y) \\
&= 0.
\end{aligned}$$

Case 2. $x < y$
If $x < z \le y$ or $x \le z < y$, then

$$\begin{aligned}[(x*y)*(x*z)]*(z*y) &= [0*0]*0\\ &= [0-0]*0\\ &= 0*0\\ &= 0-0\\ &= 0.\end{aligned}$$

If $z \le x < y$, then

$$\begin{aligned}[(x*y)*(x*z)]*(z*y) &= [0*(x-z)]*0\\ &= 0*0\\ &= 0-0\\ &= 0.\end{aligned}$$

If $x < y \le z$, then

$$\begin{aligned}[(x*y)*(x*z)]*(z*y) &= [0*0]*(z-y)\\ &= [0-0]*(z-y)\\ &= 0*(z-y)\\ &= 0.\end{aligned}$$

Therefore, in all cases, the lemma is proved. □

**Lemma 3.2** *For all* $x, y \in [0,1]$, $[x*(x*y)]*y = 0$.

*Proof*: Consider the following cases:
Case 1. If $x \ge y$, then $x \ge x - y$ so that

$$\begin{aligned}[x*(x*y)]*y &= [x*(x-y)]*y\\ &= [x-(x-y)]*y\end{aligned}$$

$$\begin{aligned} &= y*y\\ &= y-y\\ &= 0.\end{aligned}$$

Case 2. If $x < y$, then

$$\begin{aligned}[x*(x*y)]*y &= [x*0]*y\\ &= [x-0]*y\\ &= x*y\\ &= 0.\end{aligned}$$

Therefore, for all $x, y \in [0,1]$, $[x*(x*y)]*y = 0$. □

**Lemma 3.3** *For all $x, y \in [0,1]$, we have*

1. $x * x = 0$
2. *If* $x * y = 0$ *and* $y * x = 0$, *then* $x = y$,
3. $x * 0 = x$; *and*
4. $0 * x = 0$.

*Proof*:

1. $x * x = x - x = 0$.
2. If $x * y = 0$ and $y * x = 0$, then $x \leq y$ and $y \leq x$ and so $x = y$.
3. $x * 0 = x - 0 = x$.
4. $0 * x = 0$ since $x \geq 0$. □

Combining the previous lemmas, we obtain:

**Theorem 3.4** *The set $[0,1]$, together with the binary operation "$*$", is a BCK-algebra.*

Since usual multiplication is an associative binary operation on $[0,1]$, the following theorem follows.

**Theorem 3.5** *The triple $\langle [0,1], *, \cdot \rangle$ is a KS-semigroup.*

**Theorem 3.6** $[0,1]$ *is a commutative BCK-algebra*

*Proof*: Let $x, y \in [0,1]$ and consider the following cases:

Case 1. If $x \geq y$, then $x * (x * y) = x * (x - y) = x - (x - y) = y$ while $y * (y * x) = y * 0 = y - 0 = y$.

Case 2. If $x < y$, then $x * (x * y) = x * 0 = x - 0 = x$ while

$$y * (y * x) = y * (y - x) = y - (y - x) = x.$$

Hence, in either case, $x, y \in X, x * (x * y) = y * (y * x)$. □

**Corollary 3.7** *If $f : [0,1] \longrightarrow X$ is an epimorphism of KS-semigroups, then $X$ is commutative.*

*Proof*: By Theorem 3.6, $[0,1]$ is commutative and by Theorem 2.13, $f([0,1])$ is commutative. Since $f$ is onto, $f([0,1]) = X$. Thus, $X$ is commutative. □

**Theorem 3.8** *The sets* $[0,a)$ *and* $[0,a]$ *are sub KS-semigroups and are stable subsets of* $[0,1]$ *for all* $a \in [0,1]$.

*Proof*: Let $x, y \in [0,a)$. Then $0 \leq xy < x < a$ implies that $xy \in [0,a)$. Also, if $x < y$, $x * y = 0 \in [0,a)$. If $x \geq y$, $x * y = x - y \leq x < a$. Thus, $x * y \in [0,a)$. Similarly, let $x, y \in [0,a]$. Then $0 \leq xy \leq x \leq a$ implies that $xy \in [0,a]$. Also, if $x < y$, then $x * y = 0 \in [0,a]$ and if $x \geq y$, then $x * y = x - y \leq x \leq a$. Thus, $x * y \in [0,a]$. Hence, $[0,a)$ and $[0,a]$ are sub KS-semigroups of $[0,1]$. To show stability, let $x \in [0,1]$ and $y \in [0,a)$. Then $xy = yx \leq y < a$ implies $xy \in [0,a)$. Thus, $[0,a)$ is stable. Similarly, let $x \in [0,1]$ and $y \in [0,a]$. Then $xy = yx \leq y \leq a$ implies $xy \in [0,a]$. Thus, $[0,a]$ is stable. □

**Theorem 3.9** *If* $\{0\} \neq Y \subseteq [0,1]$ *is an ideal of* $[0,1]$, *then* $Y = [0,a]$ *for some* $a \in [0,1]$.

*Proof*: Let $\{0\} \neq Y$ be an ideal of $[0,1]$. Then, $0 \in Y$ and there is $0 \neq x$ in $Y$. Now, let $0 \leq z \leq x$. Then by definition of "$*$", $z * x = 0 \in Y$. Since $x \in Y$ and $Y$ is an ideal, $z \in Y$. This means that $Y$ contains all real numbers between 0 and $x$. Let $a = sup\{x : x \in Y\}$. Then $a \neq 0$ and for all $z \in [0,1]$ such that $0 \leq z < a$, $z \in Y$. Now, $a * z = a - z < a$. This means that $a * z \in Y$. Since $z \in Y$ and $Y$ is an ideal, $a \in Y$. Therefore, $Y = [0,a]$. □

The following example provides a closed subset of $[0,1]$ which is not an ideal.

**Example 3.10** Consider the subset $Y = [0,1/2]$ of $[0, 1]$. Take $y = 1/4$ in $Y$ and $x = 2/3$ in $[0,1]$. Then $x * y = 2/3 - 1/4 = 5/12 < 1/2$. Hence, $x * y \in Y$ with $y \in Y$. However, $x \notin Y$. Thus, $[0,1/2]$ is not an ideal in $[0,1]$.

**Theorem 3.11** *The only ideals in* $[0,1]$ *are* $\{0\}$ *and* $[0,1]$. *Hence,* $\{0\}$ *is a maximal ideal.*

*Proof*: Let $Y$ be an ideal in $[0,1]$. Then $Y = [0,a]$ for some $a \in [0,1]$ by Theorem 3.9. If $a = 0$, then $Y = \{0\}$. Consider $0 < a < 1$. Choose a very small $r$ such that $0 < r < a$, $a + r/2 \leq 1$ and $a - r/2 \geq 0$ and take $x = a + r/2$ and $y = a - r/2$. Then $x \in [0,1]$, $x \notin Y$, $y \in Y$. Thus, $x * y = x - y = a + r/2 - (a - r/2) = r < a$. Hence, $x * y \in Y$ with $y \in Y$. Since $Y$ is an ideal, $x \in Y$. This is a contradiction. Thus, $a = 1$, in which case, $Y = [0,1]$. □

**Theorem 3.12** *If* $Y$ *is a maximal ideal in* $[0,1]$, *then there is an isomorphism from* $[0,1]$ *to* $[0,1]/Y$.

*Proof*: By Theorem 3.11, $Y = \{0\}$ so that if $x \in [0,1]$, then

$$\begin{aligned} Y_x &= \left\{ y \in [0,1] : x\widetilde{Y}y \right\} \\ &= \{y \in [0,1] : x * y, y * x \in Y\} \\ &= \{y \in [0,1] : x * y = y * x = 0\} \\ &= \{y \in [0,1] : x = y\} \\ &= \{x\} . \end{aligned}$$

Now, consider the mapping $f : [0,1] \longrightarrow [0,1]/Y$ defined by $f(x) = Y_x$. If $x = y$, then $x * y = 0 = y * x$, which means that $x * y$ and $y * x$ are in $Y$. Thus, $x\widetilde{Y}y$ so that $Y_x = Y_y$. Hence, $f(x) = f(y)$ implying that $f$ is well-defined. Also, $f(xy) = Y_{xy} = Y_x \odot Y_y = f(x) \odot f(y)$ and

$$f(x * y) = Y_{x*y} = Y_x \otimes Y_y = f(x) \otimes f(y).$$

Thus, $f$ is a KS-semigroup homomorphism. Let $x \in ker f$. Then

$$\{x\} = Y_x = f(x) = Y_0 = Y = \{0\} ,$$

where $Y_0$ is the zero in $[0,1]/Y$. Thus, $x = 0$ and $ker f = \{0\}$. That is, $f$ is one-to-one. Next, let $Y_x \in [0,1]/Y$. Then $x \in [0,1]$ and $f(x) = Y_x$. Thus, $f$ is onto. Therefore, $f$ is an isomorphism of $[0,1]$ onto [0, 1]/Y. $\square$

**Theorem 3.13** *If $f : [0,1] \longrightarrow X$ is an epimorphism of KS-semigroups, then the only ideals in $X$ are $\{0\}$ and $X$.*

*Proof*: Let $B \subseteq X$ be an ideal in $X$. Then by Theorem 2.12, $f^{-1}(B)$ is an ideal in $[0,1]$. By Theorem 3.11, $f^{-1}(B) = \{0\}$ or $f^{-1}(B) = [0,1]$. If $f^{-1}(B) = \{0\}$, then $B = \{0\}$. If $f^{-1}(B) = [0,1]$, then $f$ onto implies that $X = f([0,1]) \subseteq f(f^{-1}(B)) \subseteq B \subseteq X$. Therefore, $B = X$. $\square$

Let $X$ be any set, $Y$ be a KS-semigroup and $F_Y^X$ be the set of all functions $f : X \longrightarrow Y$. Define the operations " $\otimes$ " and " $\odot$ " on $F_Y^X$ as follows: $(f \otimes g)(x) = f(x) * g(x)$, $(f \odot g)(x) = f(x) \cdot g(x)$ where " $*$ " and " $\cdot$ " are operations in $Y$. The next result shows that $F_Y^X$, together with "$\otimes$" and "$\odot$" is a KS-semigroup.

**Theorem 3.14** *The system $\left\langle F_Y^X, \otimes, \odot, f_0 \right\rangle$ is a KS-semigroup, where $f_0$ is the zero function.*

*Proof*: First, show that " $\otimes$ " and " $\odot$ " are binary operations on $F_Y^X$. Let $f, g \in F_Y^X$. Then $\forall x \in X, (f \otimes g)(x) = f(x) * g(x)$ and $(f \odot g)(x) = f(x) \cdot g(x)$. Since $Y$ is a KS-semigroup, "$*$" and "$\cdot$" are binary operations in $Y$, $f \otimes g$ and

$f \odot g$ are in $F_Y^X$. Next, let $f, g, h \in F_Y^X$ and $x \in X$. Then $f(x), g(x), h(x) \in Y$ and since $Y$ is a KS-semigroup,

(a) $[(f \otimes g) \otimes (f \otimes h)] \otimes (h \otimes g)(x)$

$$\begin{aligned} &= [(f \otimes g) \otimes (f \otimes h)](x) * (h \otimes g)(x) \\ &= [(f \otimes g)(x) * (f \otimes h)(x)] * (h \otimes g)(x) \\ &= [(f(x) * g(x)) * (f(x) * h(x))] * (h(x) * g(x)) \\ &= 0 \\ &= f_0(x). \end{aligned}$$

(b) $$\begin{aligned} ((f \otimes (f \otimes g)) \otimes g)(x) &= (f \otimes (f \otimes g))(x) * g(x) \\ &= (f(x) * (f \otimes g)(x)) * g(x) \\ &= (f(x) * (f(x) * g(x)) * g(x) \\ &= 0 \\ &= f_0(x). \end{aligned}$$

(c) $(f \otimes f)(x) = f(x) * f(x) = 0 = f_0(x)$.

(d) $(f_0 \otimes f)(x) = f_0(x) * f(x) = 0 = f_0(x)$.

(e) If $f \otimes g = f_0$ and $g \otimes f = f_0$, then $(f \otimes g)(x) = 0 = (g \otimes f)(x)$ so that $f(x) * g(x) = g(x) * f(x)$. Thus, $f(x) = g(x)$ for each $x \in X$ and so $f = g$. Therefore, $\langle F_Y^X, \otimes, f_0 \rangle$ is a BCK-algebra. Also, $\langle F_Y^X, \odot \rangle$ is a semigroup since

$$\begin{aligned} ((f \odot g) \odot h)(x) &= (f \odot g)(x) \cdot h(x) \\ &= (f(x) \cdot g(x)) \cdot h(x) \\ &= f(x) \cdot (g(x) \cdot h(x)) \\ &= f(x) \cdot (g \odot h)(x) \\ &= (f \odot (g \odot h))(x). \end{aligned}$$

Therefore, $\langle F_Y^X, \otimes, \odot, f_0 \rangle$ is a KS-semigroup. □

The next corollary follows from Theorems 3.5 and 3.14.

**Corollary 3.15** *Let* $X$ *be any set. The system* $\left\langle F_{[0,1]}^X, \otimes, \odot, f_0 \right\rangle$ *is a KS-semigroup.*

**Theorem 3.16** *If* $Y$ *is a KS-semigroup with unit element* $1$ *and* $X$ *is any set, then* $F_Y^X$ *has unit element* $f_1(x) = 1, \forall x \in X$.

*Proof*: By Theorem 3.14, $F_Y^X$ is a KS-semigroup. Let $g \in F_Y^X$ and $x \in X$. Then $(g \odot f_1)(x) = g(x)f_1(x) = g(x)1 = g(x)$ and

$$(f_1 \odot g)(x) = f_1(x)g(x) = 1g(x) = g(x).$$

Thus, $g \odot f_1 = f_1 \odot g = g$. Therefore, $f_1$ is the unit element of $F_Y^X$. $\square$

**Corollary 3.17** *$F_{[0,1]}^X$ has unit element $\mu_1(x) = 1, \forall x \in X$.*

*Proof*: Since in $[0,1]$ we have $x1 = 1x = x$, $[0,1]$ has unit element 1 and so by Theorem 3.16, the result follows. $\square$

**Theorem 3.18** *If $Y$ is a strong KS-semigroup, then so is $F_Y^X$.*

*Proof*: Let $f, g \in F_Y^X$. Since $Y$ is a strong KS-semigroup, by Definition 2.15,

$$(f \otimes g)(x) = f(x) * g(x) = f(x) * f(x)g(x) = f(x) * (f \odot g)(x) = (f \otimes (f \odot g))(x).$$

Therefore, $f \otimes g = f \otimes (f \odot g)$ and so $F_Y^X$ is strong. $\square$

**Theorem 3.19** *If $X$ is any set and $N$ is an ideal of a KS-semigroup $Y$, then the set $V = \left\{f \in F_Y^X : f(X) \subseteq N\right\}$ is an ideal of $F_Y^X$.*

*Proof*: Let $f \in V$, $x \in X$ and $g \in F_Y^X$. Then $(f \odot g)(x) = f(x)g(x) \in N$. Since $N$ is an ideal of $Y$, $f \odot g \in V$. Similarly, $(g \odot f)(x) = g(x)f(x) \in N$ implies that $g \odot f \in V$. Next, let $f \otimes g \in V, g \in V$ and $x \in X$. Then $(f \otimes g)(x) = f(x) * g(x) \in N$ and $g(x) \in N$. This means that $f(x) \in N$ for $N$ is an ideal. Thus, $f \in V$. Therefore, $V$ is an ideal of $F_Y^X$. $\square$

**Corollary 3.20** *The only ideals of $F_{[0,1]}^X$ are $\{f_0\}$ and $F_{[0,1]}^X$.*

*Proof*: By Theorem 3.11, the only ideals of $[0,1]$ are $\{0\}$ and $[0,1]$. Thus, by Theorem 3.19, the ideals of $F_{[0,1]}^X$ are $V_1 = \left\{f \in F_{[0,1]}^X : f(X) \subseteq \{0\}, \right\} = \{f_0\}$, and $V_2 = \left\{g \in F_{[0,1]}^X : g(X) \subseteq [0,1]\right\} = F_{[0,1]}^X$. $\square$

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