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ϕ-Complement of Intuitionistic Fuzzy Graph Structure

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Abstract

In this paper, we introduce the notion of ϕ-complement of intuitionistic fuzzy graph structure $\tilde{G}$ = (A, B_1, B_2,..., B_k) where ϕ is a permutation on {B_1, B_2,...,B_k} and obtain some results. We also define some elementary definitions like self complementary, totally self complementary, strong self complementary intuitionistic fuzzy graph structure and study their properties.

Mathematics Subject Classification: 05C72, 05C76, 05C38, 03F55

Keywords: Intuitionistic fuzzy graph structure, ϕ-complement, self complementary, totally self complementary, strong self complementary

I. Introduction

The concept of intuitionistic fuzzy graph structure $\tilde{G}$ = (A, B_1, B_2,...,B_k) is introduced and studied by the authors in [6]. Zadeh [7] in 1965 introduced the notion of fuzzy sets. Then Rosenfeld [8] gave the idea of fuzzy relations and fuzzy graph in 1975. Atanassov [5] proposed the first definition of intuitionistic fuzzy graph. E. Sampatkumar in [1] has generalized the notion of graph G = (V, E) to graph structure G = (V, R_1, R_2, ..., R_k) where R_1, R_2, ..., R_k are relations on V which are mutually disjoint and each R_i , i = 1, 2, 3, ..., k is symmetric and irreflexive. T. Dinesh and T. V. Ramakrishnan [2] introduced the notion of fuzzy graph structure and studied their properties. In this paper, we will introduce and study ϕ - complement of intuitionistic fuzzy graph structure $\tilde{G}$.

2. Preliminaries

In this section, we review some definitions that are necessary to understand the content of this paper. These are mainly taken from [2], [7], [8], [9], [10] and [11].

Definition (2.1) [11]: G = (V, R_1, R_2,...,R_k) is a graph structure if V is a non empty set and R_1, R_2,...,R_k are relations on V which are mutually disjoint such that each R_i , i=1, 2, 3, ... ,k, is symmetric and irreflexive.

Definition (2.2) [9, 10]: An intuitionistic fuzzy graph is of the form G = (V, E) where
(i) V = {v_1, v_2, ..., v_n} such that $\mu_1 : V\to [0,1]$ and $\gamma_1 : V\to [0,1]$ denote the degree of membership and non membership of the element $v_i \in V$, respectively and $0 \le \mu_1(v_i) + \gamma_1 (v_i) \le 1$, for every $v_i \in V$, (i = 1,2,….,n),
(ii) $E \subseteq V \times V$ where μ_2: V× V $\to$ [0,1] and γ_2 : V× V $\to$ [0,1] are such that
$\mu_2(v_i,v_j) \le \min\{\mu_1(v_i),\mu_1(v_j)\}$ and $\gamma_2(v_i,v_j) \le \max\{\gamma_1(v_i),\gamma_1(v_j)\}$
and $0 \le \mu_2 (v_i , v_j)+ \gamma_2 (v_i , v_j) \le 1$, for every $(v_i ,v_i) \in E$, (i, j = 1,2,….,n).

Definition (2.3) [8]: Let G = (V, R_1, R_2, ..., R_k) be a graph structure and A, B_1, B_2,….., B_k be intuitionistic fuzzy subsets (IFSs) of V, R_1, R_2, ..., R_k respectively such that

$$\mu_{B_i}(u,v) \le \mu_A(u) \wedge \mu_A(v) \text{ and } \nu_{B_i}(u,v) \le \nu_A(u) \vee \nu_A(v) \quad \forall u,v \in V \text{ and } i = 1,2,...,k.$$

Then $\tilde{G}$ = (A, B_1, B_2, …., B_k) is an IFGS of G.

Example (2.4) [8]: Consider the graph structure G = (V, R_1, R_2, R_3), where V = { u_0, u_1, u_2, u_3, u_4} and R_1 ={ (u_0, u_1), (u_0, u_2), (u_3, u_4)}, R_2 ={(u_1, u_2), (u_2, u_4)}, R_3 = {(u_2, u_3), (u_0, u_4)} are the relations on V. Let A = {< u_0, 0.5,0.4 >, < u_1, 0.6,0.3>, < u_2 , 0.2,0.6 >, <u_3,0.1,0.8> , <u_4, 0.4,0.3> } be an IFS on V and B_1 = {(u_0, u_1), 0.5,0.3 > , < (u_0, u_2), 0.1,0.3 >, < (u_3, u_4), 0.1,0.2 >}, B_2 = {< (u_1, u_2), 0.2,0.1 >, < (u_2, u_4), 0.1,0.2 >}, B_3 ={< (u_2, u_3), 0.1,0.5 >, ,< (u_0, u_4) , 0.3,0.2 >} are intuitionistic fuzzy relations on V.
Here $\mu_{B_i}(u,v) \le \mu_A(u) \wedge \mu_A(v)$ and $\nu_{B_i}(u,v) \le \nu_A(u) \vee \nu_A(v)$ $\forall$ *u, v* $\in$ V and i =1, 2, 3.
$\therefore$ $\tilde{G}$ is an intuitionistic fuzzy graph structure.

Definition (2.5) [7]: The complement of a fuzzy graph G = (σ, μ) is a fuzzy graph $\overline{G} = (\overline{\sigma}, \overline{\mu})$ where $\overline{\sigma} = \sigma$ and $\overline{\mu}(u, v) = \sigma(u) \wedge \sigma(v) - \mu(u, v)$, $\forall$ u, v $\in$ V.

Definition (2.6) [7]: Consider the fuzzy graphs $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ with $\sigma_1^* = V_1$ and $\sigma_2^* = V_2$. An isomorphism between $G_1 = (\sigma_1, \mu_1)$ and G_2 is a one to one function h from V_1 onto V_2 that satisfies $\sigma_1(u) = \sigma_2(h(u))$ and $\mu_1(u,v) = \mu_2(h(u), h(v))$, $\forall\, u,v \in V$

3. ϕ-Complement of Intuitionistic Fuzzy Graph Structure

Definition (3.1): Let $\tilde{G} = (A, B_1, B_2,...,B_k)$ be an intuitionistic fuzzy graph structure of graph structure $G = (V, R_1, R_2,...,R_k)$. Let ϕ denotes the permutation on the set $\{R_1, R_2,....,R_k\}$ and also the corresponding permutation on $\{B_1, B_2,...,B_k\}$ i.e., $\phi(B_i) = B_i^{\phi} = B_j$ (i.e., $\phi\mu_{B_i} = \mu_{B_j}$ and $\phi\nu_{B_i} = \nu_{B_j}$) if and only if $\phi(R_i) = R_j$, then the ϕ- complement of $\tilde{G}$ is denoted $\tilde{G}^{\phi}$ and is given by $\tilde{G}^{\phi} = (A, B_1^{\phi}, B_2^{\phi},...,B_k^{\phi})$ where for each $i = 1, 2, 3,\ldots, k$, we have

$$\mu_{B_i}{}^{\phi}(uv) = \mu_A(u) \wedge \mu_A(v) - \sum_{j \neq i} (\phi\mu_{B_j})(uv) \text{ and } \nu_{B_i}{}^{\phi}(uv) = \nu_A(u) \vee \nu_A(v) - \sum_{j \neq i} (\phi\nu_{B_j})(uv) .$$

Example (3.2): Consider an intuitionistic fuzzy graph structure $\tilde{G} = (A, B_1, B_2)$ such that $V = \{ u_0, u_1, u_2, u_3, u_4, u_5\}$. Let $R_1 = \{ (u_0, u_1), (u_0, u_2), (u_3, u_4)\}$, $R_2 =\{(u_1, u_2), (u_4, u_5)\}$, $A = \{< u_0, 0.8,0.2 >, < u_1, 0.9,0.1>, < u_2, 0.6,0.3 >, <u_3, 0.5,0.4> , <u_4, 0.6,0.1> , <u_5, 0.7,0.2 > \}$, $B_1 = \{ (u_0, u_1), 0.8,0.1 > , < (u_0, u_2), 0.5,0.3 >, < (u_3, u_4), 0.4,0.2 >\}$, $B_2 = \{ < (u_1, u_2), 0.6,0.2 >, <(u_4, u_5),0.5,0.1 >\}$.

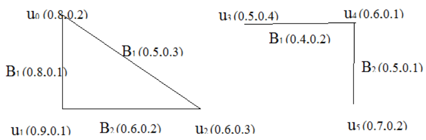


Fig 1

Let ϕ be a permutation on the set $\{B_1, B_2\}$ defined by $\phi(B_1) = B_2$ and $\phi(B_2) = B_1$, then

$$\mu_{B_1}{}^{\phi}(uv) = \mu_A(u) \wedge \mu_A(v) - \mu_{B_1}(uv) \; ; \; \nu_{B_1}{}^{\phi}(uv) = \nu_A(u) \vee \nu_A(v) - \nu_{B_1}(uv) \text{ and}$$

$$\mu_{B_2}{}^{\phi}(uv) = \mu_A(u) \wedge \mu_A(v) - \mu_{B2}(uv) \; ; \; \nu_{B_2}{}^{\phi}(uv) = \nu_A(u) \vee \nu_A(v) - \nu_{B_2}(uv).$$ Thus, we have

$$\mu_{B_1}{}^{\phi}(u_0u_1) = 0 \; ; \; \nu_{B_1}{}^{\phi}(u_0u_1) = 0.1 \text{ and } \mu_{B_1}{}^{\phi}(u_0u_2) = 0.1 \; ; \; \nu_{B_1}{}^{\phi}(u_0u_2) = 0, \mu_{B_1}{}^{\phi}(u_3u_4) = 0.1 \; ;$$
$$\nu_{B_1}{}^{\phi}(u_3u_4) = 0.2 \text{ and } \mu_{B_2}{}^{\phi}(u_1u_2) = 0 \; ; \nu_{B_2}{}^{\phi}(u_1u_2) = 0.1, \mu_{B_2}{}^{\phi}(u_4u_5) = 0.1 \; ; \; \nu_{B_2}{}^{\phi}(u_4u_5) = 0.1.$$

Remark (3.3): Here in the above example, we can check that $(\tilde{G}^{\phi})^{\phi} = \tilde{G}$ i.e., the ϕ - complement of ϕ - complement of $\tilde{G}$ is $\tilde{G}$.

Theorem (3.4): If ϕ is a cyclic permutation on $\{B_1, B_2,...,B_k\}$ of order m ($1 \leq m \leq k$), then $\tilde{G}^{\phi^m} = \tilde{G}$.

Proof. Since ϕ^m = identity permutation. Hence, $\tilde{G}^{\phi^m} = (A, B_1^{\phi^m}, B_2^{\phi^m}, ..., B_k^{\phi^m}) = (A, B_1, B_2, ..., B_k) = \tilde{G}$.

Proposition (3.5): Let $\tilde{G}$ = (A, B_1, B_2,...,B_k) be an intuitionistic fuzzy graph structure of graph structure G = (V, R_1, R_2, ..., R_k) and let ϕ and ψ be two permutations on {B_1, B_2, ..., B_k}, then

$\left(\tilde{G}^{\phi}\right)^{\psi} = \tilde{G}^{(\phi o \psi)}$. In particular $\tilde{G}^{(\phi o \psi)} = \tilde{G}$ if and only if ϕ and ψ are inverse of each other.

Proof: Straight forward.

Definition (3.6): Let $\tilde{G}$ = (A, B_1, B_2,...,B_k) and $\tilde{G}'$ = (A′, B'_1, B'_2,...,B'_k) be two IFGSs on graph structures G = (V, R_1, R_2,...,R_k) and G′ = (V′, R_1', R_2',...,R_k') respectively, then $\tilde{G}$ is isomorphic to $\tilde{G}'$ if there exists a bijective mapping f : V→ V′ and a permutation ϕ on {B_1, B_2, ...,B_k } such that ϕ(B_i) = B'_j and
(1) ∀ u ∈ V, $\mu_A(u) = \mu_{A'}(f(u))$ and $\nu_A(u) = \nu_{A'}(f(u))$
(2) ∀ (uv) ∈ R_i , $\mu_{B_i}(\mathrm{uv}) = \mu_{B'_j}(f(u)f(v))$ and $\nu_{B_i}(\mathrm{uv}) = \nu_{B'_j}(f(u)f(v))$.
In particular, if V = V′, A = A′ and B_i = B_i' for all i = 1, 2, 3, …, k, then the above two IFGSs $\tilde{G}$ and $\tilde{G}'$ are identical.

Remark (3.7): Note that identical IFGSs are always isomorphic, but converse is not true. (In example (3.10), IFGSs $\tilde{G}$ and $\tilde{G}^{\phi}$ are isomorphic but not identical).

Example (3.8): Consider the two intuitionistic fuzzy graph structures $\tilde{G}$ = (A, B_1, B_2) and $\tilde{G}'$ = (A′, B'_1, B'_2) such that V ={u_0, u_1, u_2, u_3, u_4, u_5} and V′ = { u_0', u_1', u_2', u_3', u_4', u_5'}. Let A = {< u_0, 0.8,0.2 >, < u_1, 0.9,0.1>, < u_2 , 0.6,0.3 >, <u_3, 0.5,0.4> , <u_4, 0.6,0.1> , <u_5, 0.7,0.2 > } be IFS on V and A′ = {<u_0', 0.7,0.2 > ,< u_1', 0.8,0.2 >, < u_2', 0.9,0.1>, < u_3', 0.6,0.3 >, <u_4', 0.5,0.4> , <u_5', 0.6,0.1>} be IFS on V′. Let B_1 = { (u_0, u_1), 0.8,0.1 > , < (u_0, u_2), 0.5,0.3 >, < (u_3, u_4), 0.4,0.2 >}, B_2 = { < (u_1, u_2), 0.6,0.2 >,<(u_4, u_5),0.5,0.1 > be IFRs on V and B_1' = { (u_1', u_2'), 0.8,0.1 > , < (u_1', u_3'), 0.5,0.3 >, < (u_4', u_5'), 0.4,0.2 >}, B_2' = { < (u_2', u_3'), 0.6,0.2 >,<(u_5', u_0'),0.5,0.1 > be IFRs on V′ as shown in Fig 1 and Fig 2 respectively.

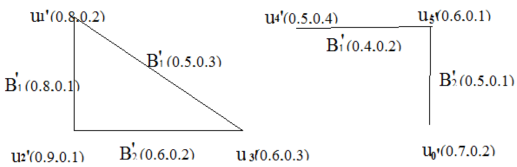


Fig 2

Then it can be easily verified that $\tilde{G}$ = (A, B_1, B_2) and $\tilde{G}'$ = (A′, B'_1, B'_2) are IFGSs. Let ϕ be a permutation on {B_1, B_2} such that $\phi(B_i) = B'_i$ and $h : V \to V'$ be a map defined by

$$h(u_k) = \begin{cases} u_{k+1}' \text{ if } k = 0,1,2,3,4 \\ u_k' \text{ if } k = 5. \end{cases}$$

Then it can be easily checked that

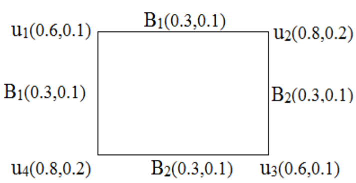
(1) $\mu_A(u_k) = \mu_{A'}(h(u_k))$ and $\nu_A(u_k) = \nu_{A'}(h(u_k))$ $\forall\, u_k \in V$

(2) $\mu_{B_i}(uv) = \mu_{B_i'}(h(u)h(v))$ *and* $\nu_{B_i}(uv) = \nu_{B_i'}(h(u)h(v)), \forall (u\, v) \in V \times V$, and i= 1, 2.

Hence $\tilde{G} \cong \tilde{G}'$.

Definition (3.9): Consider an IFGS $\tilde{G}$ of graph structure G and ϕ is a permutation on the set {B_1, B_2,...,B_k} then $\tilde{G}$ is ϕ - self complementary if $\tilde{G}$ is isomorphic to $\tilde{G}^{\phi}$ and $\tilde{G}$ is strong ϕ- self complementary if $\tilde{G}$ is identical to $\tilde{G}^{\phi}$.

Example (3.10): Consider an IFGS $\tilde{G}$ = (A, B_1, B_2) such that V ={ u_1, u_2, u_3, u_4}. Let A = {< u_1, 0.6,0.1>, < u_2 , 0.8,0.2>, <u_3, 0.6,0.1>, < u_4 , 0.8,0.2>}and B_1 = {< (u_1, u_2), 0.3,0.1>, < (u_4, u_1), 0.3,0.1>}, B_2 = {< (u_2, u_3), 0.3, 0.1>, < (u_4, u_3), 0.3, 0.1>}.



Let ϕ be a permutation on the set {B_1, B_2} defined by $\phi(B_1) = B_2$ and $\phi(B_2) = B_1$, then

$\mu_{B_1}{}^{\phi}(u_1u_2) = 0.3$; $\nu_{B_1}{}^{\phi}(u_1u_2) = 0.1$, and $\mu_{B_1}{}^{\phi}(u_4u_1) = 0.3$; $\nu_{B_1}{}^{\phi}(u_4u_1) = 0.1$

and $\mu_{B_2}{}^{\phi}(u_2u_3) = 0.3$; $\nu_{B_2}{}^{\phi}(u_2u_3) = 0.1$ and $\mu_{B_2}{}^{\phi}(u_4u_3) = 0.3$; $\nu_{B_2}{}^{\phi}(u_4u_3) = 0.1$.

Let there exists a one - one and onto map h: V →V defined by

$h(u_1) = u_3$; $h(u_2) = u_4$; $h(u_3) = u_1$ and $h(u_4) = u_2$. Then

$$\mu_A(h(u_1)) = \mu_A(u_3) = 0.6 = \mu_A(u_1) \text{ and } \nu_A(h(u_1)) = \nu_A(u_3) = 0.1 = \nu_A(u_1);$$
$$\mu_A(h(u_2)) = \mu_A(u_4) = 0.8 = \mu_A(u_2) \text{ and } \nu_A(h(u_2)) = \nu_A(u_4) = 0.2 = \nu_A(u_2);$$
$$\mu_A(h(u_3)) = \mu_A(u_1) = 0.6 = \mu_A(u_3) \text{ and } \nu_A(h(u_3)) = \nu_A(u_1) = 0.1 = \nu_A(u_3);$$
$$\mu_A(h(u_4)) = \mu_A(u_2) = 0.8 = \mu_A(u_4) \text{ and } \nu_A(h(u_4)) = \nu_A(u_2) = 0.2 = \nu_A(u_4).$$
$$\mu_{B_1}{}^{\phi}(h(u_1)h(u_2)) = \mu_{B_2}(u_3u_4) = 0.3 = \mu_{B_1}(u_1u_2) \text{ and } \nu_{B_1}{}^{\phi}(h(u_1)h(u_2)) = \nu_{B_2}(u_3u_4) = 0.1 = \nu_{B_1}(u_1u_2);$$
$$\mu_{B_1}{}^{\phi}(h(u_1)h(u_4)) = \mu_{B_2}(u_3u_2) = 0.3 = \mu_{B_1}(u_1u_4) \text{ and } \nu_{B_1}{}^{\phi}(h(u_1)h(u_4)) = \nu_{B_2}(u_3u_2) = 0.1 = \nu_{B_1}(u_1u_4);$$
$$\mu_{B_2}{}^{\phi}(h(u_2)h(u_3)) = \mu_{B_1}(u_4u_1) = 0.3 = \mu_{B_2}(u_2u_3) \text{ and } \nu_{B_2}{}^{\phi}(h(u_2)h(u_3)) = \nu_{B_1}(u_4u_1) = 0.1 = \nu_{B_2}(u_2u_3);$$
$$\mu_{B_2}{}^{\phi}(h(u_4)h(u_3)) = \mu_{B_1}(u_2u_1) = 0.3 = \mu_{B_2}(u_4u_3) \text{ and } \nu_{B_2}{}^{\phi}(h(u_4)h(u_3)) = \nu_{B_1}(u_2u_1) = 0.1 = \nu_{B_2}(u_4u_3).$$

$\therefore$ $\tilde{G}$ is ϕ- self complementary.

Definition (3.11): Consider an IFGS $\tilde{G}$ of graph structure G then

(1) $\tilde{G}$ is self complementary(SC) if $\tilde{G}$ is isomorphic to $\tilde{G}^{\phi}$ for some permutation ϕ.

(2) $\tilde{G}$ is strong self complementary(SSC) if $\tilde{G}$ is identical to $\tilde{G}^{\phi}$ for some permutation ϕ other than the identity permutation.

(3) $\tilde{G}$ is totally self complementary (TSC) if $\tilde{G}$ is isomorphic to $\tilde{G}^{\phi}$ for every permutation ϕ.

(4) $\tilde{G}$ is totally strong self complementary (TSSC) if $\tilde{G}$ is identical to $\tilde{G}^{\phi}$ for every permutation ϕ, where ϕ is a permutation on the set $\{B_1, B_2, ..., B_k\}$.

Remark (3.12): Totally self complementary $\Rightarrow$ self complementary and totally strong self complementary $\Rightarrow$ strong self complementary, but converse is not true, as is obvious from the following example (3.14).

Remark (3.13):

$$\begin{tikzcd} \text{TSSC} \arrow[r, Rightarrow] \arrow[d, Rightarrow] & \text{SSC} \arrow[d, Rightarrow] \\ \text{TSC} \arrow[r, Rightarrow] & \text{SC} \end{tikzcd}$$

Example (3.14) : Consider an intuitionistic fuzzy graph structure $\tilde{G} = (A, B_1, B_2)$ such that $V = \{u_1, u_2, u_3, u_4\}$. Let $R_1 = \{(u_1, u_2), (u_4, u_1)\}$, $R_2 = \{(u_2, u_3), (u_4, u_3)\}$, $A = \{< u_1, 0.4, 0.2 >, < u_2, 0.5, 0.1>, <u_3, 0.4, 0.2 >, < u_4, 0.5, 0.1>\}$, $B_1 = \{ < (u_1, u_2), 0.2, 0.1 >, < (u_4, u_1), 0.2, 0.1 >\}$, $B_2 = \{ < (u_2, u_3), 0.2, 0.1 >, < (u_4, u_3), 0.2, 0.1 >\}$. Let ϕ be a permutation on the set $\{B_1, B_2\}$ defined by $\phi(B_1) = B_2$ and $\phi(B_2) = B_1$, then

$\mu_{B_1}{}^{\phi}(u_1u_2)=0.2$; $\nu_{B_1}{}^{\phi}(u_1u_2)=0.1,$ and $\mu_{B_1}{}^{\phi}(u_4u_1)=0.2$; $\nu_{B_1}{}^{\phi}(u_4u_1)=0.1$ and $\mu_{B_2}{}^{\phi}(u_2u_3)=0.2$; $\nu_{B_2}{}^{\phi}(u_2u_3)=0.1$ and $\mu_{B_2}{}^{\phi}(u_4u_3)=0.2$; $\nu_{B_2}{}^{\phi}(u_4u_3)=0.1.$

Let there exists a one - one and onto map h: V $\rightarrow$V defined by
$h(u_1)=u_3; h(u_2)=u_4$; $h(u_3)=u_1$ and $h(u_4)=u_2$

$\mu_A(h(u_1))=\mu_A(u_3)=0.4=\mu_A(u_1)$ and $\nu_A(h(u_1))=\nu_A(u_3)=0.1=\nu_A(u_1)$;

$\mu_A(h(u_2))=\mu_A(u_4)=0.5=\mu_A(u_2)$ and $\nu_A(h(u_2))=\nu_A(u_4)=0.2=\nu_A(u_2)$;

$\mu_A(h(u_3))=\mu_A(u_1)=0.4=\mu_A(u_3)$ and $\nu_A(h(u_3))=\nu_A(u_1)=0.1=\nu_A(u_3)$;

$\mu_A(h(u_4))=\mu_A(u_2)=0.5=\mu_A(u_4)$ and $\nu_A(h(u_4))=\nu_A(u_2)=0.2=\nu_A(u_4)$.

$\mu_{B_2}{}^{\phi}(h(u_1)h(u_2))=\mu_{B_2}(u_3u_4)=0.2=\mu_{B_2}(u_1u_2)$ and $\nu_{B_2}{}^{\phi}(h(u_1)h(u_2))=\nu_{B_2}(u_3u_4)=0.1=\nu_{B_2}(u_1u_2)$;

$\mu_{B_2}{}^{\phi}(h(u_1)h(u_4))=\mu_{B_2}(u_3u_2)=0.2=\mu_{B_2}(u_1u_4)$ and $\nu_{B_2}{}^{\phi}(h(u_1)h(u_4))=\nu_{B_2}(u_3u_2)=0.1=\nu_{B_2}(u_1u_4)$;

$\mu_{B_1}{}^{\phi}(h(u_2)h(u_3))=\mu_{B_1}(u_4u_1)=0.2=\mu_{B_1}(u_2u_3)$ and $\nu_{B_1}{}^{\phi}(h(u_2)h(u_3))=\nu_{B_1}(u_4u_1)=0.1=\nu_{B_1}(u_2u_3)$;

$\mu_{B_2}{}^{\phi}(h(u_4)h(u_3))=\mu_{B_2}(u_2u_1)=0.2=\mu_{B_2}(u_4u_3)$ and $\nu_{B_1}{}^{\phi}(h(u_4)h(u_3))=\nu_{B_2}(u_2u_1)=0.1=\nu_{B_2}(u_4u_3)$.

$\therefore$ $\tilde{G}$ is ϕ- self complementary and hence $\tilde{G}$ is self complementary.
Let ϕ be another permutation on the set {B_1, B_2} defined by $\phi(B_1) = B_1$ and $\phi(B_2) = B_2$,
Let there exists a one - one and onto map h: V $\rightarrow$V such that
$h(u_1)=u_1$, $h(u_2)=u_4$, $h(u_3)=u_3$ and $h(u_4)=u_2$. Then $\tilde{G}$ is not isomorphic to $\tilde{G}^{\phi}$.
$\therefore$ $\tilde{G}$ is not totally self complementary.

Theorem (3.15): Let $\tilde{G}$ be self complementary IFGS, for some permutation ϕ on the set {B_1, B_2,...,B_k} then for each i = 1, 2, 3 ,….. k, we have

$$\sum_{u\neq v}\mu_{B_i}(uv)+\sum_{u\neq v}\sum_{j\neq i}(\phi\mu_{B_j})(uv)=\sum_{u\neq v}(\mu_A(u)\wedge\mu_A(v)) \quad \text{and}$$
$$\sum_{u\neq v}\nu_{B_i}(uv)+\sum_{u\neq v}\sum_{j\neq i}(\phi\nu_{B_j})(uv)=\sum_{u\neq v}(\nu_A(u)\vee\nu_A(v)) .$$

Proof: Given $\tilde{G}$ = (A, B_1, B_2,...,B_k) is ϕ- self complementary IFGS. Therefore, there exists a one - one and onto map h: V $\rightarrow$V such that
$\mu_A(h(u))=\mu_A(u)$ and $\nu_A(h(u))=\nu_A(u)$, where
$\mu_{B_j}{}^{\phi}(h(u)h(v))=\mu_{B_j}(uv)$ and $\nu_{B_j}{}^{\phi}(h(u)h(v))=\nu_{B_j}(uv)$ $\forall u, v\in V$ and $j=1,2,...,k$.

By definition of ϕ- complement of IFGS, we have

$$\mu_{B_i}{}^{\phi}(h(u)h(v)) = \mu_A\left(h(u)\right) \wedge \mu_A\left(h(v)\right) - \sum_{j\neq i}(\phi\mu_{B_j})(h(u)h(v)) \text{ and}$$

$$\nu_{B_i}{}^{\phi}(h(u)h(v)) = \nu_A(h(u)) \vee \nu_A\left(h(v)\right) - \sum_{j\neq i}(\phi\nu_{B_j})(h(u)h(v))$$

$$\Rightarrow \mu_{B_i}(uv) = \mu_A(u) \wedge \mu_A(v) - \sum_{j\neq i}(\phi\mu_{B_j})(h(u)h(v)) \text{ and}$$

$$\nu_{B_i}(uv) = \nu_A(u) \vee \nu_A(v) - \sum_{j\neq i}(\phi\nu_{B_j})(h(u)h(v)) \text{ .}$$

$$Now, \quad \sum_{u\neq v}\mu_{B_i}(uv) = \sum_{u\neq v}(\mu_A(u) \wedge \mu_A(v)) - \sum_{u\neq v}\sum_{j\neq i}(\phi\mu_{B_j})(h(u)h(v)) \text{ and}$$

$$\sum_{u\neq v}\nu_{B_i}(uv) = \sum_{u\neq v}(\nu_A(u) \vee \nu_A(v)) - \sum_{u\neq v}\sum_{j\neq i}(\phi\nu_{B_j})(h(u)h(v))$$

$$\Rightarrow \sum_{u\neq v}\mu_{B_i}(uv) = \sum_{u\neq v}(\mu_A(u) \wedge \mu_A(v)) - \sum_{u\neq v}\sum_{j\neq i}(\phi\mu_{B_j})(uv) \text{ and}$$

$$\sum_{u\neq v}\nu_{B_i}(uv) = \sum_{u\neq v}(\nu_A(u) \vee \nu_A(v)) - \sum_{u\neq v}\sum_{j\neq i}(\phi\nu_{B_j})(uv)$$

$$\Rightarrow \sum_{u\neq v}\mu_{B_i}(uv) + \sum_{u\neq v}\sum_{j\neq i}(\phi\mu_{B_j})(uv) = \sum_{u\neq v}(\mu_A(u) \wedge \mu_A(v)) \text{ and}$$

$$\sum_{u\neq v}\nu_{B_i}(uv) + \sum_{u\neq v}\sum_{j\neq i}(\phi\nu_{B_j})(uv) = \sum_{u\neq v}(\nu_A(u) \vee \nu_A(v)).$$

Remark (3.16): The result of Theorem (3.15) holds for a strong self complementary IFGS $\tilde{G}$, by using the identity mapping as the isomorphism.

Corollary (3.17): If an IFGS $\tilde{G}$ is totally self complementary, then

$$\sum_{u\neq v}\sum_{j}(\mu_{B_j})(uv) = \sum_{u\neq v}(\mu_A(u) \wedge \mu_A(v)) \text{ and } \sum_{u\neq v}\sum_{j}(\nu_{B_j})(uv) = \sum_{u\neq v}(\nu_A(u) \vee \nu_A(v))$$

Proof: By Theorem (3.15), we have

$$\sum_{u\neq v}\mu_{B_i}(uv) + \sum_{u\neq v}\sum_{j\neq i}(\phi\mu_{B_j})(uv) = \sum_{u\neq v}(\mu_A(\mathrm{u}) \wedge \mu_A(v)) \text{ and}$$

$$\sum_{u\neq v}\nu_{B_i}(uv) + \sum_{u\neq v}\sum_{j\neq i}(\phi\nu_{B_j})(uv) = \sum_{u\neq v}(\nu_A(u) \vee \nu_A(v)) \text{, hold for every permuation } \phi.$$

Using the identity permutation ϕ, we have

$$\sum_{u\neq v}\sum_{j}(\phi\mu_{B_j})(uv) = \sum_{u\neq v}(\mu_A(u) \wedge \mu_A(v)) \text{ and } \sum_{u\neq v}\sum_{j}(\phi\nu_{B_j})(uv) = \sum_{u\neq v}(\nu_A(u) \vee \nu_A(v))$$

i.e., the sum of the membership (non- membership) of all B_i- edges i= 1,2,3,….,k, is equal to the sum of the minimum (maximum) of the membership (non-membership) of the corresponding vertices.

Corollary (3.18): If IFGS $\tilde{G}$ is totally strong self complementary,then the above result also holds.

Theorem (3.19): In an IFGS $\tilde{G}$, if for all u, v ∈ V, we have
$\mu_{B_i}(uv)+\sum_{j\neq i}(\phi\mu_{B_j})(uv)=\mu_A(u)\wedge\mu_A(v)$ and $\nu_{B_i}(uv)+\sum_{j\neq i}(\phi\nu_{B_j})(uv)=\nu_A(u)\vee\nu_A(v)$,
then $\tilde{G}$ is self complementary for a permutation ϕ on the set {B_1, B_2,...,B_k}.

Proof: Let h: V →V be the identity map. Therefore,
$\mu_A(h(u))=\mu_A(u)$ and $\nu_A(h(u))=\nu_A(u)$.
By definition of ϕ- complement of IFGS, we have

$$\mu_{B_i}{}^{\phi}(h(u)h(v))=\mu_A(h(u))\wedge\mu_A(h(v))-\sum_{j\neq i}(\phi\mu_{B_j})(h(u)h(v)) = \mu_A(u)\wedge\mu_A(v)-\sum_{j\neq i}(\phi\mu_{B_j})(uv)$$

$$=(\mu_{B_i}(uv)+\sum_{j\neq i}(\phi\mu_{B_j})(uv))-\sum_{j\neq i}(\phi\mu_{B_j})(uv) = \mu_{B_i}(uv)$$

and $$\nu_{B_i}{}^{\phi}(h(u)h(v))=\nu_A(h(u))\wedge\nu_A(h(v))-\sum_{j\neq i}(\phi\nu_{B_j})(h(u)h(v))= \nu_A(u)\wedge\nu_A(v)-\sum_{j\neq i}(\phi\nu_{B_j})(uv)$$

$$=(\nu_{B_i}(uv)+\sum_{j\neq i}(\phi\nu_{B_j})(uv))-\sum_{j\neq i}(\phi\nu_{B_j})(uv)=\nu_{B_i}(uv).$$

∴ $\tilde{G}$ is ϕ- self complementary. Hence $\tilde{G}$ is self complementary for some permutation ϕ.

Corollary (3.20): In an IFGS $\tilde{G}$, if ∀ u, v ∈ V, we have
$\mu_{B_i}(uv)+\sum_{j\neq i}(\phi\mu_{B_j})(uv)=\mu_A(u)\wedge\mu_A(v)$ and $\nu_{B_i}(uv)+\sum_{j\neq i}(\phi\nu_{B_j})(uv)=\nu_A(u)\vee\nu_A(v)$,
for every permutation ϕ on the set {B_1, B_2,...,B_k} then $\tilde{G}$ is totally self complementary.

4. Conclusion

The complement of intuitionistic fuzzy graph plays an important role in the further development of the theory. Similarly the concept of ϕ-complementary IFGS is significant.

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