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Single Machine Scheduling with Deterioration Effect on Delivery Time*

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Abstract

A single machine scheduling problem with delivery time is considered. Job's delivery time has deterioration effect. For the same processing time model of minimizing different objectives, it is given optimal sequences, respectively. For the same deterioration ratio model, we also give optimal sequences for different objectives.

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1 Introduction

Scheduling theory is an important branch of the combinatorial optimization, the scheduling problem with delivery time is the bridge between scheduling theory and supply chain management. We consider the scheduling problem as follows: there are jobs of set $N = \{J_1, J_2, \ldots, J_n\}$ to be processed without preemption on a single machine. The jobs are simultaneously available at time zero. The machine can handle only one job at a time. The process can not be breaked off. The vehicles are given enough, so the completed jobs can be deliveried immediately. Denote p_j and $Q_j = q_j + b_j t$ as J_j' processing time and delivery time, respectively, here q_j is the basic delivery time, $b_j > 0$ is the deterioration ratio and t means the start delivering time. So the delivery time have deterioration effect of start delivering time.

For job J_j, let C_j and D_j donote the completion process time and completion delivery time, respectively. We denote the scheduling problems using three parameters representation $\alpha|\beta|\gamma$, where α, β, γ represents the machine environment, jobs character and objective function. For example, $1|p_j = p, Q_j = q_j + b_j t| \sum D_j$ represents a single machine, the processing time are p, delivery

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time $Q_j = q_j + b_j t$, and the objective is to minimizing total completion delivery time.

Gupta and Gupta [1], Browne and Yechiali [2] presented the single machine scheduling problem $1|p_j = b_j + \alpha_j t|C_{\max}$, respectively, where b_j is the basic processing time, $\alpha_j > 0$ is the deterioration ratio and t means the starting time. Bachman and Janiak [3] proved that the problem $1|p_j = b_j + \alpha_j t|\sum \omega_j C_j$ was NP-hard, where ω_j means the job's weight. For the same basic processing time model, Mosheiov [4] showed that the optimal sequence of problem $1|p_j = b + \alpha_j t|\sum \omega_j C_j$ is V-shaped of deterioration ratio. For the same deterioration ratio model, Mosheiov [5] proved that the optimal sequence of problem $1|p_j = b_j + \alpha t|\sum \omega_j C_j$ is Λ-shaped of processing time.

In order to consider the problems in the aspect of the delivery stage, we focus our attentions on the total delivery completion time and some other objectives with certain practical significance. We consider the same processing time model in section 2 and the same deterioration ratio model in section 3, respectively.

2 Same processing time model

We first give a lemma of rearrangement inequality, which was presented by Hardy et al.[6] .

Lemma 2.1 *If $A = \{a_1, a_2, \cdots, a_n\}$ is a permutation of a finite set of real numbers and $B = \{b_1, b_2, \cdots, b_n\}$ is a permutation of another finite set of real numbers, the quantity $a_1b_1 + a_2b_2 + \cdots + a_nb_n$ is minimized when A and B are oppositely sorted.*

Theorem 2.2 *For the problems $1|p_j = p, Q_j = q_j + b_j t|\sum D_j$ and $1|p_j = p, Q_j = q_j + b_j t|\sum Q_j$, they have same optimal schedule with jobs are arrayed in nonincreasing order of b_j.*

Proof. Consider a schedule $\pi : J_1, J_2, \ldots, J_n$,

$$C_j = jp, s_j = (j-1)p$$

$$Q_j = q_j + b_j s_j = q_j + b_j (j-1)p \tag{1}$$

$$D_j = C_j + Q_j = q_j - b_j p + p(1+b_j)j \tag{2}$$

$$\sum D_j = \sum q_j - p\sum b_j + p\sum (1+b_j)j \tag{3}$$

$$\sum Q_j = \sum q_j - p\sum b_j + p\sum b_j j \tag{4}$$

$$\sum \omega_j D_j = \sum \omega_j q_j - p\sum \omega_j b_j + p\sum \omega_j (1+b_j)j \tag{5}$$

$$\sum \omega_j Q_j = \sum \omega_j q_j - p\sum \omega_j b_j + p\sum \omega_j b_j j \tag{6}$$

According to the lemma 2.1, (3) and (4), we have the conclusion of the theorem.

□

In addition, according to the lemma 2.1, (5) and (6), we have the theorem 2.3 and theorem 2.4, respectively.

Theorem 2.3 *For the problem* $1|p_j = p, Q_j = q_j + b_j t| \sum \omega_j D_j$*, there exist an optimal schedule with jobs are arrayed in nonincreasing order of* $\omega_j(1+b_j)$*.*

Theorem 2.4 *For the problem* $1|p_j = p, Q_j = q_j + b_j t| \sum \omega_j Q_j$*, there exist an optimal schedule with jobs are arrayed in nonincreasing order of* $\omega_j b_j$*.*

Notice that $C_j - p_j$ and Q_j are the processing time and the delivery time of J_j, respectively. In practice, the costs of wait, process, and delivery are different. For the reason of processing time is fixed, we only need to use α, β to denote the cost of wait and delivery per unit time, respectively, here we can see all the jobs are equal in this case. Our objective is to minimize the total waiting cost adds total delivery cost.

Theorem 2.5 *For the problem* $1|p_j = p, Q_j = q_j + b_j t|\alpha \sum (C_j - p_j) + \beta \sum Q_j$*, there exist an optimal schedule with jobs are arrayed in nonincreasing order of* b_j*.*

Proof. Consider a schedule $\pi : J_1, J_2, \ldots, J_n$,

$$\begin{aligned} &\alpha \sum (C_j - p_j) + \beta \sum Q_j \\ =&\sum (\alpha(j-1)p + \beta q_j + \beta b_j (j-1)p) \\ =&\sum \beta q_j + p \sum (\alpha + \beta b_j)(j-1) \end{aligned}$$

So π is an optimal schedule, if the jobs are arrayed in nonincreasing order of b_j.

□

Similarly, we have optimal schedule when the jobs have different weights in theorem 2.6. Under the case of jobs have different waiting costs and delivery costs per unit time, we also have the optimal schedule for the problem with equal and different weights of jobs in theorem 2.7 and 2.8.

Theorem 2.6 *For the problem* $1|p_j = p, Q_j = q_j + b_j t|\alpha \sum \omega_j (C_j - p_j) + \beta \sum \omega_j Q_j$*, there exist an optimal schedule with jobs are arrayed in nonincreasing order of* $\omega_j(\alpha + \beta b_j)$*.*

Theorem 2.7 *For the problem* $1|p_j = p, Q_j = q_j + b_j t| \sum \alpha_j (C_j - p_j) + \sum \beta_j Q_j$*, there exist an optimal schedule with jobs are arrayed in nonincreasing order of* $\alpha_j + \beta_j b_j$*.*

Theorem 2.8 *For the problem* $1|p_j = p, Q_j = q_j + b_j t| \sum \alpha_j \omega_j (C_j - p_j) + \sum \beta_j \omega_j Q_j$*, there exist an optimal schedule with jobs are arrayed in nonincreasing order of* $\omega_j(\alpha_j + \beta_j b_j)$*.*

3 Same deterioration ratio model

In this section, we consider the same deterioration model, which means the delivery time of job J_j is $Q_j = q_j + bt$. We have a series theorems in this section. Before presenting the theorems, we first give some equations. Consider a schedule $\pi : J_1, J_2, \dots, J_n$,

$$Q_j = q_j + b\sum_{i=1}^{j-1} p_j \tag{7}$$

$$D_j = q_j + (1+b)\sum_{i=1}^{j} p_j - bp_j \tag{8}$$

$$\sum Q_j = \sum q_j - b\sum p_j + b\sum C_i \tag{9}$$

$$\sum D_j = \sum q_j - b\sum p_j + (1+b)\sum C_i \tag{10}$$

$$\sum \omega_j Q_j = \sum \omega_j q_j - b\sum \omega_j p_j + b\sum \omega_j C_i \tag{11}$$

$$\sum \omega_j D_j = \sum \omega_j q_j - b\sum \omega_j p_j + (1+b)\sum \omega_j C_i \tag{12}$$

$$\begin{aligned} \alpha\sum(C_j - p_j) + \beta\sum Q_j &= \alpha\sum_{i=1}^{j-1} p_j + \beta(\sum q_j + b\sum_{i=1}^{j-1} p_j) \\ &= \beta\sum q_j - (\alpha + b\beta)\sum p_i + (\alpha + b\beta)\sum C_i \end{aligned} \tag{13}$$

$$\begin{aligned} \alpha\sum \omega_j(C_j - p_j) + \beta\sum \omega_j Q_j &= \alpha\sum_{i=1}^{j-1} \omega_j p_j + \beta(\sum \omega_j q_j + b\sum_{i=1}^{j-1} \omega_j p_j) \\ &= \beta\sum \omega_j q_j - (\alpha + b\beta)\sum \omega_j p_i + (\alpha + b\beta)\sum \omega_j C_i \end{aligned} \tag{14}$$

$$\begin{aligned} \sum \alpha_j(C_j - p_j) + \sum \beta_j Q_j &= \sum_{i=1}^{j-1} \alpha_j p_j + \sum \beta_j q_j + b\sum_{i=1}^{j-1} \beta_j p_j \\ &= \sum \beta_j q_j - \sum(\alpha_j + b\beta_j)p_i + \sum(\alpha_j + b\beta_j)C_i \end{aligned} \tag{15}$$

$$\begin{aligned} \sum \alpha_j\omega_j(C_j - p_j) + \sum \beta_j\omega_j Q_j &= \sum_{i=1}^{j-1} \alpha_j\omega_j p_j + \sum \beta_j\omega_j q_j + b\sum_{i=1}^{j-1} \beta_j\omega_j p_j \\ &= \sum \beta_j\omega_j q_j - \sum(\alpha_j + b\beta_j)\omega_j p_i + \sum(\alpha_j + b\beta_j)\omega_j C_i \end{aligned} \tag{16}$$

Theorem 3.1 *Based on the lemma 2.1,* (9)*,* (10) *and* (13)*, the problems* $1|p_j, Q_j = q_j+bt|\sum Q_j$, $1|p_j, Q_j = q_j+bt|\sum D_j$,*and* $1|p_j, Q_j = q_j+bt|\alpha\sum(C_j-p_j)+\beta\sum Q_j$ *have same optimal schedule with jobs are arrayed in nondecreasing order of* p_j.

Theorem 3.2 *Based on the lemma 2.1,* (11)*,* (12)*, and* (14)*, the problems* $1|p_j, Q_j = q_j + bt|\sum \omega_j Q_j$, $1|p_j, Q_j = q_j + bt|\sum \omega_j D_j$, *and* $1|p_j, Q_j = q_j + bt|\alpha\sum \omega_j(C_j-p_j)+\beta\sum \omega_j Q_j$ *have same optimal schedule with jobs are arrayed in nondecreasing order of* $\frac{p_j}{\omega_j}$.

Theorem 3.3 *Based on the lemma 2.1 and* (15)*, the problems* $1|p_j, Q_j = q_j + bt|\sum \alpha_j(C_j - p_j) + \sum \beta_j Q_j$ *has an optimal schedule with jobs are arrayed in nondecreasing order of* $\frac{p_j}{\alpha_j+b\beta_j}$.

Theorem 3.4 *Based on the lemma 2.1 and* (16)*, the problems* $1|p_j, Q_j = q_j + bt|\sum \alpha_j\omega_j(C_j - p_j) + \sum \beta_j\omega_j Q_j$ *has an optimal schedule with jobs are arrayed in nondecreasing order of* $\frac{p_j}{\omega_j(\alpha_j+b\beta_j)}$.

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