International Mathematical Forum, Vol. 8, 2013, no. 32, 1577 - 1584
HIKARI Ltd, www.m-hikari.com
http://dx.doi.org/10.12988/imf.2013.36122

The Dynamics of a Delayed Pest Management SEI System with Impulsive Harvesting at Different Moments

Yongliang Li[+]

Department of Mathematics
Hubei University for Nationalities
Enshi, 445000, China

Abstract. According to biological strategy for pest control, we consider a delayed pest management SEI model with birth pulse and impulsive harvesting at different moments. We prove that all solutions of the system are uniformly ultimately bounded and get the conditions of the globally attractive infection-free boundary periodic solution of the system. Further, we obtain sufficient condition with time delay for the permanence of the system. Our results give some reasonable suggestions for pest management.

Keywords: pest management ; impulsive harvesting; extinction; permanence

1. Introduction

System with impulsive effects describing evolution processes are characterized by the fact that at certain moments of time they abruptly experience a change of state. Processes of such character are studied in almost every domain of applied science and has been studied in many investigation [1]. In [2], Xiang et al. studied a pest management SEI model.

In the following, based on the paper [2], we consider a delayed pest management SEI model with birth pulse and impulsive harvesting on susceptible at different moments

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[+] Corresponding author: *E-mail address*:jwliyongliang@163.com

$$\begin{cases} \left.\begin{aligned} &\dot{S} = -\beta S(t)I(t) - \mu S(t), \\ &\dot{E} = \beta S(t)I(t) - e^{-\mu\tau}\beta S(t-\tau)I(t-\tau) - \mu E(t), \\ &\dot{I} = e^{-\mu\tau}\beta S(t-\tau)I(t-\tau) - \mu I(t) - \alpha I(t) \end{aligned}\right\} t \neq (n+l)T, t \neq (n+1)T, \\ \left.\begin{aligned} &\Delta S(t) = a, \\ &\Delta E(t) = 0, \Delta I(t) = 0, \end{aligned}\right\} t = (n+l)T, n = 1, 2, \ldots\ldots, \\ \left.\begin{aligned} &\Delta S(t) = -pS(t), \\ &\Delta E(t) = 0, \Delta I(t) = 0, \end{aligned}\right\} t = (n+1)T, n = 1, 2, \ldots\ldots, \end{cases} \tag{1.1}$$

the initial condition for (1.1) are

$$(\eta_1(\varsigma), \eta_2(\varsigma), \eta_3(\varsigma)) \in C([-\tau, 0], R_+^3), \eta_i(0) > 0, i = 1, 2, 3. \tag{1.2}$$

Where $S(t), E(t)$ and $I(t)$ are densities of susceptible, exposed, infectious pests at time t, respectively. μ is the natural death rate of the susceptible, exposed and infectious pests. τ is the latent period of the disease, β is the contact rate, α is the death rate because of disease(the disease-related death rate). The pulse birth and impulsive harvesting occurs every T period. The constant $a > 0$ represents the birth pulse effort of pest population at $t = (n+l)T, 0 < l < 1, n \in Z_+$. β, μ, τ, T, p are positive constants. $0 \le p < 1$ represents the harvesting effort of the susceptible pests at $t = (n+1)T, n \in Z_+$. $\Delta S(t) = S(t^+) - S(t)$, $\Delta E(t) = E(t^+) - E(t), \Delta I(t) = I(t^+) - I(t)$.

Since the equation for $E(t)$ of the system (1.1) is independent of other equations. We simplify system (1.1) and restrict our attention to the following system:

$$\begin{cases} \left.\begin{aligned} &\dot{S} = -\beta S(t)I(t) - \mu S(t), \\ &\dot{I} = e^{-\mu\tau}\beta S(t-\tau)I(t-\tau) - \mu I(t) - \alpha I(t) \end{aligned}\right\} t \neq (n+l)T, t \neq (n+1)T, \\ \left.\begin{aligned} &\Delta S(t) = a, \\ &\Delta I(t) = 0, \end{aligned}\right\} t = (n+l)T, n = 1, 2, \ldots\ldots, \\ \left.\begin{aligned} &\Delta S(t) = -pS(t), \\ &\Delta I(t) = 0, \end{aligned}\right\} t = (n+1)T, n = 1, 2, \ldots\ldots, \end{cases} \tag{1.3}$$

the initial condition for (1.3) are

$$(\eta_1(\varsigma), \eta_3(\varsigma)) \in C([-\tau, 0], R_+^2), \eta_i(0) > 0, i = 1, 3. \tag{1.4}$$

2. The dynamics

Before this section, we will give some lemmas which be will be useful in the following proof.

Lemma 2.1 There exists a constant $M > 0$ such that $S(t) \le M$, $E(t) \le M$, $I(t) \le M$ for each solution $(S(t), E(t), I(t))$ of system (1.1) with all t large enough.

Lemma 2.2 [3] Consider the delay equation: $\dot{x}(t) = a_1 x(t-\tau) - a_2 x(t)$, where $a_1, a_2, \tau > 0$ for $-\tau \le t \le 0$ .We have

(i) if $a_1 < a_2$,then $\lim_{t\to\infty} x(t) = 0$, (ii) if $a_1 > a_2$,then $\lim_{t\to\infty} x(t) = +\infty$.

In this section, we will firstly obtain the sufficient condition of the stability of infection-free periodic solution of system (1.1) with (1.2).

If the absence of $I(t)$, the system (1.3) reduces to

$$\begin{cases} \dot{S}(t) = -\mu S(t), & t \ne (n+l)T, t \ne (n+1)T, \\ \Delta S(t) = a, & t = (n+l)T, \\ \Delta S(t) = -pS(t), & t = (n+1)T, n \in Z^+. \end{cases} \tag{2.1}$$

By calculation, we can get the analytic solution of system (3.1) between pulses, i.e.

$$S(t) = \begin{cases} S(nT^+) e^{-\mu(t-nT)}, & t \in [nT, (n+l)T), \\ \left(S(nT^+) e^{-\mu lT} + a\right) e^{-\mu(t-(n+l)T)}, & t \in [(n+l)T, (n+1)T). \end{cases}$$

Considering the last two equations of system (2.1), we have the stroboscopic map of system (2.1) $S((n+1)T^+) = (1-p)e^{-\mu T} S(nT^+) + a(1-p)e^{-\mu(1-l)T}$.

Let $F(S) = (1-p)e^{-\mu T} S(nT^+) + a(1-p)e^{-\mu(1-l)T}$, there exists only one positive fixed point $A_1(S^*)$,where $S^* = a(1-p)e^{-\mu(1-l)T} \big/ \left[1-(1-p)e^{-\mu T}\right]$. The positive fixed $A_1(S^*)$ is locally stable since $\left| dF(S)/dS \right|_{S=S^*} = (1-p)e^{-\mu T} < 1$.

Further, we can show $A_1(S^*)$ is globally stable, this is true when the following statements are satisfied [4]:

(i) if $S^* > S > 0$,then $S^* > F(S) > S$,(ii) if $S > S^*$,then $S > F(S) > S^*$.

By calculation, we have $dF/dS = (1-p)e^{-\mu T} > 0$,which deduces $S^* > F(S)$,when $S^* > S > 0$,if $p > 1 - e^{\mu T}$ we know $F(S) - S > \left[(1-p)e^{-\mu T} - 1\right] S^* + a(1-p)e^{-\mu(1-l)T}$ $= 0$,this yields $F(S) > S$ .So we have $S^* > F(S) > S$ under the assumption $S^* > S > 0$, otherwise $S > F(S) > S^*$ if $S > S^*$ .Thus the statement (i) and (ii) are satisfied.

Summarizing above results, we have the following propositions:

Theorem 2.1 If $p > 1 - e^{\mu T}$,the positive equilibrium $A_1(S^*)$ is globally stable, correspondingly, system (2.1) has a globally stable positive periodic solution $\tilde{S}(t)$,where

$$\tilde{S}(t) = \begin{cases} \dfrac{a(1-p)e^{-\mu(1-l)T}}{1-(1-p)e^{-\mu T}} e^{-\mu(t-nT)}, & t \in [nT, (n+l)T), \\ \dfrac{a}{1-(1-p)e^{-\mu T}} e^{-\mu(t-(n+l)T)}, & t \in [(n+l)T, (n+1)T). \end{cases}$$

Theorem 2.2 If $p > 1 - e^{\mu T}$ and $R_1 < 1$ hold,where $R_1 = \dfrac{a\beta e^{-\mu\tau}}{(\mu+\alpha)\left[1-(1-p)e^{-\mu T}\right]}$,the infection-free periodic solution $(\tilde{S}(t), 0, 0)$ of system (1.1) with (1.2) is globally attractive.

Proof. It is easy to see that the global attraction of infection-free periodic solution $(\tilde{S}(t), 0, 0)$ of system(1.1) with (1.2) is equivalent to the global attraction ofinfection-free periodic solution $(\tilde{S}(t), 0, 0)$ of system(1.3). So we only devote to system (1.3)with (1.4). Since $R_1 < 1$,so $e^{\mu\tau} > \dfrac{a\beta}{(\mu+\alpha)\left[1-(1-p)e^{-\mu T}\right]}$,we can choose ε_0 sufficiently small such that

$$e^{\mu\tau} > \beta\left[\frac{a}{1-(1-p)e^{-\mu T}} + \varepsilon_0\right] \Big/ (\mu+\alpha). \tag{2.2}$$

It follows from the first equation of system (1.3) with (1.4) that $\dot{S}(t) \le -\mu S(t)$ .So we consider the following comparison impulsive differential system

$$\begin{cases} \dot{x}(t) = -\mu x(t), & t \neq (n+l)T, t \neq (n+1)T, \\ \Delta x(t) = a, & t = (n+l)T, \\ \Delta x(t) = -px(t), & t = (n+1)T, n \in Z^+. \end{cases} \tag{2.3}$$

By $p > 1 - e^{\mu T}$ and Theorem 2.1, we obtain that the periodic solution of system (2.3)

$$\tilde{x}(t)=\begin{cases}\dfrac{a(1-p)e^{-\mu(1-l)T}}{1-(1-p)e^{-\mu T}}e^{-\mu(t-nT)}, & t\in\left[nT,(n+l)T\right),\\ \dfrac{a}{1-(1-p)e^{-\mu T}}e^{-\mu(t-(n+l)T)}, & t\in\left[(n+l)T,(n+1)T\right).\end{cases}$$

is globally stable.

By the comparison theorem of impulsive equation (see Theorem 3.1.1 in [5]), we have $S(t)<x(t)$ and $x\to\tilde{x}(t)=\tilde{S}(t)$ as $t\to\infty$ .Then there exists an integer $k_1>k_0, t>k_1$ such that $S(t)\le x(t)\le \tilde{x}(t)+\varepsilon_0, nT<t\le(n+1)T, n>k_1$.

$$S(t)<\tilde{S}(t)+\varepsilon_0\le a\Big/\left[1-(1-p)e^{-\mu T}\right]+\varepsilon_0=:\rho,\ nT<t\le(n+1)T, n>k_1.$$

From the second equation of the system (1.3), we have $\dot{I}(t)\le e^{-\mu\tau}\beta\rho I(t-\tau)-(\mu+\alpha)I(t), t>nT+\tau, n>k_1$ .Consider the following comparison differential equation $\dot{y}(t)\le e^{-\mu\tau}\beta\rho I(t-\tau)-(\mu+\alpha)I(t), t>nT+\tau, n>k_1$.

From (2.2), we have $e^{-\mu\tau}\beta\rho<\mu+\alpha$,according to Lemma 2.2 we have $\lim_{t\to\infty} y(t)=0$.

Let $(S(t), I(t))$ be the solution of system (1.3) with initial conditions (1.4) and $I(\varsigma)=\eta_3(\varsigma)(\varsigma\in[-\tau,0])$, $y(t)$ is the solution of system (3.1) with initial condition $y(\varsigma)=\eta_3(\varsigma)(\varsigma\in[-\tau,0])$ .By the comparison theorem, we have $\lim_{t\to\infty} I(t)<\lim_{t\to\infty} y(t)=0$ .Incorporating into the positivity of $I(t)$,we have $\lim_{t\to\infty} I(t)=0$ .Therefore, for any ε' (sufficiently small), there exists an integer $k_2\,(k_2T>k_1T+\tau)$ such that $I(t)<\varepsilon'$ for all $t>k_2T$.

From the first equation of the system (1.3), we have $-(\beta\varepsilon'+\mu)S(t)\le\dot{S}(t)\le-\mu S(t)$,

Then we have $y_1(t)\le S(t)\le y_2(t)$ and $y_1(t)\to\tilde{S}(t), y_2(t)\to\tilde{S}(t)$ as $t\to\infty$.

While $y_1(t)$ and $y_2(t)$ are the solutions of

$$\begin{cases}\dot{y}_i(t)=-A_i y_i(t), & t\ne(n+l)T, t\ne(n+1)T,\\ \Delta y_i(t)=a, & t=(n+l)T,\\ \Delta y_i(t)=-p y_i(t), & t=(n+1)T, n\in Z^+, i=1,2.\end{cases}\qquad(2.4)$$

Where $A_1=\beta\varepsilon'+\mu$ and $A_2=\mu$.

Similar to system (2.1), we can obtain the periodic solutions of systems (2.4)

$$\tilde{y}_i(t)=\begin{cases}\dfrac{a(1-p)e^{-A_i(1-l)T}}{1-(1-p)e^{-A_iT}}e^{-A_i(t-nT)}, \ t\in[nT,(n+l)T), \\ \dfrac{a}{1-(1-p)e^{-A_iT}}e^{-A_i(t-(n+l)T)}, t\in[(n+l)T,(n+1)T),\end{cases} \qquad i=1,2. \tag{2.5}$$

Therefore, for any $\varepsilon_1,\varepsilon_2>0$,there exists a integer $k_3, n>k_4$ such that

$$\tilde{y}_1(t)-\varepsilon_1<S(t)<\tilde{y}_2(t)+\varepsilon_2, \tag{2.6}$$

Let $\varepsilon'\to 0$,we have $\tilde{S}(t)-\varepsilon_1<S(t)<\tilde{S}(t)+\varepsilon_2$,for t large enough. Let $\varepsilon_1,\varepsilon_2\to 0$,it implies $S(t)\to\tilde{S}(t)$ as $t\to\infty$ .This completes the proof. Our next work is to investigate the permanent of the system (1.3).

Denote $R_2=\dfrac{a(1-p)\beta e^{-\mu(\tau+T)}}{(\mu+\alpha)\left[1-(1-p)e^{-\mu T}\right]}, m_2^*=\dfrac{1}{\beta T}\left[\dfrac{1}{(R_2-1)(1-p)e^{-\mu T}-R_2}\right].$

Theorem 2.3 If $R_2>1$ and $p>1-e^{\mu T}$,then there exist two positive constants m_1 and m_2 such that each positive solution $(S(t),I(t))$ of (1.3) with (1.4) satisfies $I(t)\ge m_2$ for t large enough.

Proof. The second equation of system (1.3) can be rewritten as

$$\dot{I}(t)=\left[-(\mu+\alpha)+\beta e^{-\mu\tau}S(t)\right]I(t)-\beta e^{-\mu\tau}\frac{d}{dt}\int_{t-\tau}^{t}S(u)I(u)du\,. \tag{2.7}$$

Define $V(t)=I(t)+\beta e^{-\mu\tau}\int_{t-\tau}^{t}S(u)I(u)du$ .Calculating the derivative of $V(t)$ along the solution of (1.3), it follows from(2.7) that

$$\dot{V}(t)=\left[-(\mu+\alpha)+\beta e^{-\mu\tau}S(t)\right]I(t). \tag{2.8}$$

Since $R_2>1$ and $p>1-e^{\mu T}$,from the define of m_2^*,we have

$e^{-\beta m_2^* T}=1/\left[R_2-(R_2-1)(1-p)e^{-\mu T}\right]<1$,then $m_2^*>0$ nd there exists a positive constant ε small enough such that

$$\beta\sigma e^{-\mu\tau}/(\mu+\alpha)>1, \tag{2.9}$$

Where $\sigma=a(1-p)e^{-(\beta m_2^*+\mu)T}\Big/\left[1-(1-p)e^{-(\beta m_2^*+\mu)T}\right]-\varepsilon>0$ .For any positive constant t_0 ,we claim that the inequality $I(t)<m_2$ can not hold for all $t\ge t_0$. Otherwise, there is a t_0, such that $I(t)<m_2$ for all $t\ge t_0$. It follows from the first equation of system (1.3) that for all $t\ge t_0$, $\dot{S}(t)>-(\beta m_2^*+\mu)S(t)$ .Then we have $\tilde{z}(t)\le S(t)$ where $\tilde{u}(t)$ is the unique positive periodic solution of

$$\begin{cases} \dot{u}(t) = -\left(\beta m_2^* + \mu\right)u(t), & t \neq (n+l)T, t \neq (n+1)T, \\ \Delta u(t) = a, & t = (n+l)T, \\ \Delta u(t) = -pu(t), & t = (n+1)T, n \in Z^+. \end{cases} \tag{2.10}$$

By using comparison theorem of impulsive differential equation and Theorem 2.2, for any $\varepsilon > 0$ there exists such a $t_1 (> t_0 + \tau)$,for $t \geq t_1$,

$$S(t) > \tilde{u}(t) - \varepsilon > a(1-p)e^{-\left(\beta m_2^* + \mu\right)T} \Big/ \left[1-(1-p)e^{-\left(\beta m_2^* + \mu\right)T}\right] - \varepsilon =: \sigma . \tag{2.11}$$

From (2.8) and (2.11), we have

$$\dot{V}(t) > \left[-(\mu+\alpha) + \beta\sigma e^{-\mu\tau}\right]I(t) = (\mu+\alpha)\left[\beta e^{-\mu\tau} / (\mu+\alpha) - 1\right]I(t), \quad t \geq t_1 \tag{2.12}$$

Let $m' = \min_{t\in[t_1, t_1+\tau]} I(t)$ .We will show that $I(t) \geq m'$ for all $t \geq t_1$ .Otherwise, there exists a $T_1 > 0$ such that $I(t) \geq m'$ for $t \in [t_1, t_1 + \tau + T_1]$, $I(t_1 + \tau + T_1) = m'$ and $\dot{I}(t_1 + \tau + T_1) \leq 0$. Thus from the second equation of system(1.3), (2.11) and (2.12), we easily see that $\dot{I}(t_1 + \tau + T_1) \geq (\mu+\alpha)\left[\beta e^{-\mu\tau} / (\mu+\alpha) - 1\right]m' > 0$,which is a contradiction. Hence we can get that $I(t) \geq m'$ .From (2.9) and (2.12),we have $\dot{V}(t) > (\mu+\alpha)\left[\beta e^{-\mu\tau} / (\mu+\alpha) - 1\right]m' > 0$, for all $t \geq t_1$. This implies that $V(t) \to \infty$ as $t \to \infty$. It is a contradiction to $V(t) \leq M\left(1 + M\tau e^{-\mu\tau}\right)$.Therefore, for any positive constant t_0 , the inequality $I(t) < m_2^*$ cannot hold for all $t \geq t_0$.

By the claim, we are left to consider two case. First, $I(t) \geq m_2^*$ for all t large enough, then our aim is obtained. Second, $I(t)$ is oscillatory about m_2^* . Let $m_2 = \min\left\{m_2^*/2, m_2^* e^{-(\mu+\alpha)\tau}\right\}$.In the following, we shall show that $I(t) \geq m_2$ for all t large enough. The conclusion is evident in first case. For the second case, let t^* and ω satisfy $I(t^*) = I(t^* + \omega) = m_2^*$ and $I(t) < m_2^*$ for all $t^* < t < t^* + \omega$. When t^* is large enough, the inequality $S(t) > \sigma$ hold true for $t^* < t < t^* + \omega$. Since $I(t)$ is uniformly continuous. The positive solutions of (1.3) are ultimately bounded and $I(t)$ is not effected by impulses. Hence there is a T_1 ($0 < T_1 < \tau$ and T_1 is independent of the choice of t^*)such that $I(t) > m_2^*/2$ for all $t^* < t < t^* + T_1$. If $\omega < T_1$, our aim is obtained. If $T_1 < \omega < \tau$, from the second equation of (1.3) we have that $\dot{I}(t) \geq -(\mu+\alpha)I(t)$ for $t^* < t < t^* + T_1$. Then wehave $I(t) \geq m_2^* e^{-(\mu+\alpha)\tau}$ for $t^* < t < t^* + \omega < t^* + \tau$ since $I(t^*) = m_2^*$. It is clear that $I(t) \geq m_2$ for $t^* < t < t^* + T_1$. If $\omega \geq \tau$, by the second equation of (1.3), then we have that $I(t) \ge m_2$ for $t^* < t^* + \omega$. Thus, proceeding exactly as the proof for above claim, we can obtain $I(t) \ge m_2$ for $t^* + \tau < t^* + \omega$. Since the interval $\left[t^*, t^* + \omega\right]$ is arbitrarily chosen (we only need t^* to be large), we get that $I(t) \ge m_2$ for t large enough. In view of our arguments above, the choice of m_2 is independent of the positive solution of (1.3) which satisfies that $I(t) \ge m_2$ for sufficiently large t. This completes the proof.

Theorem 2.4 If $R_2 > 1$ and $p > 1 - e^{\mu T}$, then system(1.3) with (1.4) is permanent.
Proof. Let $\left(S(t), I(t)\right)$ be any solution of system (1.3), from the first equation of system (1.3), we have $\dot{S}(t) \ge -\left(M\beta + \mu\right)S(t)$.By Theorem 2.2, we can easily prove that $\liminf_{t\to\infty} S(t) \ge a(1-p)e^{-(M\beta+\mu)T} \Big/ \left[1-(1-p)e^{-(M\beta+\mu)T}\right] =: m_1$

Set $D = \{(S(t), I(t)) \,|\, m_1 \le S(t) \le M, m_2 \le I(t) \le M\}$.Then D is a bounded compact region in which has positive distance from coordinate hyperplanes. By Theorem2.3 and Theorem 2.4, one obtains that every solution of system (1.3) with initial condition (1.4)eventually enters and remains in the region D. The proof is completed.

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Received: June 11, 2013