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Iterative Algorithms for the Generalized System of Nonlinear Quasi-Mixed Equilibrium Problems and Fixed Point Problems

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Abstract

The purpose of this paper is to introduce a new generalized system of nonlinear quasi-mixed equilibrium problems and find the common solutions of this problem and a fixed point problem. The results obtained in this paper may be viewed as an extension, refinement and improvement of the previously known results in [2-4] and others.

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1 Introduction and Preliminaries

Let $\mathcal{H}$ be a real Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and norm $\| \cdot \|$. Let $\Phi_1, \Phi_2 : \mathcal{H} \times \mathcal{H} \to \mathcal{H}$ be given two bi-functions. Let $T_i : \mathcal{H} \times \mathcal{H} \to \mathcal{H}$ be a nonlinear mapping for each $i = 1, 2$, and s, t be two fixed positive real numbers. Let $\mathcal{CC}(\mathcal{H})$ be the family of all nonempty closed convex subsets of $\mathcal{H}$ and $C_i : \mathcal{H} \to \mathcal{CC}(\mathcal{H})$ be a point-to-set mapping which associates a nonempty closed convex set $C_i(x)$ with any element x of $\mathcal{H}$, for each $i = 1, 2$. We consider the problem of finding $(x^*, y^*) \in \mathcal{H} \times \mathcal{H}$ such that $x^* \in C_1(x^*), y^* \in C_2(y^*)$ and

$$\begin{cases} \Phi_1(x^*, z) + \langle sT_1(y^*, x^*) + x^* - y^*, z - x^* \rangle + \varphi_1(z) - \varphi_1(x^*) \geq 0, \ \forall \ z \in C_1(x^*), \\ \Phi_2(y^*, z) + \langle tT_2(x^*, y^*) + y^* - x^*, z - y^* \rangle + \varphi_2(z) - \varphi_2(y^*) \geq 0, \ \forall \ z \in C_2(y^*). \end{cases} \tag{1.1}$$

Since in many important problems the closed convex set C also depends upon the solutions explicitly or implicitly, it is worth mentioning that problem (1.1) is of interest to study; see [1] for more details. Problem (1.1) is called the generalized system of nonlinear quasi-mixed equilibrium problems. The set of solutions of problem (1.1) is denoted by Ω.

Note that if $C_1(x) = C_1, C_2(x) = C_2$, for all $x \in \mathcal{H}$, then problem (1.1) is equivalent to find $(x^*, y^*) \in \mathcal{H} \times \mathcal{H}$ such that $x^* \in C_1, y^* \in C_2$ and such that

$$\begin{cases} \Phi_1(x^*, z) + \langle sT_1(y^*, x^*) + x^* - y^*, z - x^* \rangle + \varphi_1(z) - \varphi_1(x^*) \geq 0, \ \forall \ z \in C_1, \\ \Phi_2(y^*, z) + \langle tT_2(x^*, y^*) + y^* - x^*, z - y^* \rangle + \varphi_2(z) - \varphi_2(y^*) \geq 0, \ \forall \ z \in C_2. \end{cases} \tag{1.2}$$

If $\varphi_1 = \varphi_2 \equiv 0, S_1(x, y) = sT_1(y, x) + x - y, S_2(x, y) = tT_2(x, y) + y - x$, then problem (1.1) is equivalent to find $(x^*, y^*) \in \mathcal{H} \times \mathcal{H}$ such that $x^* \in C_1(x^*), y^* \in C_2(y^*)$ and

$$\begin{cases} \Phi_1(x^*, z) + \langle S_1(x^*, y^*), z - x^* \rangle \geq 0, \ \forall \ z \in C_1(x^*), \\ \Phi_2(y^*, z) + \langle S_2(x^*, y^*), z - y^* \rangle \geq 0, \ \forall \ z \in C_2(y^*), \end{cases} \tag{1.3}$$

which was considered by Suantai and Petrot [2]. Problem (1.3) is called the system of nonlinear quasi-mixed equilibrium problems.

If $\Phi_1 = \Phi_2 \equiv 0, C_1(x) = C_2(x) = \mathcal{H}$, for all $x \in \mathcal{H}$, then problem (1.1) is equivalent to find $(x^*, y^*) \in \mathcal{H} \times \mathcal{H}$ such that

$$\begin{cases} \langle sT_1(y^*, x^*) + x^* - y^*, z - x^* \rangle + \varphi_1(z) - \varphi_1(x^*) \geq 0, \ \forall \ z \in \mathcal{H}, \\ \langle tT_2(x^*, y^*) + y^* - x^*, z - y^* \rangle + \varphi_2(z) - \varphi_2(y^*) \geq 0, \ \forall \ z \in \mathcal{H}, \end{cases} \tag{1.4}$$

which is called the system of nonlinear mixed variational inequalities problems[2].

If $T_1 = T_2 = A, \varphi_1 = \varphi_2 = \varphi$, then problem (1.4) is equivalent to find $(x^*, y^*) \in \mathcal{H} \times \mathcal{H}$ such that

$$\begin{cases} \langle sA(y^*, x^*) + x^* - y^*, z - x^* \rangle + \varphi(z) - \varphi(x^*) \geq 0, \ \forall \ z \in \mathcal{H}, \\ \langle tA(x^*, y^*) + y^* - x^*, z - y^* \rangle + \varphi(z) - \varphi(y^*) \geq 0, \ \forall \ z \in \mathcal{H}. \end{cases} \tag{1.5}$$

Problem (1.5) was studied by He and Gu [3] and Petrot [4]. A special case of problem (1.5) has been studied by many authors; see [5-8] for examples. Evidently, the examples described above shown that a number of classes of variational inequalities and related optimization problems can be obtained as special of the generalized system of nonlinear quasi-mixed equilibrium problems (1.1).

Recently, Narin Petrot [4] used the resolvent operator technique to find the common solutions for the generalized system of relaxed cocoercive mixed variational inequality problems (1.5) and fixed point problems for Lipschitz mappings. Very recently, S. Suthep and N. Petrot [2] considered the system of nonlinear quasi-mixed equilibrium problems (1.3) and established the existence theorem for problem (1.3) and the uniqueness of solution.

Motivated and inspired by the above works, in this paper, we consider an iterative algorithm for approximating a common element of a generalized system of nonlinear quasi-mixed equilibrium problems and fixed point problems for Lipschitz mappings in Hilbert spaces. The results presented in this paper generalize some known results shown recently.

We need the following basic concepts and well known results.

Definition 1.1 A nonlinear mapping $T : \mathcal{H} \to \mathcal{H}$ is said to be a $\kappa-$Lipschitzian mapping if there exists a positive constant $\kappa > 0$ such that

$$\|Tx - Ty\| \le \kappa \|x - y\|, \ \forall \ x, y \in \mathcal{H}.$$

Definition 1.2 The mapping $T : \mathcal{H} \to \mathcal{H}$ is said to be
(1) $\nu-$strongly monotone if there exists a constant $\nu > 0$ such that

$$\langle Tx - Ty, x - y\rangle \ge \nu \|x - y\|^2, \ \forall \ x, y \in \mathcal{H};$$

(2) $\mu-$cocoercive if there exists a constant $\mu > 0$ such that

$$\langle Tx - Ty, x - y\rangle \ge \mu \|Tx - Ty\|^2, \ \forall \ x, y \in \mathcal{H};$$

(3) relaxed $\mu-$cocoercive if there exists a constant $\mu > 0$ such that

$$\langle Tx - Ty, x - y\rangle \ge (-\mu) \|x - y\|^2, \ \forall \ x, y \in \mathcal{H};$$

(4) relaxed $(\mu, \nu)-$cocoercive if there exist constants $\mu, \nu > 0$ such that

$$\langle Tx - Ty, x - y\rangle \ge (-\mu) \|Tx - Ty\|^2 + \nu \|x - y\|^2, \ \forall \ x, y \in \mathcal{H}.$$

From Definition 1.2, we see that the class of the relaxed $(\mu, \nu)-$cocoercive mappings is the most general class.

Definition 1.3([3]) A two-variable mapping $T : \mathcal{H} \times \mathcal{H} \to \mathcal{H}$ is said to be relaxed $(\mu, \nu)-$cocoercive if there exist constants $\mu, \nu > 0$ such that, for each x, x',

$$\langle T(x, y) - T(x', y'), x - x'\rangle \ge (-\mu) \|T(x, y) - T(x', y')\|^2 + \nu \|x - x'\|^2, \ \forall \ y, y' \in \mathcal{H}.$$

Definition 1.4([3]) A mapping $T : \mathcal{H} \times \mathcal{H} \to \mathcal{H}$ is said to be $\tau-$Lipschitz in the first variable if there exists a constant $\tau > 0$ such that for all x, x',

$$\|T(x, y) - T(x', y')\| \le \tau \|x - x'\|, \ \ \forall \ y, y' \in \mathcal{H}.$$

Lemma 1.1([9]) Let $\{a_n\}$ and $\{b_n\}$ be two nonnegative real sequences satisfying the following conditions:

$$a_{n+1} \le (1 - \lambda_{n+1}) a_n + b_n, \ \ \forall \ n \ge n_0,$$

for some $n_0 \in \mathbb{N}$, $\{\lambda_n\} \subset (0,1)$ with $\Sigma_{n=0}^{\infty}\lambda_n = \infty$, $b_n = o(\lambda_n)$. Then $\lim_{n\to\infty} a_n = 0$.

Let $\Phi : \mathcal{H} \times \mathcal{H} \to R$ and $\varphi : \mathcal{H} \to R$. We make the following assumptions for the bifunction Φ and φ:
(A1) $\Phi(x,x) = 0$, for all $x \in \mathcal{H}$;
(A2) Φ is monotone, i.e., $\Phi(x,y) + \Phi(y,x) \leq 0$ for any $x, y \in \mathcal{H}$;
(A3) for each $y \in \mathcal{H}, x \mapsto \Phi(x,y)$ is weakly upper semicontinuous;
(A4) for each $x \in \mathcal{H}, y \mapsto \Phi(x,y)$ is convex and lower semicontinuous;
(B1) for each $x \in \mathcal{H}$, there exist a bounded subset $D_x \subseteq \mathcal{H}$ and $y_x \in \mathcal{H}$ such that for any $z \in \mathcal{H} \setminus D_x$,

$$\Phi(z, y_x) + \varphi(y_x) - \varphi(z) + \langle y_x - z, z - x\rangle < 0.$$

If Φ is a bifunction from $\mathcal{H} \times \mathcal{H}$ to R, $\varphi : \mathcal{H} \to R$ and $r = 1$ in[10, Lemma 2.3], the following lemma is obtained immediately.

Lemma 1.2 Let C be a nonempty closed convex subset of $\mathcal{H}$. Let Φ be a bifunction from $\mathcal{H} \times \mathcal{H}$ to R satisfying (A1)-(A4) and let $\varphi : \mathcal{H} \to R$ be a proper lower semicontinuous and convex function with assumption (B1). Define a mapping $S^{\Phi}_{\varphi,C} : \mathcal{H} \to C$ as follows:

$$S^{\Phi}_{\varphi,C}(x) = \{z \in C : \Phi(z,y) + \varphi(y) - \varphi(z) + \langle y - z, z - x\rangle \geq 0, \ \forall\, y \in C\}$$

for all $x \in \mathcal{H}$.Then the following results hold:
(1) For each $x \in \mathcal{H}, S^{\Phi}_{\varphi,C}(x) \neq \emptyset$;
(2) $S^{\Phi}_{\varphi,C}$ is single-valued;
(3) $S^{\Phi}_{\varphi,C}$ is firmly nonexpansive, i.e., for any $x, y \in \mathcal{H}$,

$$\|S^{\Phi}_{\varphi,C}(x) - S^{\Phi}_{\varphi,C}(y)\|^2 \leq \langle S^{\Phi}_{\varphi,C}(x) - S^{\Phi}_{\varphi,C}(y), x - y\rangle.$$

Notation In what follows, for $\Gamma \subset \mathcal{H} \times \mathcal{H}$, the symbol $\Gamma \cap F(T) \neq \emptyset$ means that there exist $x^*, y^* \in \mathcal{H}$ such that $(x^*, y^*) \in \Gamma$ and $\{x^*, y^*\} \subset F(T)$, where $T : \mathcal{H} \to \mathcal{H}$ is a mapping and $F(T)$ denotes the set of fixed points of T.

2 Main results

Lemma 2.1 $(x^*, y^*) \in \Omega$ if and only if

$$\begin{cases} x^* = S^{\Phi_1}_{\varphi_1, C_1(x^*)}[y^* - sT_1(y^*, x^*)], \ s > 0, \\ y^* = S^{\Phi_2}_{\varphi_2, C_2(y^*)}[x^* - tT_2(x^*, y^*)], \ t > 0. \end{cases}$$

Remark 2.1 If $(x^*, y^*) \in \Omega \cap F(T)$, it follows from Lemma 2.1 that

$$\begin{cases} x^* = Tx^* = TS^{\Phi_1}_{\varphi_1, C_1(x^*)}[y^* - sT_1(y^*, x^*)], \ s > 0, \\ y^* = Ty^* = TS^{\Phi_2}_{\varphi_2, C_2(y^*)}[x^* - tT_2(x^*, y^*)], \ t > 0. \end{cases}$$

Based on Remark 2.1, we suggest the following iterative algorithms.
Algorithm 2.1 Let s, t be positive real numbers that appeared in problem (1.1), and for given $(x_0, y_0) \in \mathcal{H} \times \mathcal{H}$. Define $(x_n, y_n) \subset \mathcal{H} \times \mathcal{H}$ by

$$\begin{cases} x_{n+1} = (1-\alpha_n)x_n + \alpha_n T S^{\Phi_1}_{\varphi_1, C_1(x_n)}[y_n - sT_1(y_n, x_n)], \\ y_{n+1} = (1-\beta_n)y_n + \beta_n T S^{\Phi_2}_{\varphi_2, C_2(y_n)}[x_n - tT_2(x_n, y_n)]. \end{cases}$$

Algorithm 2.2 Let s, t be positive real numbers that appeared in problem (1.2), and for given $(x_0, y_0) \in \mathcal{H} \times \mathcal{H}$. Define $(x_n, y_n) \subset \mathcal{H} \times \mathcal{H}$ by

$$\begin{cases} x_{n+1} = (1-\alpha_n)x_n + \alpha_n T S^{\Phi_1}_{\varphi_1, C_1}[y_n - sT_1(y_n, x_n)], \\ y_n = (1-\beta_n)x_n + \beta_n T S^{\Phi_2}_{\varphi_2, C_2}[x_n - tT_2(x_n, y_n)]. \end{cases}$$

To prove our main results, from now on, we will assume the following condition:
Condition(A) For each $i = 1, 2$, there exists $\eta_i > 0$ such that

$$\|S^{\Phi_i}_{\varphi_i, C_i(u)} z - S^{\Phi_i}_{\varphi_i, C_i(v)} z\| \le \eta_i \|u - v\|, \ \forall\, u, v, z \in \mathcal{H}.$$

Now we state and prove the main results of this work.
Theorem 2.1 Let $\mathcal{H}$ be a real Hilbert space and $C_1, C_2 : \mathcal{H} \to \mathcal{CC}(\mathcal{H})$. For each $i = 1, 2$, let $\Phi_i : \mathcal{H} \times \mathcal{H} \to R$ be a bifunction satisfying (A1)-(A4) and let $\varphi_i : \mathcal{H} \to R$ be a proper lower semicontinuous and convex function with assumption (B1). For each $i = 1, 2$, let $T_i : \mathcal{H} \times \mathcal{H} \to \mathcal{H}$ be a two-variable relaxed $(\mu_i, \nu_i)-$cocoercive and τ_i-Lipschitz mapping in the first variable. Let $T : \mathcal{H} \to \mathcal{H}$ be a $\kappa-$Lipschitz mapping. Suppose that (x^*, y^*) is a solution of problem (1.1) and $\{x^*, y^*\} \subset F(T)$. If $\{\alpha_n\}, \{\beta_n\}$ are two sequences in [0,1] and the following conditions are satisfied:
(i) $\kappa\eta_1 < 1, \kappa\eta_2 < 1$;
(ii) $\omega_1 \ge 0, \omega_2 \ge 0, \Sigma_{n=0}^{\infty}\omega_1 = \infty$ and $\Sigma_{n=0}^{\infty}\omega_2 = \infty$,
where $\theta_1 = \sqrt{1 - 2s\nu_1 + (s^2 + 2s\mu_1)\tau_1^2}, \theta_2 = \sqrt{1 - 2t\nu_2 + (t^2 + 2t\mu_2)\tau_2^2}, \omega_1 = \alpha_n(1 - \kappa\eta_1) - \beta_n\kappa\theta_2, \omega_2 = \beta_n(1 - \kappa\eta_2) - \alpha_n\kappa\theta_1$. Then the sequences $\{x_n\}$ and $\{y_n\}$ generated by Algorithm (2.1) converge strongly to x^* and y^*, respectively.
Proof By Algorithm (2.1), Remark (2.1) and condition(A) we obtain

$$\begin{aligned} & \|x_{n+1} - x^*\| \\ \le\ & (1-\alpha_n)\|x_n - x^*\| + \alpha_n\kappa\|S^{\Phi_1}_{\varphi_1, C_1(x_n)}[y_n - sT_1(y_n, x_n)] - S^{\Phi_1}_{\varphi_1, C_1(x^*)}[y^* - sT_1(y^*, x^*)]\| \\ \le\ & (1-\alpha_n)\|x_n - x^*\| + \alpha_n\kappa\|S^{\Phi_1}_{\varphi_1, C_1(x_n)}[y_n - sT_1(y_n, x_n)] - S^{\Phi_1}_{\varphi_1, C_1(x^*)}[y_n - sT_1(y_n, x_n)]\| \\ & + \alpha_n\kappa\|S^{\Phi_1}_{\varphi_1, C_1(x^*)}[y_n - sT_1(y_n, x_n)] - S^{\Phi_1}_{\varphi_1, C_1(x^*)}[y^* - sT_1(y^*, x^*)]\| \\ \le\ & (1 - \alpha_n + \alpha_n\kappa\eta_1)\|x_n - x^*\| + \alpha_n\kappa\|(y_n - y^*) - s(T_1(y_n, x_n) - T_1(y^*, x^*))\|. \end{aligned} \tag{2.1}$$

Since T_1 be relaxed $(\mu_1, \nu_1)-$cocoercive and τ_1-Lipschitz mapping in the first variable, we have

$$\begin{aligned} & \|(y_n - y^*) - s(T_1(y_n, x_n) - T_1(y^*, x^*))\|^2 \\ = & \|y_n - y^*\|^2 - 2s\langle T_1(y_n, x_n) - T_1(y^*, x^*), y_n - y^*\rangle + s^2\|T_1(y_n, x_n) - T_1(y^*, x^*)\|^2 \\ \leq & (1 - 2s\nu_1 + (s^2 + 2s\mu_1)\tau_1^2)\|y_n - y^*\|^2. \end{aligned} \tag{2.2}$$

It follows from (2.1)and (2.2) that

$$\|x_{n+1} - x^*\| \leq (1 - \alpha_n + \alpha_n\kappa\eta_1)\|x_n - x^*\| + \alpha_n\kappa\theta_1\|y_n - y^*\|, \tag{2.3}$$

where $\theta_1 = \sqrt{1 - 2s\nu_1 + (s^2 + 2s\mu_1)\tau_1^2}$. From Algorithm (2.1), Remark (2.1) and condition(A), we get

$$\begin{aligned} \|y_{n+1} - y^*\| \leq & (1 - \beta_n + \beta_n\kappa\eta_2)\|y_n - y^*\| \\ & +\beta_n\kappa\|S^{\Phi_2}_{\varphi_2, C_2(y^*)}[x_n - tT_2(x_n, y_n)] - S^{\Phi_2}_{\varphi_2, C_2(y^*)}[x^* - tT_2(x^*, y^*)]\| \\ \leq & (1 - \beta_n(1 - \kappa\eta_2))\|y_n - y^*\| \\ & +\beta_n\kappa\|(x_n - x^*) - t(T_2(x_n, y_n) - T_2(x^*, y^*))\|. \end{aligned} \tag{2.4}$$

Since T_2 be relaxed $(\mu_2, \nu_2)-$cocoercive and τ_2-Lipschitz mapping in the first variable, we have

$$\begin{aligned} & \|(x_n - x^*) - t(T_2(x_n, y_n) - T_2(x^*, y^*))\|^2 \\ = & \|x_n - x^*\|^2 - 2t\langle T_2(x_n, y_n) - T_2(x^*, y^*), x_n - x^*\rangle + t^2\|T_2(x_n, y_n) - T_2(x^*, y^*))\|^2 \\ \leq & (1 - 2t\nu_2 + (t^2 + 2t\mu_2)\tau_2^2)\|x_n - x^*\|^2. \end{aligned} \tag{2.5}$$

Applying (2.5) into (2.4), we obtain

$$\|y_{n+1} - y^*\| \leq (1 - \beta_n + \beta_n\kappa\eta_2)\|y_n - y^*\| + \beta_n\kappa\theta_2\|x_n - x^*\|^2, \tag{2.6}$$

where $\theta_2 = \sqrt{1 - 2t\nu_2 + (t^2 + 2t\mu_2)\tau_2^2}$. Define the norm $\|\|^+$ on $\mathcal{H} \times \mathcal{H}$ by

$$\|(x, y)\|^+ = \|x\| + \|y\|, \ \forall \ (x, y) \in \mathcal{H} \times \mathcal{H}.$$

Note that $(\mathcal{H} \times \mathcal{H}, \|\|^+)$ is a Banach space and it follows from (2.3) and (2.6) that

$$\|(x_{n+1}, y_{n+1}) - (x^*, y^*)\|^+ \leq \max\{1 - \omega_1, 1 - \omega_2\}\|(x_n, y_n) - (x^*, y^*)\|^+,$$

where $\omega_1 = \alpha_n(1 - \kappa\eta_1) - \beta_n\kappa\theta_2, \omega_2 = \beta_n(1 - \kappa\eta_2) - \alpha_n\kappa\theta_1$. By conditions (i)-(ii) and Lemma 1.1, we obtain

$$\|(x_{n+1}, y_{n+1}) - (x^*, y^*)\|^+ \to 0(n \to \infty),$$

that is $x_n \to x^*$ and $y_n \to y^*$ as $n \to \infty$.

Remark 2.1 Theorem 2.1 extends the results in [2].

Remark 2.2 If $\beta_n = \alpha_n,\ \forall\ n \geq 0$ in Algorithm 2.1, conditions (i) and (ii) can be replaced by (i$'$) $\kappa\eta_1 + \kappa\theta_1 < 1, \kappa\eta_2 + \kappa\theta_2 < 1$ and (ii$'$) $\Sigma_{n=0}^{\infty}\alpha_n = \infty$, respectively.

Theorem 2.2 Let $\mathcal{H}$ be a real Hilbert space and $\Phi_i : \mathcal{H} \times \mathcal{H} \to R$ be a bifunction satisfying (A1)-(A4) and let $\varphi_i : \mathcal{H} \to R$ be a proper lower semi-continuous and convex function with assumption (B1). For each $i = 1, 2$, let $T_i : \mathcal{H} \times \mathcal{H} \to \mathcal{H}$ be a two-variable relaxed $(\mu_i, \nu_i)-$cocoercive and τ_i-Lipschitz mapping in the first variable. Let $T : \mathcal{H} \to \mathcal{H}$ be a $\kappa-$Lipschitz mapping. Suppose that (x^*, y^*) is a solution of problem (1.2) and $\{x^*, y^*\} \subset F(T)$. If $\{\alpha_n\}, \{\beta_n\}$ are two sequences in [0,1] satisfying the following conditions:
(i)$\kappa\sqrt{1 - 2s\nu_1 + (s^2 + 2s\mu_1)\tau_1^2} < 1, \kappa\sqrt{1 - 2t\nu_2 + (t^2 + 2t\mu_2)\tau_2^2} < 1$;
(ii)$\Sigma_{n=0}^{\infty}\alpha_n = \infty$ and $\beta_n \to 1 (n \to \infty)$.
Then the sequences $\{x_n\}$ and $\{y_n\}$ generated by Algorithm (2.2) converge strongly to x^* and y^*, respectively.

Proof It follows from (2.3),(2.5) and Algorithm (2.1) that

$$\begin{aligned}
\|x_{n+1} - x^*\| &\leq (1 - \alpha_n)\|x_n - x^*\| + \alpha_n\kappa\|S_{\varphi_1, C_1}^{\Phi_1}[y_n - sT_1(y_n, x_n)] - S_{\varphi_1, C_1}^{\Phi_1}[y^* - sT_1(y^*, x^*)]\| \\
&\leq (1 - \alpha_n)\|x_n - x^*\| + \alpha_n\kappa\theta_1\|y_n - y^*\|, \qquad (2.7) \\
\|y_n - y^*\| &\leq (1 - \beta_n)\|x_n - y^*\| + \beta_n\kappa\|(x_n - x^*) - t(T_2(x_n, y_n) - T_2(x^*, y^*))\| \\
&\leq (1 - \beta_n)\|x_n - x^*\| + (1 - \beta_n)\|x^* - y^*\| + \beta_n\kappa\theta_2\|x_n - x^*\| \\
&= (1 - \beta_n(1 - \kappa\theta_2))\|x_n - x^*\| + (1 - \beta_n)\|x^* - y^*\|. \qquad (2.8)
\end{aligned}$$

From (2.7) and (2.8) we obtain

$$\|x_{n+1} - x^*\| \leq [1 - \alpha_n(1 - \kappa\theta_1(1 - \beta_n(1 - \kappa\theta_2)))]\|x_n - x^*\| + \alpha_n\kappa\theta_1(1 - \beta_n)\|x^* - y^*\|.$$

Hence by conditions (i)-(ii) and Lemma 1.1, we have $x_n \to x^*(n \to \infty)$. Furthermore, by condition (ii) and (2.8) we get $y_n \to y^*(n \to \infty)$.

Remark 2.3 Theorem 2.2 extends and improves the main results in [3,4,6].

Remark 2.4 Similar to Theorem 3.4 in [2], we can also consider stability of the iterative Algorithm 2.1 and Algorithm 2.2.

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