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A Review: Hemodynamics of Cerebral Aneurysm with Mathematical Modeling

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Abstract

Hemodynamic parameters of cerebral aneurysm play an important role in the progression, growth and rupture of an aneurysm. Therefore, the knowledge of hemodynamic parameters may provide physicians with understanding of aneurysm initiation, growth and rupture. Progression of medical imaging technology and improvement of the computer has enabled to predict the hemodynamics of aneurysm with increase accuracy using computational analysis. In this paper, computational hemodynamic studies on cerebral aneurysm initiation, growth and rupture are reviewed. A mathematical model to govern the flow in an aneurysm is also presented. A mathematical model and the computational of hemodynamics in cerebral aneurysm are expected to provide for planing and decision for treatment.

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1 Introduction

Cerebral aneurysm is a bulge in the vessel wall of an artery located in the brain. Aneurysm can be developed due to weakness of the arterial wall. If it becomes large enough, it can rupture and spill blood into the surrounding tissue. There are three types of cerebral aneurysms, separated by their geometry [1]. A saccular aneurysm (or berry aneurysm) is the most common type of aneurysm. It is rounded look like berry that is attached to an artery by a neck or stem. A less common type of aneurysm is a fusiform aneurysm. It is spindle-shaped. It is formed though the widening of the vessel wall. Another type of aneurysm is a giant aneurysm. It is a berry aneurysm but it large. It occurs at the bifurcation of an artery.

Since the initiation of aneurysm is not well understood, there is no known the prevention for this vascular disease. The numerous studies have been performed to provide the detailed hemodynamic information of the artery. Many studies have attempted to identify appropriate hemodynamic properties correlated with the aneurysm initiation. Hemodynamic properties such as the blood pressure, velocity of blood flow and the wall shear stress are performed to be linked to the progression of the aneurysm [2, 3]. Wall shear stress is considered to be the effect of the development of the cerebral aneurysm. The wall shear stress acts directly on the endothelium cell or a growth of mechanism of an aneurysm. Furthermore, high wall shear stress magnitude or high spatial and temporal variation of wall shear stress might mechanically damage the inner wall artery [4, 5, 6, 7, 8].

For the aneurysm growth, many computational fluid dynamic (CFD) studies have been performed to predict the hemodynamics of the aneurysms using not only the ideal curved and bifurcation [9, 10, 11, 12], but also aneurysm models based on data from medical imaging technology [13, 14, 15]. Atherosclerotic wall changes due to a low wall shear stress have been studied. The complex flow patterns and the low wall shear stress may be correlated with aneurysm growth. Several theoretical studies related to the growth of an aneurysm [10, 11] have been presented. Kroon et al. (2009,[11]) proposed a new theoretical model for the growth of saccular cerebral aneurysm. The model is able to predict wall shear stresses correlated with the experiment. The model is also used to predict future growth of aneurysm.

Rupture of cerebral aneurysm occurs when arteries wall tension exceeds the mechanical strength of the wall. Wall tension is proportional to intramural high blood pressure, large aneurysm and thin wall. Moreover, the hemodynamic may affect the wall remodeling process. Therefore, hemodynamic including high blood pressure and wall shear stress may directly influence wall rupture and low wall shear stress and flow distribution may affect the aneurysm wall weakening [3, 16]. Nieto et al. (2000, [17]) studied a nonlinear biomathematical model for study the intracranial aneurysms. The results show that a sudden change in blood pressure and turbulent flow inside aneurysm affect to rupture of aneurysm. The CFD studies of wall shear stress have been performed on the rupture of aneurysm. Shojima et al. (2004 [18]) simulated the wall shear stress in human cerebral aneurysm. They concluded that the high wall shear stress may not be correlated with aneurysm rupture. This results agree with results obtained from Valencia et al. (2008 [19]). However, pressure may be the hemodynamic effect on aneurysm rupture. Many studies demonstrated that the flow pattern around the aneurysm elevated blood pressure at aneurysm [20, 21, 22]. A large blood pressure along a wall affects on wall shear stress in an aneurysm. Therefore, hypertension may affect the aneurysm rupture.

In order to investigate hemodynamics in cerebral aneurysm, some mathematical and experiment studies have been proposed [23, 24, 25, 26, 27]. However, experiments have been conducted only for flows in a simple geometry. It is difficult to measure detailed characteristics of the flow by experiment. Recent progress in medical imaging technology and advancements of computer technology have enabled computational fluid dynamics (CFD) analysis to predict the hemodynamics of aneurysms. Over the last two decades, extensive research has been carried out to study various phenomena occurring in the blood flow problem, including experimental, analytical and numerical studies [23, 24, 25, 26, 27]. Numerical investigation has been used under various conditions in simulating blood flow. Studies for both normal and aneurysm vessels have been carried out for idealized arteries, idealized arterial bifurcations, branchings arteries. Most analyses assume the fluid to be Newtonian, a generally valid approximation for the rheological behavior of blood in the larger blood vessels. There has been some work on the flow of non-Newtonian fluids [28].

2 Mathematical Modeling Hemodynamics

Precise analysis of blood flow through arteries requires coupling of the blood flow with the elastic deformation of the blood vessel. To capture the main feature of blood flow through aneurysm arteries and to keep the model simple, the effect of the deformation of blood vessels on blood flow is neglected. It has been generally accepted that human blood behaves as a Newtonian fluid when the shear rate is greater than 100 s^{-1}. However, when the shear rate is lower than 100 s^{-1}, blood behaves as a non-Newtonian fluid, and the shear stresses depend nonlinearly on the deformation rate. In pulsatile blood flow, the instantaneous shear rate over a cardiac cycle may vary from zero to more than 1000 s^{-1} depending on the problem under examination. If human blood is modelled as a non-Newtonian fluid, the stress-deformation rate relation is described by

$$\sigma = -pI + 2\eta(\dot{\gamma})\mathbf{D}, \tag{1}$$

where p is the pressure and D is the rate of deformation tensor given by

$$\mathbf{D} = \frac{1}{2}\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right),$$

η and $\dot{\gamma}$ denote respectively the viscosity of blood and shear rate. Various non-Newtonian models have been proposed to describe the relation between η and $\dot{\gamma}$. The Carreau's shear-thinning model

$$\eta = \eta_\infty + (\eta_0 - \eta_\infty)\left[1 + (\lambda\dot{\gamma})^2\right]^{(n-1)/2},$$

in which $\dot{\gamma} = \sqrt{2tr(\mathbf{D}^2)}$ is a scalar measure of the rate of deformation tensor. The shear rate is corresponding to

$$\dot{\gamma} = \sqrt{2u_{1x}^2 + 2u_{2y}^2 + 2u_{3z}^2 + (u_{1y} + u_{2x})^2 + (u_{2z} + u_{3y})^2 + (u_{1z} + u_{3x})^2}.$$

η_0 and η_∞ denote the zero shear viscosity and the infinite shear viscosity. The consistency index, n, is a parameter whose value is between 0 and 1.

The equations governing the blood flow include the constitutive equation (1) and the following continuity and stress equations of motion:

$$\nabla \cdot \mathbf{u} = 0, \tag{2}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = \frac{1}{\rho} \nabla \cdot \sigma, \tag{3}$$

where ρ denotes the blood density.

By substituting equation (1) into (3), we have the following Navier-Stokes equations

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = \frac{1}{\rho} \nabla \cdot [-pI + \eta(\nabla \mathbf{u} + (\nabla \mathbf{u})^T)]. \tag{4}$$

Using appropriates boundary conditions, the governing equations can be solved numerically by using the finite element, finite difference, and more recently finite volume methods. [29, 30, 31].

3 Conclusion

The control of flow pattern of blood through cerebral aneurysm plays an important role in the formation, progression and rupture of cerebral aneurysm. To understand aneurysm progression, growth and rupture of aneurysm, knowledge of hemodynamic parameters is reviewed. Mathematical model to simulation blood flow in cerebral aneurysm has many advantage. A mathematical model to govern the flow in an aneurysm is presented. Governing equations consist of Navier-Stokes equations. The governing equations with appropriate boundary conditions lead to a suitable mathematical model. Finite element, finite difference, and more recently finite volume methods have been used for flow calculations. Although the mathematical model can do some experiment, the development of a mathematical model and numerical techniques still require for future study.

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