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Optimal Reserve Inventory between Two Machines when the Repair Time has Change of Distribution after a Change Point

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Abstract

In this paper an inventory model is derived under a situation that when two machines M_1 and M_2 are in series and the output of machine M_1 is the input for machine M_2. The purpose of this model is to derive an optimum reserve inventory for the machine M_2 when the machine M_1 goes to repair state or down state. It is considered that the repair time or the down time of M_1 is taken as a random variable and undergoes a change of distribution at a particular point called the change point. Numerical Illustrations are provided by considering suitable situations.

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1. Introduction

MODEL - I

A model in which two machines in series is considered. The output of machine M_1 is the input for machine M_2. Whenever machine M_1 is at fault i.e not working, then the machine M_2 may go to an idle state if enough reserve inventories for M_2 is not maintained.

It is to be noted that if there is excess stock, then there will be cost of inventory or if there is inadequate stock, then there will be shortage of inventory since there is an idle time cost for machine M_2.

Hence optimal reserve inventory is suggested.

For example in Gold mines, the minerals containing gold are excavated from the mines and processed to extract pure gold. If there is a breakdown of machine M_1 installed in mines, the processing of extracting gold is not possible. The process state i.e M_2 will become idle. The idle time of M_2 is costlier. Hence a reserve inventory between M_1 and M_2 is suggested.

In paper [2], the authors have discussed the breakdown time distribution undergoes a change of parameter. It is discussed that the change of parameter occurs after the change point or truncation point.

ASSUMPTIONS OF THE MODEL - I

- When machine M_1 goes to downstate, the supply to the machine M_2 is from the reserve inventory
- The consumption rate of M_2 is constant
- The repair time of M_1 is a random variable
- Within a short duration the reserve inventory 'S' is replenished after each breakdown of M_1.

Notations employed in this study

X: continuous r.v denotes the breakdown time of M_1 whose p.d.f is f(x) and c.d.f is F(x).
τ : change point or truncation point is a random variable denoting the duration of breakdown of M_1 and its pdf is g(.) with cdf G(.)
S : reserve inventory
$\hat{S}$: optimum reserve inventory
μ : mean time interval between breakdowns of machine M_1

d : cost per unit time of idle time of machine M_2
h : cost per unit time of holding one unit of reserve inventory
r : consumption rate per unit time of machine M_2

Diagram of a Model - I

2. Overview of the Basic Model

The idle time of M_2 is given by

$$t = \begin{cases} 0 & \text{if } \tau \leq \frac{S}{r} \\ \tau - \frac{S}{r} & \text{if } \tau > \frac{S}{r} \end{cases} \qquad 2.1$$

From [1], the expected cost of overages and shortages in inventory is given by the equation

$$E(C) = hS + \frac{d}{\mu}\int_{\frac{S}{r}}^{\infty}\left(\tau - \frac{S}{r}\right) g(\tau)d\tau \qquad 2.2$$

where $\frac{1}{\mu}$ = the average number of breakdowns of M_1 per unit time.
h = inventory holding cost.
d = shortage cost.

The optimal reserve size $\hat{S}$ can be obtained by solving the equation $\frac{dE(C)}{dS} = 0$

Hence $$G\left[\frac{\hat{S}}{r}\right] = 1 - \left[\frac{r\mu h}{d}\right] \qquad 2.3$$

This is the basic model discussed in [1].

This model has been modified and studied in a different technique in the papers [2] - [6] by introducing the Setting the Clock Back to Zero (SCBZ) property.

A different version of the above model is discussed here by using the concept of change of distribution at a change point. The concept of change of distribution at a change point is discussed in [7].

3. Change of Breakdown Time Distribution after a Change Point

Let X be a breakdown time r.v distributed according to a probability law and $x \in (0, \infty)$. Let τ be a fixed point and $\tau \in (0, \infty)$. If the breakdown time falls below τ then X is distributed according to the probability law $f_1(x, \theta)$ where 'θ' is a parameter and if it is greater than τ then X is distributed according to the probability law $f_2(x, k)$ where 'k' is a parameter i.e the breakdown time undergoes a change after τ and hence τ is called a change point or truncation point. The idea of change of parameter is discussed in [8] and it can be further developed as follows.

The p.d.f of X is

$$f(x) = \begin{cases} f_1(x) & when\ x \leq \tau \\ \overline{F_1}(\tau) f_2(x-\tau) & when\ x > \tau \end{cases}$$

where $F_1(\tau)$ is c.d.f of $f_1(x)$ and $\overline{F_1(\tau)} = 1 - F_1(\tau)$.

MODEL - II

Let us consider a new model with the same assumptions and notations of model I

Suppose that the r.v X follows exponential distribution with parameter 'θ' before 'τ' and gamma 2 distribution with parameter 'k' after 'τ'. The p.d.f of X is given by

$$f(x) = \begin{cases} \theta e^{-\theta x} & if\ 0 < x \leq \tau \\ \dfrac{e^{-\theta\tau} 2^k (x-\tau)^{k-1} e^{-2(x-\tau)}}{\Gamma k} & if\ x > \tau \end{cases} \qquad 3.1$$

It can be verified that f(x) is a proper p.d.f

4. Setting the Clock Back to Zero Property (SCBZ property)

This property is the generalization of the memory less property (MLP) which is enjoyed by the exponential distribution. This property is introduced in [9]. The random variable X is said to satisfy SCBZ property if

$$\frac{S(x+\tau,\theta,k)}{S(\tau,\theta)} = S(x,k)$$

where $S(\tau,\theta)$ is the survivor function and $S(\tau,\theta) = 1 - F(x,\theta)$.

Let us show that equation 3.1 satisfies SCBZ property.

$S(x,k) = P(X > x)$

$$= \int_x^\infty f(x)dx$$

$$= \int_x^\tau f_1(x)dx + \int_\tau^\infty f_2(x)dx$$

$$= \int_x^\tau \theta e^{-\theta x}dx + \int_\tau^\infty \frac{e^{-\theta\tau}2^k(x-\tau)^{k-1}e^{-2(x-\tau)}}{\Gamma k}dx$$

$$= (e^{-\theta x} - e^{-\theta\tau}) + \frac{e^{-\theta\tau}2^k}{\Gamma k}\,\mathrm{I} \qquad 4.1$$

where $I = \int_\tau^\infty (x-\tau)^{k-1}e^{-2(x-\tau)}dx$

put y = x – τ $\therefore dy = dx$

Limits : when x = τ , y = 0 ; when x = ∞, y = ∞.

$$I = \int_{0\tau}^\infty y^{k-1}e^{-2y}dy$$

$$= \frac{\Gamma k}{2^k}$$

Thus equation 4.1 becomes

$$S(x,k) = (e^{-\theta x} - e^{-\theta\tau}) + \frac{e^{-\theta\tau}2^k}{\Gamma k}\frac{\Gamma k}{2^k} = e^{-\theta x}$$

$$S(x,k) = e^{-\theta x}$$

Now

$$\frac{S(x+\tau,\theta,k)}{S(\tau,\theta)} = \frac{e^{-\theta(x+\tau)}}{e^{-\theta\tau}} = e^{-\theta x} = S(x,k)$$

Thus f(x) possesses SCBZ property.

Let us calculate E(X) of model II

$$E(X) = \int_0^\tau x\theta e^{-\theta x}dx + \int_\tau^\infty x\frac{e^{-\theta\tau}2^k(x-\tau)^{k-1}e^{-2(x-\tau)}}{\Gamma k}dx$$

After integration

$$E(X) = \frac{1}{\theta} + e^{-\theta\tau}\left(\frac{k}{2} - \frac{1}{\theta}\right)$$

Substituting $\mu = E(X)$ in the equation 2.3, we get

$$G\left(\frac{\hat{S}}{r}\right) = 1 - \frac{hr}{d}\left[\frac{1}{\theta} + e^{-\theta\tau}\left(\frac{k}{2} - \frac{1}{\theta}\right)\right]$$

Special Case

Let g(τ) ~ exp(λ)

$$G\left(\frac{\hat{S}}{r}\right) = P\left(\tau \leq \frac{\hat{S}}{r}\right)$$

$$1 - e^{-\lambda\left(\frac{\hat{S}}{r}\right)} = 1 - \frac{hr}{d}\left[\frac{1}{\theta} + e^{-\theta\tau}\left(\frac{k}{2} - \frac{1}{\theta}\right)\right]$$

After simplification

$$\hat{S} = \frac{r}{\lambda}\, log_e\left[\frac{hr}{d}\left(\frac{1}{\theta} + e^{-\theta\tau}\left(\frac{k}{2} - \frac{1}{\theta}\right)\right)\right]^{-1}$$

5. Numerical Illustration and Discussion

The following illustrations are taken up and discussed.

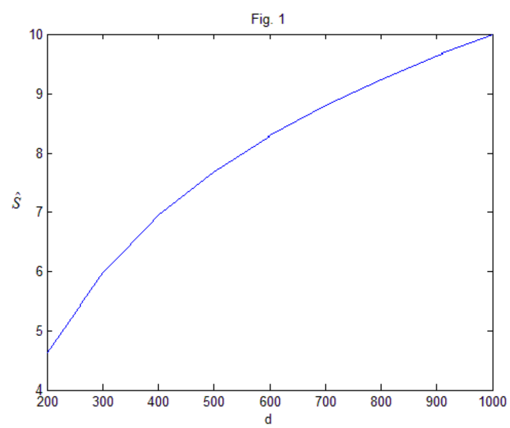
Consider an inventory system with the following parametric values in the proper units.

Example :(i)

Changes in 'd' and the other parameters kept fixed.
$h = 10,\ \tau = 20\ ,\ \lambda = 1.5,\ r = 5,\ \theta = 1.0,\ k = 1.2,$

Table (5.1)

d	200	300	400	500	600	700	800	900	1000
$\hat{S}$	4.6210	5.9725	6.9315	7.6753	8.2830	8.7969	9.2420	9.6346	9.9858

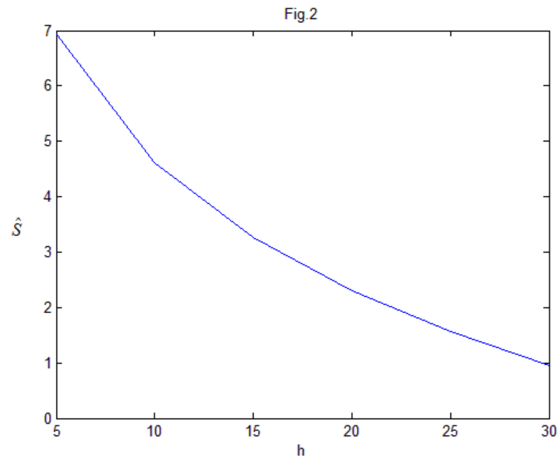


Example : (ii)

Changes in h and the other parameters kept fixed.
$d = 200,\quad \tau = 20,\quad \lambda = 1.5,\quad r = 5,\quad \theta = 1.0,\quad k = 1.2$

Table (5.2)

h	5	10	15	20	25	30
$\hat{S}$	6.9315	4.6210	3.2694	2.3105	1.5667	0.9589



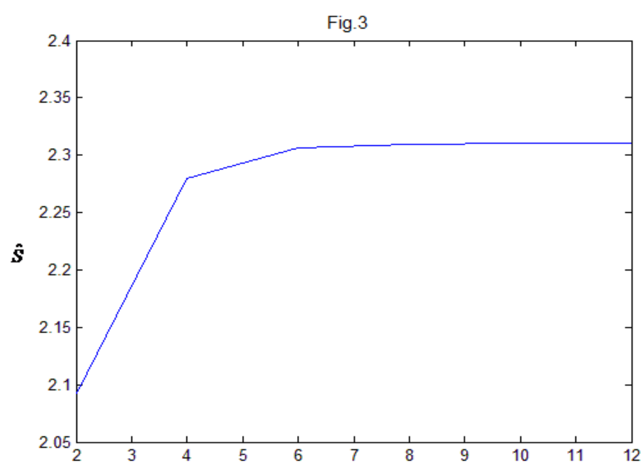
Example :(iii)

Changes in $'\tau'$ and the other parameters kept fixed.

$d = 200, \quad h = 20, \quad \lambda = 1.5, \quad r = 5, \quad \theta = 1.0, \quad k = 3$

Table (5.3)

τ	2	4	6	8	10	12
$\hat{S}$	2.0922	2.2801	2.3064	2.3099	2.3104	2.3105



CONCLUSIONS

The following conclusions may be drawn from the analysis of the data used

1. From example (i) it is understood that if the shortage cost 'd' increases, the optimal reserve $\hat{S}$ also increases when all other constants are kept fixed. Therefore, it is recommended that if the shortage cost is larger, the reserve inventory should also be larger.

2. From example (ii) it is clear that if the holding cost 'h' increases, there is a drop in the value of $\hat{S}$ when all other parameters are kept fixed. Hence the smaller inventory is suggested if the inventory holding cost 'h' is larger.

3. From example (iii) If the duration of breakdown time 'τ' increases then the size of the inventory is also increases. However, if the repair time is considerably large, the role of 'τ' may be more significant and it depends upon the values of 'r' also.

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