International Mathematical Forum, Vol. 7, 2012, no. 34, 1669 - 1677

On the Difference Space $m(M, A, \phi, p)(\Delta_u^r, q, \mu)$ Defined by an Orlicz Function and an Infinite Matrix

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Abstract

The purpose of this paper is to define and study the space $m(M, A, \phi, p)(\Delta_u^r, q, \mu)$ defined by an Orlicz function and infinite matrix, where $A = (a_{ik})$ is an infinite matrix of complex numbers, $u = (u_i)$ is an arbitrary sequence such that $u_i \neq 0$, $i = 1, 2, 3, \cdots$, and for any sequence $x = (x_i)$, the difference sequence Δx is given by $\Delta x = (\Delta x_i)_{i=1}^{\infty} = (x_i - x_{i+1})_{i=1}^{\infty}$. We also study some inclusion relations involving the space $m(M, A, \phi, p)(\Delta_u^r, q, \mu)$ and some other results.

Mathematics Subject Classification: 40A05,40C05, 40A45

Keywords: Difference sequence, lacunary sequence, statistical convergence and sequence of modulus functions

1 Introduction

Let w, l_∞ and l_p denote the spaces of all, bounded, and p absolutely summable sequences respectively. Also, φ_s denotes the set of all subsets of $\mathbb{N}$, those do not contain more than s elements. Further (ϕ_n) will denote a nondecreasing sequence of positive real numbers such that $n\phi_{n+1} \leq (n+1)\phi_n$, for all $n \in \mathbb{N}$. The class of all sequences satisfying this property is denoted by Φ.

The space $m(\phi)$ was defined and studied by Sargent [16] who studied some of it's properties and obtained it's relationship with the space l_p. Later on, it was investigated by Rath [13], Rath and Tripathy [14], Tripathy and Sen [19], Tripathy and Mahanta [18] and others.

An Orlicz function is a function $M : [0, \infty) \to [0, \infty)$ which is continuous, nondecreasing, and convex with $M(0) = 0$, $M(x) > 0$ for $x > 0$ and $M(x) \to \infty$, as $x \to \infty$, (see [7]).

If convexity of M is replaced by $M(x+y) \leq M(x)+M(y)$, then it is called a modulus function, defined and studied by Nakano [11], Ruckle [15], Maddox [9] and others.

An Orlicz function M is said to satisfy the Δ_2-condition for all values of l, if there exist a constant $K > 0$ such that $M(2l) \leq KM(l)(l \geq 0)$ (see Krasnoselskii and Rutickii [7]).

An Orlicz function M can always be represented in the following integral form $M(x) = \int\limits_0^x q(t)dt$, where q, known as the kernel of M, is right-differentiable for $t \geq 0, q(0) = 0, q(t) > 0$ for $t > 0, q$ is nondecreasing, and $q(t) \rightarrow \infty$ as $t \rightarrow \infty$.

Lindenstrauss and Tzafriri [8] used the idea of Orlicz function to define what is called an Orlicz sequence space :

$$l_M = \{x = (x_i) \in w : \sum_{k=1}^{\infty} M(\frac{\mid x_i \mid}{\rho}) < \infty, \text{ for some } \rho \ > 0\}$$

which is a Banach space with the norm :

$$\parallel x \parallel_M = \inf\{\rho \ > 0 : \sum_{k=1}^{\infty} M(\frac{\mid x_i \mid}{\rho}) \leq 1\}.$$

The space l_M is closely related to the space l_p which is an Orlicz space with $M(x) = x^p, 1 \leq p < \infty$.

Different Orlicz sequence spaces were studied by several mathematicians as Bhardwaj and Singh [3], Bilgen [4], Güngör et al [6], Tripathy and Mahanta [18], Esi and Et [5], Parashar and Choudhary [12] and many others.

The following inequality will be used throughout the paper :

$$\mid a_i + b_i \mid^{p_i} \leq \max(1, 2^{H-1})(\mid a_i \mid^{p_i} + \mid b_i \mid)^{p_i},$$

where a_i and b_i are complex numbers and $H = \sup p_i < \infty$.

The sequence space $m(\phi)$ was introduced and studied by Sargent [16]. Later on, it was investigated by Tripathy [17], Tripathy and Sen [19] and Malkowsky and Mursaleen [10]. In 2003, Tripathy and Mahanta [18] defined and studied the sequence space $m(M, \Delta, \phi)$.

Let $A = (a_{ik})$ be an infinite matrix of complex numbers. We write $Ax = (A_i(x))$ if $A_i(x) = \sum_{k=1}^{\infty} a_{ik}x_k$ converges for each i.

Let $p = (p_i)$ be a bounded sequence of positive real numbers such that $0 < h = \inf p_i \leq p_i \leq \sup p_i < \infty$. Then Altun and Bilgen [1] introduced the space

$$m(M, A, \phi, p) = \{x \in w : \sup_{s \geq 1, \sigma \in \varphi_s} \frac{1}{\phi_s} \sum_{i \in \sigma} M(\frac{| A_i(x) |}{\rho})^{p_i} < \infty, \text{ for some } \rho > 0\}.$$

Let r be a fixed positive integer and $0 \leq p < \infty$, then Colak and Et [2] defined and discussed the sequence space

$$m(\phi, p)(\Delta^r) = \{x \in w : \sup_{s \geq 1, \sigma \in \varphi_s} \frac{1}{\phi_s} \sum_{i \in \sigma} | \Delta^r x_i |^p < \infty\}.$$

Now, if $u = (u_i)$ is any sequence such that $u_i \neq 0$ for each $i, w(X)$ denotes the space of all sequences with elements in X, where (X, q) denotes a seminormed space, seminormed by q, and μ is any real number such that $\mu \geq 0$, then we define the following sequence spaces :

$$m(M, A, \phi, p)(\Delta_u^r, q, \mu)$$

$$= \{x \in w : \sup_{s \geq 1, \sigma \in \varphi_s} \frac{1}{\phi_s} \sum_{i \in \sigma} i^{-\mu} M(q(\frac{| A_i(\Delta_u^r x) |}{\rho})^{p_i} < \infty, \text{ for some } \rho > 0\}.$$

where

$$\Delta_u^0 x = u_i x_i,$$

$$\Delta_u^1 x = u_i x_i - u_{i+1} x_{i+1},$$

$$\Delta_u^2 x = \Delta(\Delta_u^1 x),$$

$$\vdots$$

$$\Delta_u^r x = \Delta(\Delta_u^{r-1} x),$$

so that

$$\Delta_u^r x = \Delta_{u_i}^r x_i = \sum_{j=0}^{r} (-1)^j \binom{r}{j} u_{i+j} x_{i+j}.$$

2 Main Results

We prove the following theorems.

Theorem 2.1. The space $m(M, A, \phi, p)(\Delta_u^r, q, \mu)$ is linear space over the complex field $\mathbb{C}$.

Proof. Let $x = (x_i), y = (y_i) \in m(M, A, \phi, p)(\Delta_u^r, q, \mu)$ and $\alpha, \beta \in \mathbb{C}$. Then there exists some positive ρ_1 and ρ_2 such that :

$$\sup_{s\geq 1, \sigma\in\varphi_s} \frac{1}{\phi_s}\sum_{i\in\sigma} i^{-\mu} M(q(\frac{\mid A_i(\Delta_u^r x)\mid}{\rho_1})^{p_i} < \infty$$

and

$$\sup_{s\geq 1, \sigma\in\varphi_s} \frac{1}{\phi_s}\sum_{i\in\sigma} i^{-\mu} M(q(\frac{\mid A_i(\Delta_u^r y)\mid}{\rho_2})^{p_i} < \infty$$

Define $\rho = \max(2 \mid \alpha \mid \rho_1, 2 \mid \beta \mid \rho_2)$. Then we have

$$\begin{aligned}
& \sup_{s\geq 1, \sigma\in\varphi_s} \frac{1}{\phi_s}\sum_{i\in\sigma} i^{-\mu} M(q(\frac{\mid A_i(\alpha\Delta_u^r x + \beta\Delta_u^r y)\mid}{\rho})^{p_i} \\
= & \sup_{s\geq 1, \sigma\in\varphi_s} \frac{1}{\phi_s}\sum_{i\in\sigma} i^{-\mu} M(q(\frac{\mid A_i(\alpha\Delta_u^r x) + A_i(\beta\Delta_u^r y)\mid}{\rho})^{p_i} \\
\leq & \max(1, 2^{H-1}) \sup_{s\geq 1, \sigma\in\varphi_s} \frac{1}{\phi_s}\sum_{i\in\sigma} i^{-\mu} M(q(\frac{\mid A_i(\alpha\Delta_u^r x)\mid}{\rho_1})^{p_i} \\
& + \sup_{s\geq 1, \sigma\in\varphi_s} \frac{1}{\phi_s}\sum_{i\in\sigma} i^{-\mu} M(q(\frac{\mid A_i(\beta\Delta_u^r y)\mid}{\rho_2})^{p_i} \\
< & \infty.
\end{aligned}$$

Hence $\alpha x + \beta y \in m(M, A, \phi, p)(\Delta_u^r, q, \mu)$.

Theorem 2.2. Let $Ax \to \infty$, as $x \to \infty$. Then the space $m(M, A, \phi, p)(\Delta_u^r, q, \mu)$ is a linear topological space paranormed by

$$g(x) = \{\rho^{\frac{p_k}{H}} : [\sup_{s\geq 1, \sigma\in\varphi_s} \frac{1}{\phi_s}\sum_{i\in\sigma} i^{-\mu} M(q(\frac{\mid A_i(\Delta_u^r x)\mid}{\rho})^{p_i}]^{1/H} \leq 1, k = 1, 2, 3, \cdots\}.$$

Proof. Clearly $g(x) = g(-x)$. Since $M(0) = 0$, we see that $Ax = 0$ for $x = 0$, therefore $g(x) = 0$.

Let $x = (x_i)$, $y = (y_i) \in m(M, A, \phi, p)(\Delta_u^r, q, \mu)$. Then there exists some $\rho_1 > 0$ and $\rho_2 > 0$ such that :

$$\sup_{s\geq 1,\sigma\in\varphi_s} \frac{1}{\phi_s} \sum_{i\in\sigma} i^{-\mu} M(q(\frac{\mid A_i(\Delta_u^r x) \mid}{\rho})^{p_i} \leq 1$$

and

$$\sup_{s\geq 1,\sigma\in\varphi_s} \frac{1}{\phi_s} \sum_{i\in\sigma} i^{-\mu} M(q(\frac{\mid A_i(\Delta_u^r y) \mid}{\rho})^{p_i} \leq 1.$$

Let $\rho = \rho_1 + \rho_2$. Then we have

$$\begin{aligned} & \sup_{s\geq 1,\sigma\in\varphi_s} \frac{1}{\phi_s} \sum_{i\in\sigma} i^{-\mu} M(q(\frac{\mid A_i(\Delta_u^r (x+y)) \mid}{\rho})^{p_i} \\ \leq \ & (\frac{\rho_1}{\rho_1+\rho_2})^h \sup_{s\geq 1,\sigma\in\varphi_s} \frac{1}{\phi_s} \sum_{i\in\sigma} i^{-\mu} M(q(\frac{\mid A_i(\Delta_u^r x) \mid}{\rho_1})^{p_i} \\ & +(\frac{\rho_2}{\rho_1+\rho_2})^h \sup_{s\geq 1,\sigma\in\varphi_s} \frac{1}{\phi_s} \sum_{i\in\sigma} i^{-\mu} M(q(\frac{\mid A_i(\Delta_u^r y) \mid}{\rho_2})^{p_i}. \end{aligned}$$

Hence, we get that $g(x + y) \leq g(x) + g(y)$.

Finally, for $\lambda \in \mathbb{C}$, without loss of generality let $\lambda \neq 0$, then the continuity of the scalar multiplication follows from the following inequality :

$$\begin{aligned} g(\lambda x) &= \{\rho^{\frac{p_k}{H}} : [\sup_{s\geq 1,\sigma\in\varphi_s} \frac{1}{\phi_s} \sum_{i\in\sigma} i^{-\mu} M(q(\frac{\mid A_i(\lambda\Delta_u^r x) \mid}{\rho})^{p_i}]^{1/H} \leq 1, k = 1, 2, 3, \cdots\} \\ &= \{\mid \lambda \mid \rho_1^{\frac{p_k}{H}} : [\sup_{s\geq 1,\sigma\in\varphi_s} \frac{1}{\phi_s} \sum_{i\in\sigma} i^{-\mu} M(q(\frac{\mid A_i(\lambda\Delta_u^r x) \mid}{\rho_1})^{p_i}]^{1/H} \leq 1, k = 1, 2, 3, \cdots\} \end{aligned}$$

where $\rho =\mid \lambda \mid \rho_1$.

Hence, we get that $g(\lambda x) \leq \max(1, \mid \lambda \mid)\ g(x)$.

Theorem 2.3. The space $m(M, A, \phi^1, p)(\Delta_u^r, q, \mu) \subseteq m(M, A, \phi^2, p)(\Delta_u^r, q, \mu)$ if and only if $\sup_{s\geq 1} \frac{\phi_s^1}{\phi_s^2} < \infty$.

Proof. Let $x = (x_i) \in m(M, A, \phi^1, p)(\Delta_u^r, q, \mu)$ and $T = \sup_{s\geq 1} \frac{\phi_s^1}{\phi_s^2}$. Then we can write :

$$\begin{aligned} & \sup_{s\geq 1,\sigma\in\varphi_s} \frac{1}{\phi_s^2} \sum_{i\in\sigma} i^{-\mu} M(q(\frac{\mid A_i(\Delta_u^r x) \mid}{\rho})^{p_i} \\ \leq \ & \sup_{s\geq 1} \frac{\phi_s^1}{\phi_s^2} \sup_{s\geq 1,\sigma\in\varphi_s} \frac{1}{\phi_s^1} \sum_{i\in\sigma} i^{-\mu} M(q(\frac{\mid A_i(\Delta_u^r x) \mid}{\rho})^{p_i} \\ = \ & T \sup_{s\geq 1,\sigma\in\varphi_s} \frac{1}{\phi_s^1} \sum_{i\in\sigma} i^{-\mu} M(q(\frac{\mid A_i(\Delta_u^r x) \mid}{\rho})^{p_i}. \end{aligned}$$

Therefore $x \in m(M, A, \phi^2, p)(\Delta_u^r, q, \mu)$.

Conversely, let $m(M, A, \phi^1, p)(\Delta_u^r, q, \mu) \subseteq m(M, A, \phi^2, p)(\Delta_u^r, q, \mu)$ and $x \in m(M, A, \phi^1, p)(\Delta_u^r, q, \mu)$. Then there exists $\rho > 0$ such that

$$\sup_{s\geq 1, \sigma\in\varphi_s} \frac{1}{\phi_s^1} \sum_{i\in\sigma} i^{-\mu} M(q(\frac{| A_i(\Delta_u^r x) |}{\rho})^{p_i} < \infty.$$

Suppose that $\sup_{s\geq 1} \frac{\phi_s^1}{\phi_s^2} = \infty$. Then there exists a sequence of positive natural numbers (s_j) such that $\lim_{j\to\infty} \frac{\phi_{s_j}^1}{\phi_{s_j}^2} = \infty$. Hence we can write

$$\begin{aligned} & \sup_{s\geq 1, \sigma\in\varphi_s} \frac{1}{\phi_s^2} \sum_{i\in\sigma} i^{-\mu} M(q(\frac{| A_i(\Delta_u^r x) |}{\rho})^{p_i} \\ \geq \; & \sup_{s\geq 1, \sigma\in\varphi_s} \frac{1}{\phi_s^1} \sum_{i\in\sigma} i^{-\mu} M(q(\frac{| A_i(\Delta_u^r x) |}{\rho})^{p_i} = \infty. \end{aligned}$$

Therefore $x \notin m(M, A, \phi^2, p)(\Delta_u^r, q, \mu)$ which is a contradiction. Hence $\sup_{s\geq 1} \frac{\phi_s^1}{\phi_s^2} < \infty$.

Proposition 2.4. Let M be an Orlicz function which satisfies the Δ_2-condition. Then $m(M, A, \phi^1, p)(\Delta_u^r, q, \mu) = m(M, A, \phi^2, p)(\Delta_u^r, q, \mu)$
if and only if $\sup_{s\geq 1} \frac{\phi_s^1}{\phi_s^2} < \infty$ and $\sup_{s\geq 1} \frac{\phi_s^2}{\phi_s^1} < \infty$.

Theorem 2.5. Let M and M_1 be Orlicz functions which satisfies the Δ_2-condition. Then $m(M, A, \phi, p)(\Delta_u^r, q, \mu) \subseteq m(M \circ M_1, A, \phi, p)(\Delta_u^r, q, \mu)$

Proof. Let $x \in m(M, A, \phi, p)(\Delta_u^r, q, \mu)$ and $\varepsilon > 0$ be given and choose δ with $0 < \delta < 1$ such that $M(t) < \varepsilon$ for $0 \leq t \leq \delta$. write

$$\begin{aligned} & \sup_{s\geq 1, \sigma\in\varphi_s} \frac{1}{\phi_s} \sum_{i\in\sigma} i^{-\mu} M(M_1(q(\frac{| A_i(\Delta_u^r x) |}{\rho})))^{p_i} \\ = \; & \sup_{s\geq 1, \sigma\in\varphi_s} \frac{1}{\phi_s} \sum_{1} i^{-\mu} M(M_1(q(\frac{| A_i(\Delta_u^r x) |}{\rho})))^{p_i} \\ & + \sup_{s\geq 1, \sigma\in\varphi_s} \frac{1}{\phi_s} \sum_{2} i^{-\mu} M(M_1(q(\frac{| A_i(\Delta_u^r x) |}{\rho})))^{p_i}, \end{aligned}$$

where the summation $\sum_1$ is over $M_1(q(\frac{|A_i(\Delta_u^r x)|}{\rho})) \leq \delta$ and the summation $\sum_2$ is over $M_1(q(\frac{|A_i(\Delta_u^r x)|}{\rho})) > \delta$.

Since M is continuous, we have

$$\begin{aligned}
&\sup_{s\geq 1,\sigma\in\varphi_s}\frac{1}{\phi_s}\sum_1 i^{-\mu}M(M_1(q(\frac{\mid A_i(\Delta_u^r x)\mid}{\rho})))^{p_i}\\
\leq\ &\max\{1,M(1)^H\}\sup_{s\geq 1,\sigma\in\varphi_s}\frac{1}{\phi_s}\sum_1 i^{-\mu}M_1(q(\frac{\mid A_i(\Delta_u^r x)\mid}{\rho}))^{p_i}\\
\leq\ &\max\{1,M(1)^H\}\sup_{s\geq 1,\sigma\in\varphi_s}\frac{1}{\phi_s}\sum_{i\in\sigma} i^{-\mu}M_1(q(\frac{\mid A_i(\Delta_u^r x)\mid}{\rho}))^{p_i}.
\end{aligned}$$

For $M_1(q(\frac{|A_i(\Delta_u^r x)|}{\rho}))>\delta$, we use the fact that $M_1(q(\frac{|A_i(\Delta_u^r x)|}{\rho}))<M_1(q(\frac{|A_i(\Delta_u^r x)|}{\rho}))\delta^{-1}\leq 1+M_1(q(\frac{|A_i(\Delta_u^r x)|}{\rho}))\delta^{-1}$.

Since M satisfies the Δ_2-condition, then there exists $L>1$ such that

$$\begin{aligned}
&M(M_1(q(\frac{\mid A_i(\Delta_u^r x)\mid}{\rho})))\\
<\ &M(1+M_1(q(\frac{\mid A_i(\Delta_u^r x)\mid}{\rho}))\delta^{-1})\\
\leq\ &\frac{1}{2}M(2)+\frac{1}{2}M(2M_1(q(\frac{\mid A_i(\Delta_u^r x)\mid}{\rho}))\delta^{-1})\\
\leq\ &\frac{1}{2}LM(2)M_1(q(\frac{\mid A_i(\Delta_u^r x)\mid}{\rho}))\delta^{-1}\\
&+\frac{1}{2}LM(2)M_1(q(\frac{\mid A_i(\Delta_u^r x)\mid}{\rho}))\delta^{-1}\\
=\ &LM(2)\delta^{-1}M_1(q(\frac{\mid A_i(\Delta_u^r x)\mid}{\rho})).
\end{aligned}$$

Now, we see that

$$\begin{aligned}
&\sup_{s\geq 1,\sigma\in\varphi_s}\frac{1}{\phi_s}\sum_2 i^{-\mu}M(M_1(q(\frac{\mid A_i(\Delta_u^r x)\mid}{\rho})))^{p_i}\\
\leq\ &\max\{1,(LM(2)\delta^{-1})^H\}\sup_{s\geq 1,\sigma\in\varphi_s}\frac{1}{\phi_s}\sum_2 i^{-\mu}M_1(q(\frac{\mid A_i(\Delta_u^r x)\mid}{\rho}))^{p_i}\\
\leq\ &\max\{1,(LM(2)\delta^{-1})^H\}\sup_{s\geq 1,\sigma\in\varphi_s}\frac{1}{\phi_s}\sum_{i\in\sigma} i^{-\mu}M_1(q(\frac{\mid A_i(\Delta_u^r x)\mid}{\rho}))^{p_i}
\end{aligned}$$

Hence

$$\begin{aligned}
&\sup_{s\geq 1,\sigma\in\varphi_s}\frac{1}{\phi_s}\sum_{i\in\sigma} i^{-\mu} M(M_1(q(\frac{\mid A_i(\Delta_u^r x)\mid}{\rho})))^{p_i}\\
\leq\ &\max\{1, M(1)^H\}\sup_{s\geq 1,\sigma\in\varphi_s}\frac{1}{\phi_s}\sum_{i\in\sigma} i^{-\mu} M_1(q(\frac{\mid A_i(\Delta_u^r x)\mid}{\rho}))^{p_i}\\
&+\max\{1, (LM(2)\delta^{-1})^H\}\sup_{s\geq 1,\sigma\in\varphi_s}\frac{1}{\phi_s}\sum_{i\in\sigma} i^{-\mu} M_1(q(\frac{\mid A_i(\Delta_u^r x)\mid}{\rho}))^{p_i}.
\end{aligned}$$

Therefore $x \in m(M\circ M_1, A, \phi, p)(\Delta_u^r, q, \mu)$.

Theorem 2.6. The sequence space $m(M, A, \phi, p)(\Delta_u^r, q, \mu)$ is solid.

Proof. Let $\alpha = (\alpha_i)$ be a sequence of scalars such that $\mid \alpha_i \mid \leq 1$, for all $i \in \mathbb{N}$. Then we get that

$$\begin{aligned}
&\sup_{s\geq 1,\sigma\in\varphi_s}\frac{1}{\phi_s}\sum_{i\in\sigma} i^{-\mu} M(q(\frac{\mid A_i(\alpha_i\Delta_u^r x_i)\mid}{\rho})^{p_i}\\
\leq\ &\sup_{s\geq 1,\sigma\in\varphi_s}\frac{1}{\phi_s}\sum_{i\in\sigma} i^{-\mu} M(q(\frac{\sup\mid \alpha_i\mid A_i(\Delta_u^r x_i)\mid}{\rho})^{p_i}\\
\leq\ &\sup_{s\geq 1,\sigma\in\varphi_s}\frac{1}{\phi_s}\sum_{i\in\sigma} i^{-\mu} M(q(\frac{\mid A_i(\Delta_u^r x_i)\mid}{\rho})^{p_i}.
\end{aligned}$$

Then the result follows from the above inequality.

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Received: January, 2012