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A Study on Anti-Fuzzy Normal Subsemiring of a Semiring

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Abstract. In this paper, we made an attempt to study the algebraic nature of anti- fuzzy normal subsemiring of a semiring.

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INTRODUCTION

There are many concepts of universal algebras generalizing an associative ring (*R ; + ; .*). Some of them in particular, nearrings and several kinds of semirings have been proven very useful. An algebra (R ; +, .) is said to be a semiring if (R ; +) and (R ; .) are semigroups satisfying a. (b+c) = a. b+a. c and (b+c) .a = b. a+c. a for all a, b and c in R. A semiring R is said to be additively commutative if a+b = b+a for all a, b in R. A semiring R may have an identity 1, defined by 1. a = a = a. 1 and a zero 0, defined by 0+a = a = a+0 and a.0 = 0 = 0.a for all a in R. After the introduction of fuzzy sets by L.A. Zadeh [16], several researchers explored on the generalization of the concept of fuzzy sets. The notion of fuzzy subnearrings and ideals was introduced by S. Abou Zaid [11]. In this paper, we introduce the some theorems in anti-fuzzy normal subsemiring of a semiring.

1. PRELIMINARIES:

1.1 Definition: Let X be a non–empty set. A **fuzzy subset A** of X is a function A : X → [0, 1].

1.2 Definition: Let R be a semiring. A fuzzy subset A of R is said to be a **fuzzy subsemiring (FSSR)** of R if it satisfies the following conditions:

(i) $\mu_A(x + y) \geq \min\{\mu_A(x), \mu_A(y)\}$,

(ii) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R.

1.3 Definition: Let R be a semiring. A fuzzy subset A of R is said to be **an anti-fuzzy subsemiring (AFSSR)** of R if it satisfies the following conditions:

(i) $\mu_A(x + y) \leq \max\{\mu_A(x), \mu_A(y)\}$,

(ii) $\mu_A(xy) \leq \max\{\mu_A(x), \mu_A(y)\}$, for all x and y in R.

1.4 Definition: Let R be a semiring. An anti-fuzzy subsemiring A of R is said to be an **anti-fuzzy normal subsemiring (AFNSSR)** of R if it satisfies the following conditions:

(i) $\mu_A(x+y) = \mu_A(y+x)$,

(ii) $\mu_A(xy) = \mu_A(yx)$, for all x and y in R.

1.5 Definition: Let A and B be fuzzy subsets of sets G and H, respectively. The anti-product of A and B, denoted by AxB, is defined as AxB= $\{\langle(x, y),\mu_{AxB}(x,y)\rangle$ / for all x in G and y in H$\}$, where $\mu_{AxB}(x, y) = \max\{\mu_A(x), \mu_B(y)\}$.

1.6 Definition: Let A be a fuzzy subset in a set S, the anti-strongest fuzzy relation on S, that is a fuzzy relation on A is V given by $\mu_V(x, y) = \max\{\mu_A(x), \mu_A(y)\}$, for all x and y in S.

1.7 Definition: Let (R, +, ·) and (R^{I}, +, ·) be any two semirings. Let $f : R \rightarrow R^{I}$ be any function and A be an anti-fuzzy subsemiring in R, V be an anti-fuzzy subsemiring in f(R) = R^{I}, defined by $\mu_V(y) = \inf_{x\in f^{-1}(y)} \mu_A(x)$, for all x in R and y in R^{I}. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

1.8 Definition: Let A be a fuzzy subset of X. For α in [0, 1], the lower level subset of A is the set $A_\alpha = \{x\in X : \mu_A(x) \leq \alpha\}$.

2. PROPERTIES OF ANTI-FUZZY NORMAL SUBSEMIRING OF A SEMIRING

2.1 Theorem: Union of any two anti-fuzzy subsemiring of a semiring R is an anti-fuzzy subsemiring of R.

2.2 Theorem: The union of a family of anti-fuzzy subsemirings of semiring R is an anti-fuzzy subsemiring of R.

2.3 Theorem: If A and B are any two anti-fuzzy subsemirings of the semirings R_1 and R_2 respectively, then anti-product AxB is an anti-fuzzy subsemiring of R_1xR_2.

2.4 Theorem: Let A be a fuzzy subset of a semiring R and V be the strongest anti-fuzzy relation of R. Then A is an anti-fuzzy subsemiring of R if and only if V is an anti-fuzzy subsemiring of RxR.

2.5 Theorem: Let A be an anti-fuzzy subsemiring of a semiring H and f is an isomorphism from a semiring R onto H.Then A◦f is an anti-fuzzy subsemiring of R.

2.6 Theorem: Let A be an anti-fuzzy subsemiring of a semiring H and f is an anti-isomorphism from a semiring R onto H. Then A◦f is an anti-fuzzy subsemiring of R.

2.7 Theorem: Let (R, +, .) and (R^{l}, +, .) be any two semirings. The homomorphic image of an anti-fuzzy subsemiring of R is an anti-fuzzy subsemiring of R^{l}.

2.8 Theorem: Let (R, +, .) and (R^{l}, +, .) be any two semirings. The homomorphic preimage of an anti-fuzzy subsemiring of R^{l} is an anti-fuzzy subsemiring of R.

2.9 Theorem: Let (R, +, .) and (R^{l}, +, .) be any two semirings. The anti-homomorphic image of an anti-fuzzy subsemiring of R is an anti-fuzzy subsemiring of R^{l}.

2.10 Theorem: Let (R, +, .) and (R^{l}, +, .) be any two semirings. The anti-homomorphic preimage of an anti-fuzzy subsemiring of R^{l} is an anti-fuzzy subsemiring of R.

2.11 Theorem: Let A be an anti-fuzzy subsemiring of a semiring R. Then for α in [0,1], A_α is a lower level subsemiring of R.

2.12 Theorem: Let A be an anti-fuzzy subsemiring of a semiring R. Then two lower level subsemiring $A_{\alpha 1}$, $A_{\alpha 2}$ and α_1, α_2 are in [0,1] with $\alpha_1 < \alpha_2$ of A are equal iff there is no x in R such that $\alpha_2 > \mu_A(x) > \alpha_1$.

2.13 Theorem: Let A be an anti-fuzzy subsemiring of a semiring R. If any two lower level subsemirings of A belongs to R, then their intersection is also lower level subsemiring of A in R.

2.14 Theorem: Let A be an anti-fuzzy subsemiring of a semiring R. If $\alpha_i \in [0,1]$, and $A_{\alpha i}$, $i \in I$ is a collection of lower level subsemirings of A, then their intersection is also a lower level subsemiring of A.

2.15 Theorem: Let A be an anti-fuzzy subsemiring of a semiring R. If any two lower level subsemirings of A belongs to R, then their union is also a lower level subsemiring of A in R.

2.16 Theorem: Let A be an anti-fuzzy subsemiring of a semiring R. If $\alpha_i \in [0,1]$, and $A_{\alpha i}$, $i \in I$ is a collection of lower level subsemirings of A, then their union is also a lower level subsemiring of A.

2.17 Theorem: The homomorphic image of a lower level subsemiring of an anti-fuzzy subsemiring of a semiring R is a lower level subsemiring of an anti-fuzzy subsemiring of a semiring R^{l}.

2.18 Theorem: The homomorphic pre-image of a lower level subsemiring of an anti-fuzzy subsemiring of a semiring R^{l} is a lower level subsemiring of an anti-fuzzy subsemiring of a semiring R.

2.19 Theorem: The anti-homomorphic image of a lower level subsemiring of an anti-fuzzy subsemiring of a semiring R is a lower level subsemiring of an anti-fuzzy subsemiring of a semiring R^{l}.

2.20 Theorem: The anti-homomorphic pre-image of a lower level subsemiring of an anti-fuzzy subsemiring of a semiring R^{l} is a lower level subsemiring of an anti-fuzzy subsemiring of a semiring R.

2.21 Theorem: Let (R, +, .) be a semiring. If A and B are two anti-fuzzy normal subsemirings of R, then their union $A \cup B$ is an anti-fuzzy normal subsemiring of R.

Proof: Let x and y∈R. Let A ={⟨ x, $\mu_A(x)$ ⟩ / x∈R} and B = {⟨ x, $\mu_B(x)$ ⟩ / x∈R} be anti-fuzzy normal subsemirings of a semiring R. Let C = A ∪ B and C={⟨ x, $\mu_C(x)$⟩ / x∈R}. Then, Clearly C is an anti-fuzzy subsemiring of a semiring R, since A and B are two anti-fuzzy subsemirings of a semiring R. And, (i) $\mu_C(x+y)$ = max {$\mu_A(x+y)$,$\mu_B(x+y)$} = max {$\mu_A(y+x)$,$\mu_B(y+x)$} = $\mu_C(y+x)$,for all x and y in R. Therefore, $\mu_C(x+y) = \mu_C(y+x)$, for all x and y in R.(ii) $\mu_C(xy)$ = max {$\mu_A(xy)$, $\mu_B(xy)$} = max {$\mu_A(yx)$, $\mu_B(yx)$}= $\mu_C(yx)$, for all x and y in R. Therefore, $\mu_C(xy) = \mu_C(yx)$, for all x and y in R. Hence A∪B is an anti-fuzzy normal subsemiring of a semiring R.

2.22 Theorem: Let R be a semiring. The union of a family of anti-fuzzy normal subsemirings of R is an anti-fuzzy normal subsemiring of R.

Proof: Let $\{A_i\}_{i\in I}$ be a family of anti-fuzzy normal subsemirings of a semiring R and let A = $\bigcup_{i\in I} A_i$. Then for x and y in R. Clearly the union of a family of anti-fuzzy subsemirings of a semiring R is an anti-fuzzy subsemiring of a semiring R.(i) $\mu_A(x+y) = \sup_{i\in I} \mu_{A_i}(x+y) = \sup_{i\in I} \mu_{A_i}(y+x) = \mu_A(y+x)$, for all x and y in R. Therefore, $\mu_A(x+y) = \mu_A(y+x)$, for all x and y in R. (ii) $\mu_A(xy) = \sup_{i\in I} \mu_{A_i}(xy) = \sup_{i\in I} \mu_{A_i}(yx) = \mu_A(yx)$, for all x and y in R. Therefore, $\mu_A(xy) = \mu_A(yx)$, for all x and y in R. Hence the union of a family of anti-fuzzy normal subsemirings of a semiring R is an anti-fuzzy normal subsemiring of a semiring R.

2.23 Theorem: Let A and B be anti-fuzzy subsemiring of the semirings G and H, respectively. If A and B are anti-fuzzy normal subsemirings, then AxB is an anti- fuzzy normal subsemiring of GxH.

Proof: Let A and B be anti-fuzzy normal subsemirings of the semirings G and H respectively. Clearly AxB is an anti-fuzzy subsemiring of GxH. Let x_1 and x_2 be in G, y_1 and y_2 be in H. Then (x_1, y_1) and (x_2, y_2) are in GxH. Now, $\mu_{AxB}[(x_1, y_1) + (x_2, y_2)] = \mu_{AxB}(x_1+x_2, y_1+y_2)$ = max{$\mu_A(x_1+x_2)$, $\mu_B(y_1+y_2)$} = max{$\mu_A(x_2+x_1)$,$\mu_B(y_2+y_1)$ = $\mu_{AxB}(x_2+x_1,y_2+y_1) = \mu_{AxB}[(x_2,y_2)+(x_1,y_1)]$. Therefore, $\mu_{AxB}[(x_1,y_1)+(x_2,y_2)] = \mu_{AxB}[(x_2,y_2)+(x_1,y_1)]$. And, $\mu_{AxB}[(x_1,y_1)(x_2,y_2)] = \mu_{AxB}(x_1x_2, y_1y_2)$ = max {$\mu_A(x_1x_2)$, $\mu_B(y_1y_2)$} = max {$\mu_A(x_2x_1)$, $\mu_B(y_2y_1)$}= $\mu_{AxB}(x_2x_1, y_2y_1) = \mu_{AxB}[(x_2,y_2)(x_1,y_1)]$. Therefore, $\mu_{AxB}[(x_1, y_1)(x_2, y_2)] = \mu_{AxB}[(x_2, y_2)(x_1, y_1)]$. Hence AxB is an anti-fuzzy normal subsemiring of GxH.

2.24 Theorem: Let A be a fuzzy subset in a semiring R and V be the strongest anti-fuzzy relation on R. Then A is an anti-fuzzy normal subsemiring of R if and only if V is an anti-fuzzy normal subsemiring of RxR.

Proof: Suppose that A is a anti-fuzzy normal subsemiring of R. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in RxR. Clearly V is a anti-fuzzy subsemiring of RxR. We have, $V(x+y) = V[(x_1,x_2)+(y_1,y_2)] = V((x_1+y_1, x_2+y_2)) = A((x_1+y_1)) \wedge A((x_2+y_2)) = A((y_1+x_1)) \wedge A((y_2+x_2)) = V((y_1+x_1, y_2+x_2)) = V[(y_1, y_2)+(x_1, x_2)] = V(y+x)$ Therefore,$V(x+y)=V(y+x)$, for all x and y in RxR. We have, V(xy) =

$V[(x_1,x_2)(y_1,y_2)] = V((x_1y_1,x_2y_2)) = A((x_1y_1)) \wedge A((x_2y_2)) = A((y_1x_1)) \wedge A((y_2x_2)) = V((y_1x_1, y_2x_2)) = V[(y_1, y_2)(x_1, x_2)] = V(yx)$ Therefore, $V(xy) = V(yx)$, for all x and y in RxR.This proves that V is a anti-fuzzy normal subsemiring of RxR. Conversely, assume that V is a anti-fuzzy normal subsemiring of RxR, then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in RxR, we have $A(x_1+y_1) \wedge A(x_2+y_2) = V((x_1+y_1, x_2+y_2)) = V[(x_1, x_2)+(y_1, y_2)] = V(x+y) = V(y+x) = V[(y_1, y_2)+(x_1, x_2)] = V((y_1+x_1, y_2+x_2)) = A(y_1+x_1) \wedge A(y_2+x_2)$. We get, $A((x_1+y_1)) = A(y_1+x_1)$, for all x_1 and y_1 in R. And $A(x_1y_1) \wedge A(x_2y_2) = V((x_1y_1, x_2y_2)) = V[(x_1, x_2)(y_1, y_2)] = V(xy) = V(yx) = V[(y_1, y_2)(x_1, x_2)] = V((y_1x_1, y_2x_2)) = A(y_1x_1) \wedge A(y_2x_2)$. We get, $A((x_1y_1)) = A(y_1x_1)$, for all x_1 and y_1 in R. Hence A is a anti-fuzzy normal subsemiring of R.

2.25 Theorem: Let (R, +, .) and (R^l, +, .) be any two semirings. The homomorphic image of an anti-fuzzy normal subsemiring of R is an anti-fuzzy normal subsemiring of R^l.

Proof: Let (R, +, .) and (R^l, +, .) be any two semirings and $f : R \rightarrow R^l$ be a homomorphism. Then, $f(x+y) = f(x) + f(y)$ and $f(xy) = f(x) f(y)$, for all x and y in R. Let V = f(A), where A is an anti-fuzzy normal subsemiring of a semiring R. We have to prove that V is an anti-fuzzy normal subsemiring of a semiring R^l. Now, for f(x), f(y) in R^l, clearly V is an anti-fuzzy subsemiring of a semiring R^l, since A is an anti-fuzzy subsemiring of a semiring R. Now, $\mu_v(f(x) + f(y)) = \mu_v(f(x+y)) \leq \mu_A(x+y) = \mu_A(y+x) \geq \mu_v(f(y+x)) = \mu_v(f(y)+f(x))$, which implies that $\mu_v(f(x)+f(y)) = \mu_v(f(y)+(f(x))$, for all f(x) and f(y) in R^l. Again, $\mu_v(f(x)f(y)) = \mu_v(f(xy)) \leq \mu_A(xy) = \mu_A(yx) \geq \mu_v(f(yx)) = \mu_v(f(y) f(x))$, which implies that $\mu_v(f(x)f(y)) = \mu_v(f(y) f(x))$, for all f(x) and f(y) in R^l. Hence V is an anti-fuzzy normal subsemiring of a semiring R^l.

2.26 Theorem: Let (R, +, .) and (R^l, +, .) be any two semirings. The homomorphic preimage of an anti-fuzzy normal subsemiring of R^l is an anti-fuzzy normal subsemiring of R.

Proof: Let (R, +, .) and (R^l, +, .) be any two semirings and $f : R \rightarrow R^l$ be a homomorphism. Then, $f(x+y) = f(x) + f(y)$ and $f(xy) = f(x) f(y)$, for all x and y in R. Let V = f(A), where V is an anti-fuzzy normal subsemiring of a semiring R^l. We have to prove that A is an anti-fuzzy normal subsemirring of a semiring R. Let x and y in R. Then, clearly A is an anti-fuzzy subsemiring of a semiring R, since V is an anti-fuzzy subsemiring of a semiring R^l. Now, $\mu_A(x + y) = \mu_v(f(x + y)) = \mu_v(f(x) + f(y)) = \mu_v(f(y) + f(x)) = \mu_v(f(y + x)) = \mu_A(y + x)$, which implies that $\mu_A(x + y) = \mu_A(y + x)$, for all x and y in R. Again, $\mu_A(xy) = \mu_v(f(xy)) = \mu_v(f(x)f(y)) = \mu_v(f(y)f(x)) = \mu_v(f(yx)) = \mu_A(yx)$, which implies that $\mu_A(xy) = \mu_A(yx)$, for all x and y in R. Hence A is an anti-fuzzy normal subsemiring of a semiring R.

2.27 Theorem: Let (R, +, .) and (R^l, +, .) be any two semirings. The anti-homomorphic image of an anti-fuzzy normal subsemiring of R is an anti-fuzzy normal subsemiring of R^l.

Proof: Let (R, +, .) and (R^l, +, .) be any two semirings and $f : R \rightarrow R^l$ be an anti-homomorphism. Then, $f(x + y) = f(y) + f(x)$ and $f(xy) = f(y) f(x)$, for all x and y in R. Let V = f(A), where A is an anti-fuzzy normal subsemiring of a semiring R. We have to prove that V is an anti-fuzzy normal subsemiring of a semiring R^l. Now, for f(x) and f(y) in R^l, clearly V is an anti-fuzzy subsemiring of a semiring R^l, since A is an anti-fuzzy subsemiring of a semiring R. Now, $\mu_v(f(x)+f(y)) = \mu_v(f(y + x)) \leq \mu_A(y + x) = \mu_A(x+y) \geq \mu_v(f(x+y)) = \mu_v(f(y) + f(x))$, which implies that $\mu_v(f(x) + f(y)) = \mu_v(f(y) + f(x))$, for all f(x) and f(y) in R^l. Again, $\mu_v(f(x)f(y)) = \mu_v(f(yx)) \leq \mu_A(yx) = \mu_A(xy) \geq \mu_v(f(xy)) = \mu_v(f(y) f(x))$, which implies that $\mu_v(f(x)f(y)) = \mu_v(f(y) f(x))$, for all f(x) and f(y) in R^l. Hence V is an anti-fuzzy normal subsemiring of a semiring R^l.

2.28 Theorem: Let (R, +, .) and (R^l, +, .) be any two semirings. The anti-homomorphic preimage of an anti-fuzzy normal subsemiring of R^l is an anti-fuzzy normal subsemiring of R.

Proof: Let (R, +, .) and (R^l, +, .) be any two semirings and $f : R \rightarrow R^l$ be an anti-homomorphism. Then, f(x+y) = f(y) + f(x) and f(xy) = f(y) f(x), for all x and y in R. Let V = f(A), where V is an anti-fuzzy normal subsemiring of a semiring R^l. We have to prove that A is an anti-fuzzy normal subsemiring of a semiring R. Let x and y in R, then clearly A is an anti-fuzzy subsemiring of a semiring R, since V is an anti-fuzzy subsemiring of a semiring R^l. Now, $\mu_A(x+y) = \mu_v(f(x+y)) = \mu_v(f(y) + f(x)) = \mu_v(f(x) + f(y)) = \mu_v(f(y+x)) = \mu_A(y + x)$, which implies that $\mu_A(x + y) = \mu_A(y + x)$, for all x and y in R. Again, $\mu_A(xy) = \mu_v(f(xy)) = \mu_v(f(y)f(x)) = \mu_v(f(x)f(y)) = \mu_v(f(yx)) = \mu_A(yx)$, which implies that $\mu_A(xy) = \mu_A(yx)$, for all x and y in R. Hence A is an anti-fuzzy normal subsemiring of a semiring R.

<u>In the following Theorem ◦ is the composition operation of functions:</u>

2.29 Theorem: Let A be an anti-fuzzy subsemiring of a semiring H and f is an isomorphism from a semiring R onto H. If A is an anti-fuzzy normal subsemiring of the semiring H, then A◦f is an anti-fuzzy normal subsemiring of the semiring R.

Proof: Let x and y in R and A be an anti-fuzzy normal subsemiring of a semiring H. Then we have, clearly A◦f is an anti-fuzzy subsemiring of a semiring R. Now, $(\mu_A \circ f)(x+y) = \mu_A(f(x + y)) = \mu_A(f(x) + f(y)) = \mu_A(f(y) + f(x)) = \mu_A(f(y + x)) = (\mu_A \circ f)(y + x)$, which implies that $(\mu_A \circ f)(x + y) = (\mu_A \circ f)(y + x)$, for all x and y in R. And, $(\mu_A \circ f)(xy) = \mu_A(f(xy)) = \mu_A(f(x)f(y)) = \mu_A(f(y)f(x)) = \mu_A(f(yx)) = (\mu_A \circ f)(yx)$, which implies that $(\mu_A \circ f)(xy) = (\mu_A \circ f)(yx)$, for all x and y in R. Hence A◦f is an anti-fuzzy normal subsemiring of a semiring R.

2.30 Theorem: Let A be an anti-fuzzy subsemiring of a semiring H and f is an anti-isomorphism from a semiring R onto H. If A is an anti-fuzzy normal subsemiring of the semiring H, then A◦f is an anti-fuzzy normal subsemiring of the semiring R.

Proof: Let x and y in R and A be an anti-fuzzy normal subsemiring of a semiring H.

Then we have, clearly A◦f is an anti-fuzzy subsemiring of a semiring R. Now, $(\mu_A \circ f)(x + y) = \mu_A(f(x + y)) = \mu_A(f(y) + f(x)) = \mu_A(f(x) + f(y)) = \mu_A(f(y + x)) = (\mu_A \circ f)(y + x)$, which implies that $(\mu_A \circ f)(x + y) = (\mu_A \circ f)(y + x)$, for all x and y in R. And, $(\mu_A \circ f)(xy) = \mu_A(f(xy)) = \mu_A(f(y)f(x)) = \mu_A(f(x)f(y)) = \mu_A(f(yx)) =$

($\mu_A \circ f$) (yx), which implies that ($\mu_A \circ f$) (xy) = ($\mu_A \circ f$) (yx) , for all x and y in R. Hence A∘f is an anti-fuzzy normal subsemiring of a semiring R.

2.31Theorem: Let A be an anti-fuzzy normal subsemiring of a semiring R. Then for α in [0,1], A_α is a lower level subsemiring of R.

Proof: It is trivial.

2.32 Theorem: Let A be an anti-fuzzy normal subsemiring of a semiring R. Then two lower level subsemiring $A_{\alpha 1}$, $A_{\alpha 2}$ and α_1, α_2 are in [0,1] with $\alpha_1 < \alpha_2$ of A are equal iff there is no x in R such that $\alpha_2 > \mu_A(x) > \alpha_1$.

Proof: It is trivial.

2.33 Theorem: Let A be an anti-fuzzy normal subsemiring of a semiring R. If any two lower level subsemirings of A belongs to R, then their intersection is also lower level subsemiring of A in R.

Proof: It is trivial.

2.34 Theorem: Let A be an anti-fuzzy normal subsemiring of a semiring R. If $\alpha_i \in [0,1]$, and $A_{\alpha i}$, $i \in I$ is a collection of lower level subsemirings of A, then their intersection is also a lower level subsemiring of A.

Proof: It is trivial.

2.35 Theorem: Let A be an anti-fuzzy normal subsemiring of a semiring R. If any two lower level subsemirings of A belongs to R, then their union is also a lower level subsemiring of A in R.

Proof: It is trivial.

2.36 Theorem: Let A be an anti-fuzzy normal subsemiring of a semiring R. If $\alpha_i \in [0,1]$, and $A_{\alpha i}$, $i \in I$ is a collection of lower level subsemirings of A, then their union is also a lower level subsemiring of A.

Proof: It is trivial.

2.37 Theorem: The homomorphic image of a lower level subsemiring of an anti-fuzzy normal subsemiring of a semiring R is a lower level subsemiring of an anti-fuzzy normal subsemiring of a semiring R^l.

Proof: It is trivial.

2.38 Theorem: The homomorphic pre-image of a lower level subsemiring of an anti-fuzzy normal subsemiring of a semiring R^l is a lower level subsemiring of an anti-fuzzy normal subsemiring of a semiring R.

Proof: It is trivial.

2.39 Theorem: The anti-homomorphic image of a lower level subsemiring of an anti-fuzzy normal subsemiring of a semiring R is a lower level subsemiring of an anti-fuzzy normal subsemiring of a semiring R^l.

Proof: It is trivial.

2.40 Theorem: The anti-homomorphic pre-image of a lower level subsemiring of an anti-fuzzy normal subsemiring of a semiring R^l is a lower level subsemiring of an anti-fuzzy normal subsemiring of a semiring R.

Proof: It is trivial.

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