International Mathematical Forum, Vol. 7, 2012, no. 28, 1381 - 1391

Some Results on Geometric Mean Graphs

S. Somasundaram

Department of Mathematics
Manonmaniam Sundaranar University
Tirunelveli - 627 012
Tamil Nadu, India
somumsu@rediffmail.com

P. Vidhyarani

Department of Mathematics
KG College of Arts and Science
Coimbatore - 641 035
Tamil Nadu, India
vidhyaranip@gmail.com

S. S. Sandhya

Department of Mathematics
Shree Ayyappa College
Nagarcoil - 629 807
Tamil Nadu, India
sssandhya2009@gmail.com

Abstract

A graphs G= (V, E) with p vertices and q edges is said to be a Geometric mean graph if it is possible to label the vertices $x \in V$ with distinct labels f(x) from 1, 2 … q+1 in such way that when each edge e = uv is labeled with f (uv) = $\lfloor \sqrt{f(u)f(v)} \rfloor$ or $\lceil \sqrt{f(u)f(v)} \rceil$ then the edge labels are distinct. Here we prove that $C_m \cup P_n$, $C_m \cup C_n$, nK_3, $nK_3 \cup P_n$, $nK_3 \cup C_m$,crown, square of a path are geometric mean graphs.

Keywords: Graph, geometric mean graph, crown, square of a path, union of graphs.

Introduction

The graphs considered here will be finite, undirected and simple graph without isolated vertices. The vertex set is denoted by V(G) and edge set is denoted by E(G). The union of two graphs G_1 and G_2 has vertex set $V(G_1) \cup V(G_2)$ and edge set $E(G_1) \cup E(G_2)$. The join G_1+G_2 of two graphs G_1 and G_2 has vertex set $V(G_1) \cup V(G_2)$ and edge set $E(G_1+G_2) = E(G_1) \cup E(G_2) \cup \{uv; u \in V(G_1)$ and $v \in V(G_2)\}$. mG denotes the disjoint union of m copies of G. Terms are not defined here are used in the sense of Harary [1].

Mean labeling was introduced by S.Somasundaram and R. Ponraj [2] in 2003 and their behaviour studied in [2] and [3]. Harmonic mean labeling was introduced by S.Somasundram, R.Ponraj and S.S. Sandhya [4] and their behaviour studied in [4] and [5]. Geometric mean labeling was introduced by S.Somasundram, P.Vidhyarani and R.Ponraj in [6]. In this paper we investigate geometric mean labeling of union of some graphs.

2. Geometric Mean Labeling

Definition 2.1

A graph G = (V, E) with p vertices and q edges said to be a Geometric mean graphs if it is possible to label the vertices $x \in V$ with distinct elements f(x) from 1, 2… q +1 in such a way that when edge e =uv is labeled with $\lceil \sqrt{f(u)f(v)} \rceil$ (or) $\lfloor \sqrt{f(u)f(v)} \rfloor$ then the resulting edge labels are distinct. In this case f is called a Geometric mean labeling of G.

Note 2.2

In a Geometric mean labeling of graphs, the vertices get labels from 1, 2… q+1 and edge from 1, 2 …q.

C_m and P_n are geometric mean graphs [6].

Now we prove the following:

Theorem: 2.3

$C_m \cup P_n$ is a geometric mean graph for $m \geq 3$, $n \geq 1$.

Proof:

Let C_m be the cycle $u_1u_2 .. u_mu_1$ and P_n be the path $v_1v_2 \ldots v_n$.

Define a function

$f: V(C_m \cup P_n) \to \{1, 2 \ldots\ldots q+1\}$ by

$f(u_i) = i \qquad 1 \leq i \leq m$

$f(v_i) = m+i \qquad 1 \leq i \leq n$

Then the set of labels of the edges of C_m is $\{1, 2\ldots, m\}$. The set of labels of the edges of P_n is $\{m+1, m+2 \ldots m+n-1\}$.

Hence $C_m \cup P_n$ is a Geometric mean graph of $m \geq 3$ and $n \geq 1$.

Example

Geometric Mean labeling of $C_5 \cup P_6$ is given below.

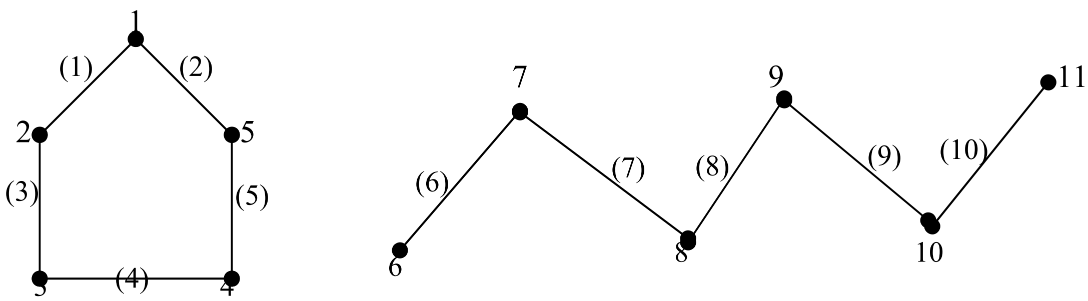

Figure-1

Next we have the following:

Theorem 2.4

$C_m \cup C_n$ is a geometric mean graph for $m \geq 3$, $n \geq 3$.

Proof:

Let C_m be the cycle of the vertices $u_1, u_2, u_3 \ldots u_m, u_1$ and C_n be the cycle of the vertices $v_1, v_2 \ldots v_n, v_1$.

Define a function

$f: V(C_m \cup C_n) \to \{1, 2 \ldots\ldots q+1\}$ by

$f(u_i) = i, \quad 1 \leq i \leq m$

$f(v_i) = m+i, \quad 1 \leq i \leq n$

Then the set of labels of the edges of C_m is $\{1, 2, 3 \ldots\ldots m\}$. The set of labels of the edges of C_n is $\{m+1, m+2 \ldots m+n\}$.

Hence $C_m \cup C_n$ is a geometric mean graph.

Example:

Geometric mean labeling of $C_5 \cup C_7$ is given below

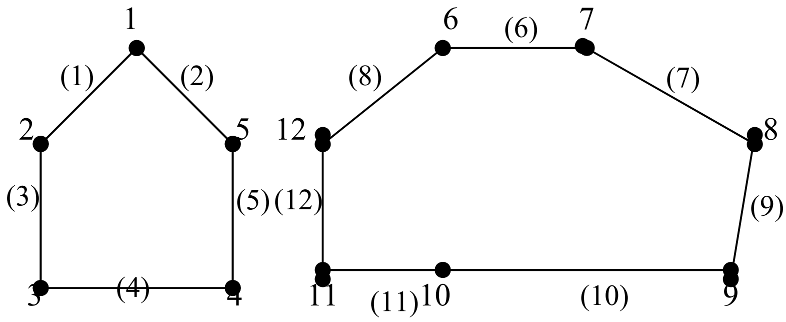

Figure 2

Note: 2.5

$C_m \cup P_n$ and $C_m \cup C_n$ are mean graphs [2] as well as Harmonic mean graphs [5].

Theorem 2.6

nK_3 is a geometric mean graph for $n \geq 1$

Proof:

Let the vertex set of nK_3 be $V = V_1 \cup V_2 \quad \ldots \cup V_n$ where $V_i = \{v_i^1, v_i^2, v_i^3\}$ and the edge set

$E = E_1 \cup E_2 \cup \ldots E_n$ where $E_i = \{ e_i^1, e_i^2, e_i^3\}$

Define

$f : V(nK_3) \rightarrow \{1, 2, 3 \ldots\ldots q+1\}$ by

$f(V_i^j) = 3(i-1)+j \qquad 1 \leq i \leq n,\ 1 \leq j \leq 3.$

The set of labels of the edges of nK_3 is $\{1, 2, 3 \ldots\ldots 3n\}$.

Hence nK_3 is a geometric mean graph for $n \geq 1$

Example:

Geometric mean labeling of $3K_3$ is given below

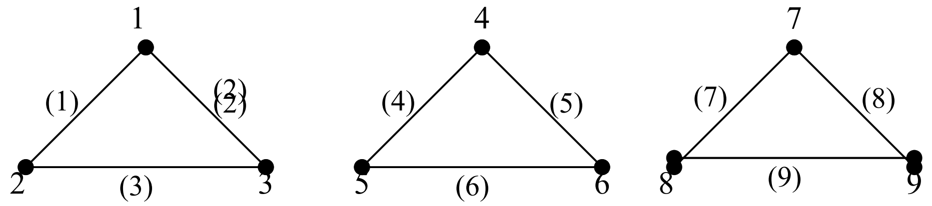

Figure 3

Now we have

Theorem 2.7

$nK_3 \cup P_m$ is a geometric mean graph for $n \geq 1$, $m \geq 1$.

Proof:

Let the vertex set of nK_3 be $U = U_1 \cup U_2 \cup U_3 .. \cup U_n$

Where $U_i = \{ u_i^1, u_i^2, u_i^3\}$ and the edge set $E = E_1 \cup E_2 \cup E_3 \ldots \cup E_n$.

Where $E_i = \{ e_i^1, e_i^2, e_i^3\}$

Let P_m be the path $v_1 v_2 .. v_m$

Define a function

$f: V (nK_3 \cup P_m) \rightarrow \{1, 2, 3 \ldots.. . q + 1\}$ by

$f (u_i^j) = 3(i-1)+j \qquad 1 \leq i \leq n$

$f (v_i) = 3n+i \qquad 1 \leq j \leq m.$

The set of labels of the edges of nK_3 is $\{1, 2, 3 \ldots.. . 3n\}$.

The set of label of the edges of P_m is $\{3n+1, 3n+2, .. . 3n+m-1\}$.

Hence $nK_3 \cup P_m$ is geometric mean graph for $n \geq 1$, $m \geq 1$.

Example:

Geometric mean labeling of $2K_3 \cup P_5$ is given below.

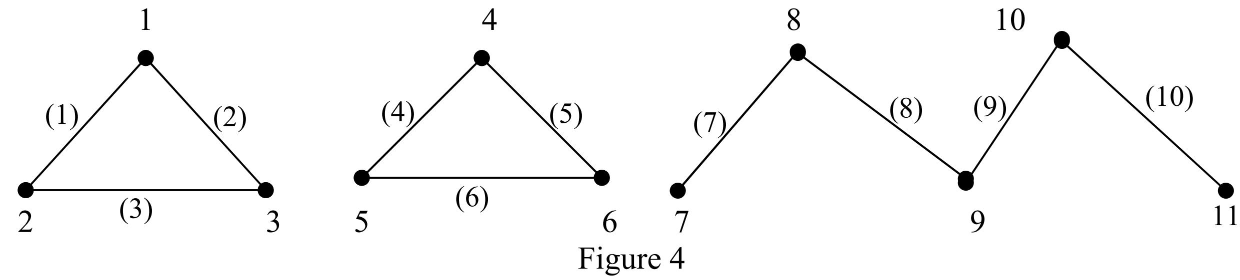

Figure 4

Theorem 2.8

$nK_3 \cup C_m$ is a geometric mean graph for $n \geq 1$, $m \geq 3$.

Proof:

Let nK_3 be the n copies of K_3 graph. Let the vertex set of nK_3 be $U = U_1 \cup U_2 \ldots \cup U_n$ Where $U_i = \{ u_i^1, u_i^2, u_i^3 \}$ and the edge set $E = E_1 \cup E_2 \ldots \cup E_n$ where

$E_i = \{ e_i^1, e_i^2, e_i^3 \}$ **.** C_m be the cycle of $v_1 v_2 .. v_n v_1$

Let us define a function

$$f : V(nK_3 \cup C_m) \rightarrow \{1, 2, \ldots\ldots q+1\}$$

by $f(u_i^j) = 3(i-1)+j$ $\quad 1 \leq i \leq n, 1 \leq j \leq 3$.

$f(v_i) = 3n+i$ $\quad 1 \leq i \leq m$.

The set of label of the edges of nK_3 is $\{1, 2, 3 \ldots\ldots 3n\}$ and

The set of label of the edges of C_m is $\{3n+1, 3n+2, \ldots 3n+m\}$.

Hence $nK_3 \cup C_m$ is geometric mean graph for $n \geq 1$, $m \geq 3$.

Example:

Geometric mean labeling of $2K_3 \cup C_6$ is given below.

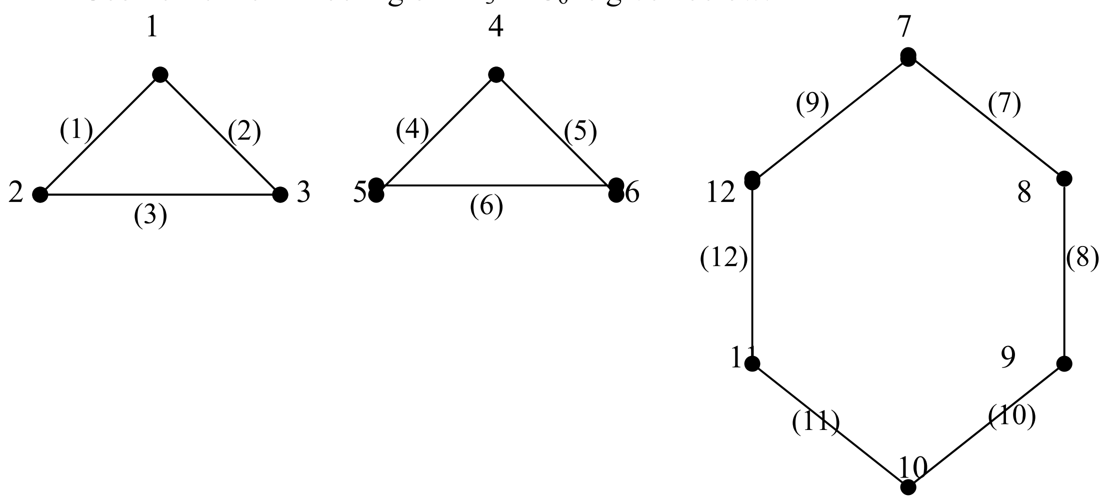

Figure 5

Note 2.9

nK_3, $nK_3 \cup P_m$, $nK_3 \cup C_m$ are also Harmonic mean graphs [5].

Theorem 2.10

mC_n is a Geometric mean graph for $m \geq 1$, $n \geq 3$

Proof:

Let the vertex set of mC_n be $V = V_1 \cup V_2 \cup \ldots \cup V_m$

Where $Vi = \{ v_i^1, v_i^2, \ldots v_i^n \}$ and the edge set $E = E_1 \cup E_2 \ldots \cup E_m$ where $Ei = \{ e_i^1, e_i^2, \ldots e_i^n \}$ and

Define $f: V(mC_n) \to \{1, 2, \ldots . q+1\}$ by

$f(V_i^j) = n(i-1)+j \qquad 1 \leq i \leq m, \ 1 \leq j \leq n.$

The set of labels of the edges of mC_n is $\{1, 2, 3 \ldots . mn\}$

Hence mC_n is geometric mean graph.

Example:

Geometric mean labeling of $3C_5$ is given below:

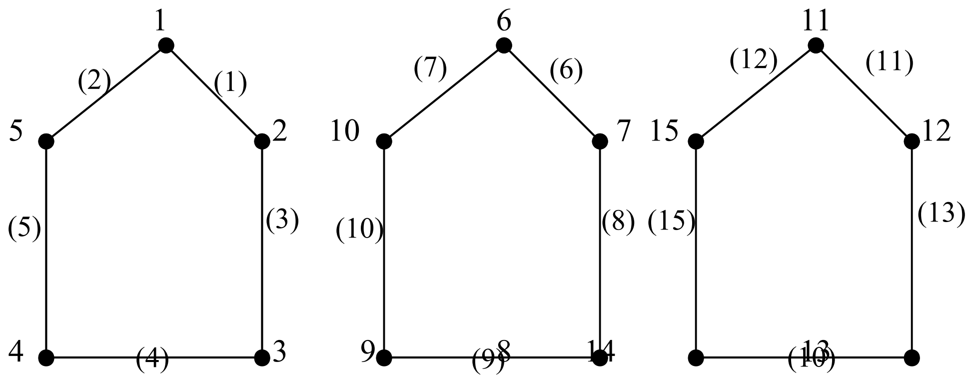

Figure 6

Note: 2.11

mC_4 is also Harmonic mean graphs [5] .

Definition 2.12

The square G^2 of a graph G has $v(G^2) = V(G)$ with u, v adjacent in G^2 whenever $d(u,v) \leq 2$ in G. The power of G^3, G^4 …. of G are similarly defined.

Theorem: 2.13

The graph P_n^2 is a geometric mean graph.

Proof:

Let $u_1, u_2 \ldots\ldots u_n$ be the path P_n

P_n^2 has n vertices and 2n-3 edges.

Define f : $V(P_n^2) \rightarrow \{1, 2\ldots. q+1)$

by $f(u_1) = 1$

$f(u_i) = 2i-2, \quad 2 \leq i \leq n$

The label of the edge u_iu_{i+1} is 2i -1 $(1 \leq i \leq n-1)$

The label of the edge $u_i u_{i+2}$ is 2i $(1 \leq i \leq n-2)$

Hence P_n^2 is geometric mean graph.

Example

Geometric mean labeling of P_5^2 is given below

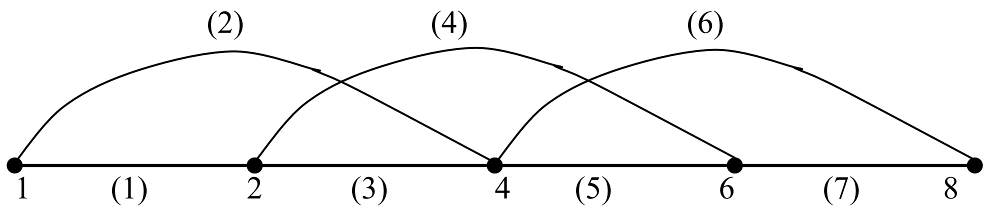

Figure 7

Definition 2.14

The corona $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G (which has P vertices and P copies of G_2) and then joining the i^{th} vetex of G_1 to every vertices in the i^{th} copy of G_2. Here we restrict ourselves to corona with cycles.

The graph $C_n \odot K_1$ is called a crown.

Theorem: 2.15:

The crown $C_n \odot K_1$ is a geometric mean graph for all $n \geq 3$.

Proof:

Let C_n be the cycle $u_1\, u_2 \ldots u_n\, u_1$ and let v_i be the vertex adjacent to u_i $(1 \leq i \leq n)$

Define a function

$f: V(C_n \odot K_1) \rightarrow \{1, 2, 3, \ldots . q+1\}$

by $f(u_i) = 2i-1$ for $1 \leq i \leq 2$.

$f(u_i) = 2i+1$ for $3 \leq i \leq n$.

$f(v_i) = 2i$ for $1 \leq i \leq n$.

Then the edge labels are distinct.

Hence crown is a geometric mean graph for all $n \geq 3$.

Example: Geometric mean labeling of $C_5 \odot K_1$ is given below

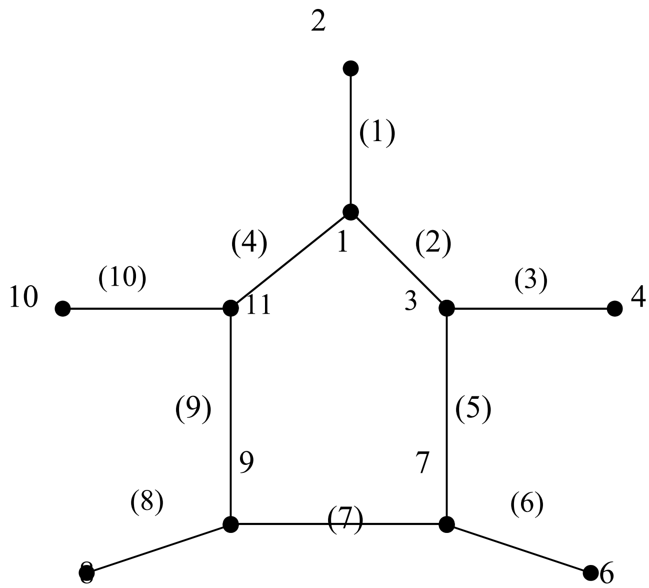

Figure 8

Acknowledgement: The authors wish to express their thanks to Dr. R. Ponraj, Department of Mathematics, Sri Paramakalyani College of Arts and Science, Alwarkurichi-627 412 for his valuable comments in revising the paper.

References

1. F. Harary, Graph theory, Narosan Publishing House Reading, New Delhi 1998.
2. S. Somasundaram and R. Ponraj, "Mean labelings of graphs", National academy Science letters, **26** (2003), 210-213.
3. S. Somasundaram and R. Ponraj, "Some Results on Mean graph", Pure and Applied Mathematics Science, **58** (2003), 29-35.
4. S. Somasundaram, R. Ponraj and S.S. Sandhya "Harmonic mean labelings of Graphs", Communicated
5. S. Somasundaram, R. Ponraj and S.S. Sandhya, "Some Results on Harmonic Mean graphs",Communicated.
6. S. Somasundaram, P.Vidhyarani and R. Ponraj, "Geometric Mean Labelings of Graphs", Bulletin of Pure and Applied Sciences, **30E** (2011), 153-160 (Accepted).

Received: November, 2011