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# Slant Helices According to Type-1 Bishop Frame in Weyl Space

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**Abstract**

In this paper, we have defined slant helices according to Bishop frame (type-1 Bishop frame) in three dimensional Weyl space $W_3$. Besides, we have given necessary and sufficient conditions of a curve to be slant helix in $W_3$.

**Mathematics Subject Classification:** 53B25, 53A25.

**Keywords:** Weyl space, type-1 Bishop frame, slant helix

# 1 Introduction

A manifold with a conformal metric $g_{ij}$ and a symmetric connection $\nabla_k$ satisfying the compatibility condition

$$\nabla_k g_{ij} - 2\mathrm{T}_k g_{ij} = 0 \tag{1}$$

is called a Weyl space which will be denoted by $W_3(g_{ij}, \mathrm{T}_k)$. The vector field $\mathrm{T}_k$ is named the complementary vector field. Under a renormalization of the metric tensor $g_{ij}$ in the form

$$\overset{v}{g}_{ij} = \lambda^2 g_{ij} \tag{2}$$

the complementary vector field $\mathrm{T}_k$ is transformed by the law

$$\overset{v}{\mathrm{T}}_k = \mathrm{T}_k + \partial_k \ln \lambda \tag{3}$$

where $\lambda$ is a scalar function [11].

If under the transformation (2), the quantity $A$ is called a satellite of $g_{ij}$ with weight $\{p\}$.

The prolonged derivative and prolonged covariant derivative of $A$ are, respectively, defined ( [6], [12])

$$\dot{\partial}_k A = \partial_k A - p\mathrm{T}_k A \tag{4}$$

and

$$\dot{\nabla}_k A = \nabla_k A - p\mathrm{T}_k A. \tag{5}$$

The $\underset{r}{v}^i (i, r = 1, 2, 3)$ be the contravariant components of the vector field $\underset{r}{v}$ in $W_3(g_{ij}, \mathrm{T}_k)$. Suppose that the vector fields $\underset{r}{v}$ are normalized by the conditions $g_{ij}\underset{r}{v}^i\underset{r}{v}^j = 1$.

The prolonged covariant derivative of the vector field $\underset{r}{v}$ is given by [13]

$$\dot{\nabla}_k \underset{r}{v}^i = \underset{r}{\overset{s}{\mathrm{T}}}_k \underset{s}{v}^i \quad (s = 1, 2, 3). \tag{6}$$

The quantities

$$\underset{rs}{\overset{q}{\tau}} = \underset{r}{\overset{q}{\mathrm{T}}}_k \underset{s}{v}^k \quad (q = 1, 2, 3; \quad r \neq s) \tag{7}$$

and

$$\underset{s}{\overset{r}{\varsigma}} = \underset{s}{\overset{r}{\mathrm{T}}}_k \underset{s}{v}^k \tag{8}$$

are called the Chebyshev curvature of the first kind and geodesic curvature of the net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v})$ [13], respectively.

Since the net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v})$ is an orthogonal net, we have [13]

$$\underset{r}{\overset{r}{\mathrm{T}}}_k = 0, \quad \underset{r}{\overset{p}{\mathrm{T}}}_k + \underset{p}{\overset{r}{\mathrm{T}}}_k = 0 \quad (r \neq p). \tag{9}$$

# 2 Preliminaries

Izumiya and Takeuchi [7] defined slant helices which are generalizations of the notion of general helices. Kula and Yaylı [9] investigated the spherical indicatrix of slant helices and showed that the tangent and binormal indicatrix of them are spherical helices. Also, Kula et al. [10] gave some characterizations for slant helices in Euclidean 3-space. Bï£¡kçï£¡ and Karacan defined slant helices according to Bishop frame [4]. Ali and Turgut [1] extended the notion of slant helix from Euclidean 3-space to Euclidean $n$-space. Ali and Turgut [2] researched the position vector of a timelike slant helix in Minkowski 3-space $\mathrm{E}_1^3$. Recently, Doḡan studied the proof of theorem which characterizes a slant helix [5].

# 3 Slant Helices According to Type-1 Bishop Frame in Weyl Space

Let $C$ be a curve in three dimensional Weyl space $W_3$. Let $\underset{1}{v}$, $\underset{2}{v}$ and $\underset{3}{v}$ be the tangent vector field, principal normal vector field and binormal vector field of $C$ at a point $P$. Let they constitute Frenet frame: $\{\underset{1}{v}, \underset{2}{v}, \underset{3}{v}\}$. Since this frame is an orthonormal basis, these vector fields are normalized by the conditions $g_{ij}\underset{1}{v}^i\underset{1}{v}^j = g_{ij}\underset{2}{v}^i\underset{2}{v}^j = g_{ij}\underset{3}{v}^i\underset{3}{v}^j = 1$ and besides the conditions $g_{ij}\underset{1}{v}^i\underset{2}{v}^j = g_{ij}\underset{1}{v}^i\underset{3}{v}^j = g_{ij}\underset{2}{v}^i\underset{3}{v}^j = 0$ are satisfied. These two conditions are also expressed as $g_{ij}\underset{r}{v}^i\underset{s}{v}^j = \delta^r_s \quad (r, s = 1, 2, 3)$.

We can write Frenet formulas for these vector fields in the following form:

$$\underset{1}{v}^k\dot{\nabla}_k\underset{1}{v}^i = \kappa_1\underset{2}{v}^i \tag{10}$$

$$\underset{1}{v}^k\dot{\nabla}_k\underset{2}{v}^i = -\kappa_1\underset{1}{v}^i + \kappa_2\underset{3}{v}^i \tag{11}$$

$$\underset{1}{v}^k\dot{\nabla}_k\underset{3}{v}^i = -\kappa_2\underset{2}{v}^i. \tag{12}$$

Multiplying (10) by $g_{ij}\underset{2}{v}^j$ and summing on $i$ and $j$, we get

$$\overset{p}{\underset{1}{\mathrm{T}}}_k\underset{1}{v}^k\underset{p}{v}^i\underset{2}{v}^j g_{ij} = \kappa_1 \tag{13}$$

or

$$\overset{2}{\underset{1}{\mathrm{T}}}_k\underset{1}{v}^k = \kappa_1 \tag{14}$$

or

$$\overset{2}{\underset{1}{\text{Ƨ}}} = \kappa_1 \tag{15}$$

where $\overset{1}{\underset{1}{T}}_k = 0$, $g_{ij}\underset{2}{v}^i\underset{2}{v}^j = 1$ and $g_{ij}\underset{3}{v}^i\underset{2}{v}^j = 0$.

Multiplying (12) by $g_{ij}\underset{2}{v}^j$ and summing on $i$ and $j$, we have

$$\overset{p}{\underset{3}{\mathrm{T}}}_k\underset{1}{v}^k\underset{p}{v}^i\underset{2}{v}^j g_{ij} = -\kappa_2 \tag{16}$$

or

$$\overset{2}{\underset{3}{\mathrm{T}}}_k\underset{1}{v}^k = -\kappa_2 \tag{17}$$

or

$$-\overset{2}{\underset{3}{\tau}}_1 = \kappa_2 \tag{18}$$

where $g_{ij}\underset{1}{v}^i\underset{2}{v}^j = 0$, $g_{ij}\underset{2}{v}^i\underset{2}{v}^j = 0$ and $\overset{3}{\underset{3}{\mathrm{T}}}_k = 0$.

The Bishop frame [3] or parallel transport frame is an alternative approach to define a moving frame that is well defined even when the curve has vanishing second derivative. Let us denote the type-1 Bishop frame by $\{\underset{1}{v}^i, \underset{1}{n}^i, \underset{2}{n}^i\}$. The derivative formulas of this frame defined as

$$\underset{1}{v}^k \dot{\nabla}_k \underset{1}{v}^i = k_1 \underset{1}{n}^i + k_2 \underset{2}{n}^i \tag{19}$$

or

$$\underset{1}{v}^k \dot{\nabla}_k \underset{1}{n}^i = -k_1 \underset{1}{v}^i \tag{20}$$

or

$$\underset{1}{v}^k \dot{\nabla}_k \underset{2}{n}^i = -k_2 \underset{1}{v}^i \tag{21}$$

where $k_1$ and $k_2$ are named as Bishop curvatures or natural curvatures [8].

Multiplying (19) by $g_{ij}\underset{1}{n}^i$ and summing on $i$ and $j$, we have

$$\overset{p}{\underset{1}{\mathrm{T}}}_k \underset{1}{v}^k \underset{p}{v}^i \underset{1}{n}^j g_{ij} = -k_1 \tag{22}$$

or

$$\overset{2}{\underset{1}{\mathrm{T}}}_k \underset{1}{v}^k \underset{2}{v}^i \underset{1}{n}^j g_{ij} + \overset{3}{\underset{1}{\mathrm{T}}}_k \underset{1}{v}^k \underset{3}{v}^i \underset{1}{n}^j g_{ij} = k_1 \tag{23}$$

or

$$\overset{2}{\underset{1}{\text{Ƨ}}} \cos\theta - \overset{3}{\underset{1}{\text{Ƨ}}} \sin\theta = k_1 \tag{24}$$

where $\theta = \sphericalangle(\underset{2}{v}^i, \underset{1}{n}^i)$, $g_{ij}\underset{1}{n}^i\underset{1}{n}^j = 1$, and $g_{ij}\underset{2}{n}^i\underset{1}{n}^j = 0$.

Multiplying (19) by $g_{ij}\underset{2}{n}^j$ and summing on $i$ and $j$, we get

$$\overset{p}{\underset{1}{\mathrm{T}}}_k \underset{1}{v}^k \underset{p}{v}^i \underset{2}{n}^j g_{ij} = k_2 \tag{25}$$

or

$$\overset{2}{\underset{1}{\mathrm{T}}}_k \underset{1}{v}^k \underset{2}{v}^i \underset{2}{n}^j g_{ij} + \overset{3}{\underset{1}{\mathrm{T}}}_k \underset{1}{v}^k \underset{3}{v}^i \underset{2}{n}^j g_{ij} = k_2 \tag{26}$$

or

$$\overset{2}{\underset{1}{\text{Ƨ}}} \sin\theta + \overset{3}{\underset{1}{\text{Ƨ}}} \cos\theta = k_2 \tag{27}$$

where $g_{ij}\underset{2}{n}^i\underset{2}{n}^j = 1$.

(24) and (27) are the quantities in Weyl space that correspond to natural curvatures.

On the other hand, $\underset{1}{v}^k \dot{\nabla}_k \underset{1}{v}^i = \kappa_1 \underset{2}{v}^i$. Multiplying this equality by $g_{ij}\underset{1}{n}^j$ and using (22), we get

$$k_1 = \kappa_1 \cos\theta. \tag{28}$$

Multiplying the same equality by $g_{ij}\underset{2}{n}^j$ and using (25), we get

$$k_2 = \kappa_1 \sin\theta. \tag{29}$$

From (28) and (29), we have

$$k_1^2 + k_2^2 = \kappa_1^2. \tag{30}$$

Since $\underset{2}{v}^i$ is orthogonal to $\underset{1}{v}^i$, $\underset{2}{v}^i$ can be written in the following form:

$$\underset{2}{v}^i = \alpha \underset{1}{n}^i + \beta \underset{2}{n}^i \tag{31}$$

or

$$\underset{2}{v}^i = \cos\theta \underset{1}{n}^i + \sin\theta \underset{2}{n}^i \tag{32}$$

where $\theta = \sphericalangle(\underset{2}{v}^i, \underset{1}{n}^i)$.

Using $\underset{3}{v}^k = \varepsilon_{ijk}\underset{1}{v}^i\underset{2}{v}^j$ and (32) and right-hand rule, we get

$$\underset{3}{v}^k = \varepsilon_{ijk}\underset{1}{v}^i(\cos\theta\underset{1}{n}^j + \sin\theta\underset{2}{n}^j) = \cos\theta\underset{2}{n}^k - \sin\theta\underset{1}{n}^k. \tag{33}$$

From (12), (32) and (33), we have

$$\underset{1}{v}^l \dot{\nabla}_l \underset{3}{v}^k = -(\underset{1}{v}^l \dot{\nabla}_l \theta)(\sin\theta\underset{2}{n}^k + \cos\theta\underset{1}{n}^k) \tag{34}$$

$$-\kappa_2 \underset{2}{v}^k = -(\underset{1}{v}^l \dot{\nabla}_l \theta)\underset{2}{v}^k \tag{35}$$

$$\kappa_2 = \underset{1}{v}^l \dot{\nabla}_l \theta. \tag{36}$$

The relation among the vector fields of Frenet frame and type-1 Bishop frame is in the following form:

$$\begin{pmatrix} \underset{1}{v}^i \\ \underset{2}{v}^i \\ \underset{3}{v}^i \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \underset{1}{v}^i \\ \underset{1}{n}^i \\ \underset{2}{n}^i \end{pmatrix}. \tag{37}$$

**Definition 3.1.** *Let $C$ be a curve in $W_3$. $C$ is called a slant helix if the vector field $\underset{1}{n}^i$ has constant angle $\theta$ with some fixed vector field $u$, i.e., $g_{ij}\underset{1}{n}^i u^j = \cos\varphi$ where $u$ is normalized in the form $g_{ij}u^i u^j = 1$.*

**Theorem 3.2.** *Let $C$ be a curve which has non-zero natural curvatures in $W_3$. $C$ is a slant helix if and only if $\frac{k_1}{k_2}$ is constant.*

*Proof.* Let $C$ be a slant helix. Then

$$g_{ij}\underset{1}{n}^i u^j = \cos\varphi = \text{constant} \tag{38}$$

is satisfied by means of Definition 3.1 where $\varphi = \sphericalangle(\underset{1}{n}^i, u^j)$.

By taking prolonged covariant derivative of (38) in the direction of $\underset{1}{v}^k$, we get

$$g_{ij}(\underset{1}{v}^k \dot{\nabla}_k \underset{1}{n}^i)u^j = 0 \tag{39}$$

or

$$g_{ij}\underset{1}{v}^i u^j = 0 \quad (k_1 \neq 0). \tag{40}$$

Again, by taking prolonged covariant derivative of (40) in the direction of $\underset{1}{v}^k$, we have

$$g_{ij}(\underset{1}{v}^k \dot{\nabla}_k \underset{1}{v}^i)u^j = 0 \tag{41}$$

or

$$k_1 g_{ij}\underset{1}{n}^i u^j + k_2 g_{ij}\underset{2}{n}^i u^j = 0 \tag{42}$$

or

$$k_1 \cos\varphi + k_2 \sin\varphi = 0 \tag{43}$$

or

$$\frac{k_1}{k_2} = -\frac{\sin\varphi}{\cos\varphi} = -\tan\varphi = \text{constant}. \tag{44}$$

Conversely, suppose that $\frac{k_1}{k_2} = -\tan\varphi$.

From (40), we write

$$u^j = \alpha\underset{1}{n}^j + \beta\underset{2}{n}^j \tag{45}$$

or

$$u^j = \cos\varphi\underset{1}{n}^j + \sin\varphi\underset{2}{n}^j \tag{46}$$

where $\varphi = \sphericalangle(\underset{1}{n}^j, u^j)$.

Taking prolonged covariant derivative of (46) in the direction of $\underset{1}{v}^k$, we get

$$\underset{1}{v}^k \dot{\nabla}_k u^j = -(k_1\cos\varphi + k_2\sin\varphi)\underset{1}{v}^j. \tag{47}$$

Since $\frac{k_1}{k_2} = -\tan\varphi$, we obtain $k_1\cos\varphi + k_2\sin\varphi = 0$.

That is, $\underset{1}{v}^k \dot{\nabla}_k u^j = 0$ is valid. $u$ is a constant vector field.

The proof is completed. □

**Remark 3.3.** *The equation* (12) *is equivalent to*

$$\overset{2}{\underset{1}{\mathfrak{z}}}\cos(\theta - \varphi) - \overset{3}{\underset{1}{\mathfrak{z}}}\sin(\theta - \varphi) = 0$$

*by means of* (24) *and* (27).

**Theorem 3.4.** *$C$ is a slant helix if and only if*

$$det(\underset{1}{v}^k\dot{\nabla}_k\underset{1}{n}^i, \underset{1}{v}^l\dot{\nabla}_l(\underset{1}{v}^k\dot{\nabla}_k\underset{1}{n}^i), \underset{1}{v}^h\dot{\nabla}_h(\underset{1}{v}^l\dot{\nabla}_l(\underset{1}{v}^k\dot{\nabla}_k\underset{1}{n}^i))) = 0.$$

*Proof.* Let $C$ be a slant helix. Then, $\frac{k_1}{k_2}$ = constant from Theorem 3.1.

On the other hand, let us calculate the following derivatives of $\underset{1}{n}^i$:

$$\underset{1}{v}^k\dot{\nabla}_k\underset{1}{n}^i = -k_1\underset{1}{v}^i \tag{48}$$

$$\underset{1}{v}^l\dot{\nabla}_l(\underset{1}{v}^k\dot{\nabla}_k\underset{1}{n}^i) = -(\underset{1}{v}^l\dot{\nabla}_l k_1)\underset{1}{v}^i - k_1^2\underset{1}{n}^i - k_1k_2\underset{2}{n}^i \tag{49}$$

$$\begin{aligned}\underset{1}{v}^h\dot{\nabla}_h(\underset{1}{v}^l\dot{\nabla}_l(\underset{1}{v}^k\dot{\nabla}_k\underset{1}{n}^i)) &= [-\underset{1}{v}^h\dot{\nabla}_h(\underset{1}{v}^l\dot{\nabla}_l k_1) + k_1^3 + k_1k_2^2]\underset{1}{v}^i - \\ &-3k_1(\underset{1}{v}^h\dot{\nabla}_h k_1)\underset{1}{n}^i + [-2k_2\underset{1}{v}^l\dot{\nabla}_l k_1 - k_1\underset{1}{v}^h\dot{\nabla}_h k_2]\underset{2}{n}^i.\end{aligned} \tag{50}$$

$$\begin{aligned}&\det(\underset{1}{v}^k\dot{\nabla}_k\underset{1}{n}^i, \underset{1}{v}^l\dot{\nabla}_l(\underset{1}{v}^k\dot{\nabla}_k\underset{1}{n}^i), \underset{1}{v}^h\dot{\nabla}_h(\underset{1}{v}^l\dot{\nabla}_l(\underset{1}{v}^k\dot{\nabla}_k\underset{1}{n}^i))) = \\ &= \begin{vmatrix} -k_1 & 0 & 0 \\ -\underset{1}{v}^l\dot{\nabla}_l k_1 & -k_1^2 & -k_1k_2 \\ -\underset{1}{v}^h\dot{\nabla}_h(\underset{1}{v}^l\dot{\nabla}_l k_1) + k_1^3 + k_1k_2^2 & -3k_1\underset{1}{v}^h\dot{\nabla}_h k_1 & -2k_2\underset{1}{v}^l\dot{\nabla}_l k_1 - k_1\underset{1}{v}^h\dot{\nabla}_h k_2 \end{vmatrix} \\ &= k_1^3(\underset{1}{v}^h\dot{\nabla}_h\frac{k_1}{k_2})k_2^2.\end{aligned} \tag{51}$$

Since $\frac{k_1}{k_2}$ = constant, $\underset{1}{v}^h\dot{\nabla}_h\frac{k_1}{k_2} = 0$ is valid. Then

$$\det(\underset{1}{v}^k\dot{\nabla}_k\underset{1}{n}^i, \underset{1}{v}^l\dot{\nabla}_l(\underset{1}{v}^k\dot{\nabla}_k\underset{1}{n}^i), \underset{1}{v}^h\dot{\nabla}_h(\underset{1}{v}^l\dot{\nabla}_l(\underset{1}{v}^k\dot{\nabla}_k\underset{1}{n}^i))) = 0$$

is obtained.

Conversely, suppose that

$$\det(\underset{1}{v}^k\dot{\nabla}_k\underset{1}{n}^i, \underset{1}{v}^l\dot{\nabla}_l(\underset{1}{v}^k\dot{\nabla}_k\underset{1}{n}^i), \underset{1}{v}^h\dot{\nabla}_h(\underset{1}{v}^l\dot{\nabla}_l(\underset{1}{v}^k\dot{\nabla}_k\underset{1}{n}^i))) = 0.$$

Then $\underset{1}{v}^h\dot{\nabla}_h\frac{k_1}{k_2} = 0$ is obtained. That is $\frac{k_1}{k_2}$ = constant. $C$ is a slant helix.

□

**Theorem 3.5.** *$C$ is a slant helix if and only if*

$$det(\underset{1}{v}^k\dot{\nabla}_k\underset{2}{n}^i, \underset{1}{v}^l\dot{\nabla}_l(\underset{1}{v}^k\dot{\nabla}_k\underset{2}{n}^i), \underset{1}{v}^h\dot{\nabla}_h(\underset{1}{v}^l\dot{\nabla}_l(\underset{1}{v}^k\dot{\nabla}_k\underset{2}{n}^i))) = 0.$$

*Proof.* Let $C$ be a slant helix. Then, $\frac{k_1}{k_2}$ = constant from Theorem 3.1.

Let us calculate the following derivatives of $\underset{2}{n}^i$:

$$\underset{1}{v}^k\dot{\nabla}_k\underset{2}{n}^i = -k_2\underset{1}{v}^i \tag{52}$$

$$\underset{1}{v}^l\dot{\nabla}_l(\underset{1}{v}^k\dot{\nabla}_k\underset{2}{n}^i) = -(\underset{1}{v}^l\dot{\nabla}_l k_2)\underset{1}{v}^i - k_1k_2\underset{1}{n}^i - k_2^2\underset{2}{n}^i \tag{53}$$

$$\begin{aligned}\underset{1}{v}^h\dot{\nabla}_h(\underset{1}{v}^l\dot{\nabla}_l(\underset{1}{v}^k\dot{\nabla}_k\underset{2}{n}^i)) = {} & [-\underset{1}{v}^h\dot{\nabla}_h(\underset{1}{v}^l\dot{\nabla}_l k_2) + k_1^2k_2 + k_2^3]\underset{1}{v}^i + \\ & [-2k_1\underset{1}{v}^h\dot{\nabla}_h k_2 - k_2\underset{1}{v}^h\dot{\nabla}_h k_1]\underset{1}{n}^i - 3k_2(\underset{1}{v}^h\dot{\nabla}_h k_2)\underset{2}{n}^i.\end{aligned} \tag{54}$$

$$\begin{aligned}&\det(\underset{1}{v}^k\dot{\nabla}_k\underset{2}{n}^i, \underset{1}{v}^l\dot{\nabla}_l(\underset{1}{v}^k\dot{\nabla}_k\underset{2}{n}^i), \underset{1}{v}^h\dot{\nabla}_h(\underset{1}{v}^l\dot{\nabla}_l(\underset{1}{v}^k\dot{\nabla}_k\underset{2}{n}^i))) = \\ &= \begin{vmatrix} -k_2 & 0 & 0 \\ -\underset{1}{v}^l\dot{\nabla}_l k_2 & -k_1k_2 & -k_2^2 \\ -\underset{1}{v}^h\dot{\nabla}_h(\underset{1}{v}^l\dot{\nabla}_l k_2) + k_1^2k_2 + k_2^3 & -2k_1\underset{1}{v}^h\dot{\nabla}_h k_2 - k_2\underset{1}{v}^h\dot{\nabla}_h k_1 & -3k_1\underset{1}{v}^h\dot{\nabla}_h k_2 \end{vmatrix} \\ &= k_2^5(\underset{1}{v}^h\dot{\nabla}_h\frac{k_1}{k_2}).\end{aligned} \tag{55}$$

Since $\frac{k_1}{k_2}$ = constant, $\underset{1}{v}^h\dot{\nabla}_h\frac{k_1}{k_2} = 0$ is valid. Then

$$\det(\underset{1}{v}^k\dot{\nabla}_k\underset{2}{n}^i, \underset{1}{v}^l\dot{\nabla}_l(\underset{1}{v}^k\dot{\nabla}_k\underset{2}{n}^i), \underset{1}{v}^h\dot{\nabla}_h(\underset{1}{v}^l\dot{\nabla}_l(\underset{1}{v}^k\dot{\nabla}_k\underset{2}{n}^i))) = 0$$

is obtained.

Conversely, suppose that

$$\det(\underset{1}{v}^k\dot{\nabla}_k\underset{2}{n}^i, \underset{1}{v}^l\dot{\nabla}_l(\underset{1}{v}^k\dot{\nabla}_k\underset{2}{n}^i), \underset{1}{v}^h\dot{\nabla}_h(\underset{1}{v}^l\dot{\nabla}_l(\underset{1}{v}^k\dot{\nabla}_k\underset{2}{n}^i))) = 0.$$

Then $\underset{1}{v}^h\dot{\nabla}_h\frac{k_1}{k_2} = 0$, i.e., $\frac{k_1}{k_2}$ = constant. $C$ is a slant helix.

The proof is completed. □

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