International Mathematical Forum, Vol. 7, 2012, no. 25, 1209 - 1222

Spanning Tree Algorithm for Families of Chained Graphs

D. Lotfi, M. El Marraki and D. Aboutajdine

LRIT Unité associée au CNRST(URAC29)- Faculty of Sciences
University of Mohammed V-Agdal, B.P.1014 RP, Rabat, Morocco
doun.lotfi@gmail.com, marraki@fsr.ac.ma, aboutaj@fsr.ac.ma

Abstract

Enumeration of trees is a new line of research in graph theory; many researchers worked on this area, starting with the Matrix Tree Theorem given by Kirchhoff who defined the number of spanning trees in graph G as the determinant of its Laplacian matrix, since this later is easy to compute but it cannot give the recurrences of spanning trees. In this paper, we present two different methods to derive recursive functions, the deletion and contraction method and the splitting method that calculate the number of spanning trees in some families of graph, many recursions are given for the wheel graph, fan graph and family of corn graphs. The purpose of this paper is to propose an algorithm that calculates the number of spanning trees in a chained graph. Finally, we give the complexity of the chained wheel graph and the chained corn graph as applications of the spanning trees algorithm.

Mathematics Subject Classification: 05C85, 05C30

Keywords: planar graph, complexity, spanning tree, grid graph, wheel graph, fan graph

1 Introduction

The most classical theories of interest concerning spanning trees is the complexity or the number of spanning trees of a given graph. Kirchhoff [7] gave a theorem called the Matrix Tree Theorem that determinates the number of spanning trees in a graph G as the value of any cofactor of the matrix $D_G - A_G$ such that D_G is the degree matrix (A diagonal matrix $D_G = diag(d_1, ..., d_n)$ corresponding to the graph that has the vertex degree of d_i in the ith entry) and A_G is the adjacency matrix of G (Boolean matrix such that the position a_{ij} is equal to the number of edges between the ith vertex and jth vertex).

The disadvantage of this method is that it can not produce the recurrence that gives the sequence of numbers. The motivation in this paper is the following classes of graph:

- Fan graph F_n,
- Wheel graph W_n,
- Corn graph N_n, with n is the number of petals.

All these classes of graphs can formed which we called the k chained graph, a chain that contains k graph mentioned above. For each of these families of graph, an explicit recurrence is given and its corresponding algorithm to count the number of spanning trees.

2 History and outline

The first historical result is the Cayley's Theorem [3], which gives an explicit formula to count the number of spanning trees of a planar graph C. Later in 2008, Bogdanowicz [1] gave a formula for the number of spanning trees in a Fan graph. In 2009, Haghighi and Bibak [6] used the Cayley's Theorem to find a recursive function that calculates the number of spanning trees in Fan and Wheel graph. Recently, El Marraki, Lotfi and Modabish [8],[12] gave a new method to find explicit recursions counting the number of spanning trees for the planar graph case. In the latest paper and this one, the main objective is the same: Our aim is to find a recursive function to count the number of spanning trees for some families of graph. The first result in this paper is to give recursive functions that calculates the complexity of chained wheel graph and chained corn graph. As a consequence, we propose an algorithm counting the two complexities which is based on calculating the power of a specific matrix.

3 Basic Vocabulary

We shall use the standard terminology of graph theory, as it is introduced in most books on the theory of graphs [2], [5], [13]. In this paper, a graph implies a finite simple graph which has neither loops or multiple edges.

Definition 3.1 *A graph is said to be planar if it can be illustrated in the plane so that arcs representing edges meet only at points representing vertices. Informally, G is planar if it can be drawn in the plane without any edge crossings. It can also be called as a map.*

A graph G can have many planar drawing, it depends on the visual field through we can observe the graph. The following figure illustrates a planar embedding of a graph G.

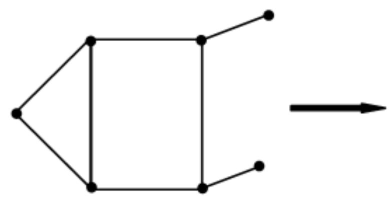


Figure 1: Planar embedding of a graph

Euler [13] gave a formula that relates the number of vertices, edges and faces of a planar graph:$|V_{\mathcal{M}}| + |F_{\mathcal{M}}| - |E_{\mathcal{M}}| = 2$. (eg.,The previous planar graph (See Figure 1) has 7 vertices, 8 edges and 3 faces).

Definition 3.2 *Spanning Tree of undirected connected graph G is a tree formed by all the vertices and some or all the edges of G.*

The following figure gives some spanning trees of a graph G.

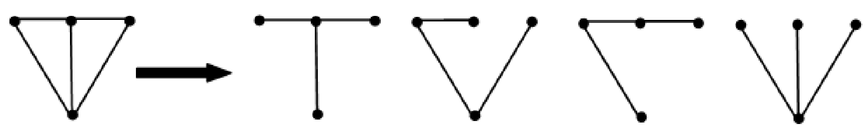


Figure 2: Some spanning trees of graph G

Definition 3.3 *Let G be a graph, the complexity of G is the number of trees spanning G, it is denoted by $\tau(G)$ and it depends mostly on the number of edges in G (eg., the complexity of the graph mentioned above (See Figure 2) is 8).*

Throughout the paper, a graph implies a finite connected indirected graph, the connectivity is that there must be a path between each pair of vertices. Note that the exact geometric positions of vertices or the lengths of the edges are not important. We will find many recursions in this paper, all concerning the spanning trees in the fan graph, wheel graph and corn graph.

4 Spanning trees Recursions

The first recursive formula that calculates the number of spanning trees in graph G was given by Cayley [3] as a special case. Note that Cayley's formula count the number of trees spanning the graph G that satisfies the criteria of planarity, connectivity and disorientation.

4.1 Spanning trees recursions by deletion and contraction

As we mentioned before, a connected graph may have an exponential number of spanning trees by deleting some edges and preserving connectivity. We shall now define the proprieties of deletion and contraction an edge, the deletion is the graph $(V, E-e)$ and G/e is the contraction obtained from $G-e$ by fusing two vertices v_1 and v_2 (See Figure 3).

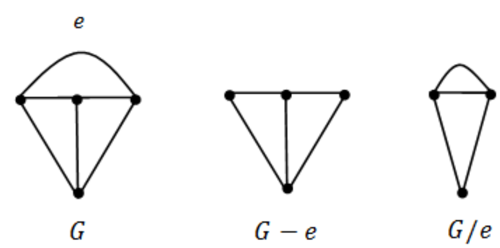


Figure 3: Graph G, $G-e$ and G/e

Theorem 4.1 *Let G be a graph, $e \in G$ then the spanning trees of G that don't contain the edge e and the spanning trees of the deletion $G-e$ are in bijection, alternatively, the spanning trees of G that contain e and the spanning trees of the contraction G/e are in bijection.*

Let $T \in \tau(G)$, $\{T \in \tau(G) \quad | \quad e \notin T\} \longleftrightarrow \tau(G{-}e)$ and $\{T \in \tau(G) \quad | \quad e \in T\} \longleftrightarrow \tau(G/e)$.

For any graph G and edge e, the deletion and contraction recursion that was proved by Kirchhoff [7] is given by the following formula:

$$\tau(G) = \tau(G-e) + \tau(G/e)$$

4.1.1 Complexity of the corn graph

Let N_p be the corn graph that contains $p+2$ vertices with p is the number of petals as illustrated in the following figure 4 (a).

Lemma 4.2 *The complexity of the corn graph N_p is given by the following formula*

$$\tau(N_p) = \frac{1}{\sqrt{5}}\left(\frac{\phi^3 - \phi^{p+2}}{1-\phi} - \frac{\bar{\phi}^3 - \bar{\phi}^{p+2}}{1-\bar{\phi}}\right) + 5, \quad p \geq 2$$

with $\phi = \frac{3+\sqrt{5}}{2}$ and $\bar{\phi} = \frac{3-\sqrt{5}}{2}$.

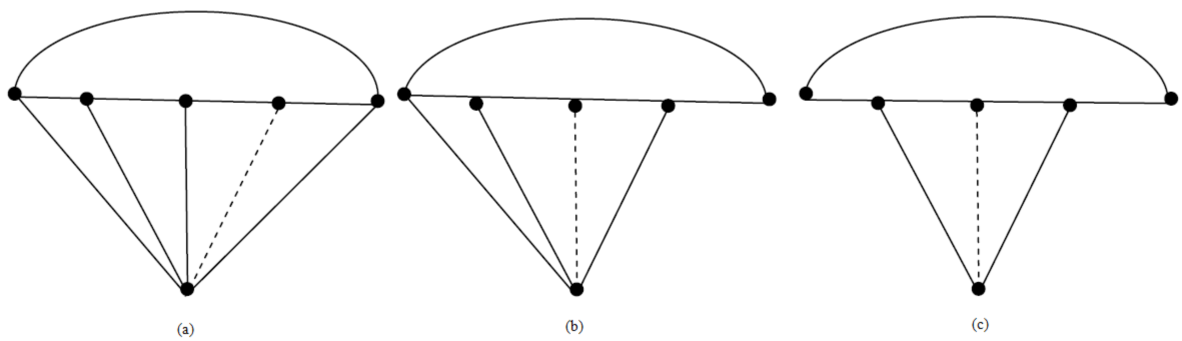


Figure 4: Corn Graphs N_p, $N_p^{'}$ and $N_p^{''}$

Proof 4.3 *Let N_p be the corn graph and F_p be the fan graph (See figure 6), from Theorem 4.1 we get*

$$\begin{aligned}
\tau(N_p) &= \tau(F_p) + \tau(N_{p-1}) \\
&= \tau(F_p) + \tau(F_{p-1}) + ... + \tau(F_4) + \tau(F_3) + \tau(F_2) + \tau(N_1) \\
&= \frac{1}{\sqrt{5}}((\phi^{p+1} + \phi^p + \phi^{p-1} + ... + \phi^2) - (\bar{\phi}^{p+1} + \bar{\phi}^p + \bar{\phi}^{p-1} + ... + \bar{\phi}^2)) + 5 \\
&= \frac{1}{\sqrt{5}}(\frac{\phi^3 - \phi^{p+2}}{1-\phi} - \frac{\bar{\phi}^3 - \bar{\phi}^{p+2}}{1-\bar{\phi}}) + 5, \quad p \geq 2, \quad \phi = \frac{3+\sqrt{5}}{2} \quad \textit{and} \quad \bar{\phi} = \frac{3-\sqrt{5}}{2}.
\end{aligned}$$

Let $N_p^{'}$ be the corn graph obtained by deleting the last edge from the graph N_p (See figure 4 (b)).

Lemma 4.4 *The complexity of the corn graph $N_p^{'} = N_p - e$ is given by the following formula:*

$$\tau(N_p^{'}) = \tau(F_{p-1}) + \tau(N_{p-1}), \quad p \geq 2.$$

Let $N_p^{''}$ be the corn graph obtained by deleting the first and the last edges from the corn graph N_p (See figure 4 (c)).

Lemma 4.5 *The complexity of the corn graph $N_p^{''}$ is given by the following formula:*

$$\tau(N_p^{''}) = \tau(F_{p-2}) + \tau(N_{p-1}^{''}), \quad p \geq 2.s$$

The proof of the Lemma 4.4 and 4.5 is similar to the previous one. Note that we use this result in section 6 to calculate the number of spanning trees in the chained corn graph.

4.2 Spanning trees recursions by splitting

Definition 4.6 *Let G has a separation pair x, y (an edge $e = xy$ or a simple path $p = x, a, b, ..., y$ such that all of its vertices have degree 2 except x and y) then we can split G into two graphs G_1 and G_2, called split graphs, we shall denote $G = G_1 \ddagger G_2$ such that G_1 is obtained from $G_1^{'}$ by adding a new simple path $p = x, a, b, ..., y$, similarly, we define the split graph G_2 (See figure 5) [13].*

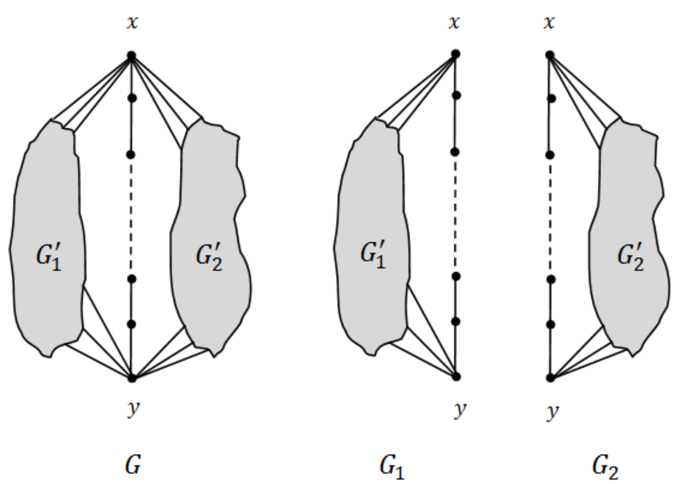


Figure 5: Graph $G = G_1 \ddagger G_2$

Theorem 4.7 *Let G be a planar graph that can be split in two graphs G_1 and G_2, $G = G_1 \ddagger G_2$ such that $\ddagger$ is a simple path that contains k edges (See figure 5), the number of spanning trees in the graph G is given by the following equation:*

$$\tau(G) = \tau(G_1) \times \tau(G_2) - k^2 \tau(G_1^{'}) \times \tau(G_2^{'}) \quad [11]$$

Examples: Let W_{p+q+1} be the wheel graph defined by $p + q$ petals and contains $p + q + 1$ vertices, $2(p + q)$ edges and $p + q + 1$ faces, and let F_{p+1} be the fan graph that contains $p-1$ petals, $p+1$ vertices, $2p-1$ edges and p faces (See figure 6). In [12] and [14], the authors give the complexity of the wheel and fan graphs by using the deletion and contraction method or the splitting method studied above.

The number of spanning trees in W_{p+q+1} and F_{p+1} is given by the following equations:

$$\tau(\mathcal{W}_{p+q+1}) = (\tfrac{3+\sqrt{5}}{2})^{p+q} + (\tfrac{3-\sqrt{5}}{2})^{p+q} - 2, \quad p, q \geq 2.$$
$$\tau(\mathcal{F}_{p+1}) = \tfrac{1}{\sqrt{5}}((\tfrac{3+\sqrt{5}}{2})^{p-1} - (\tfrac{3-\sqrt{5}}{2})^{p-1}), \quad p \geq 2.$$

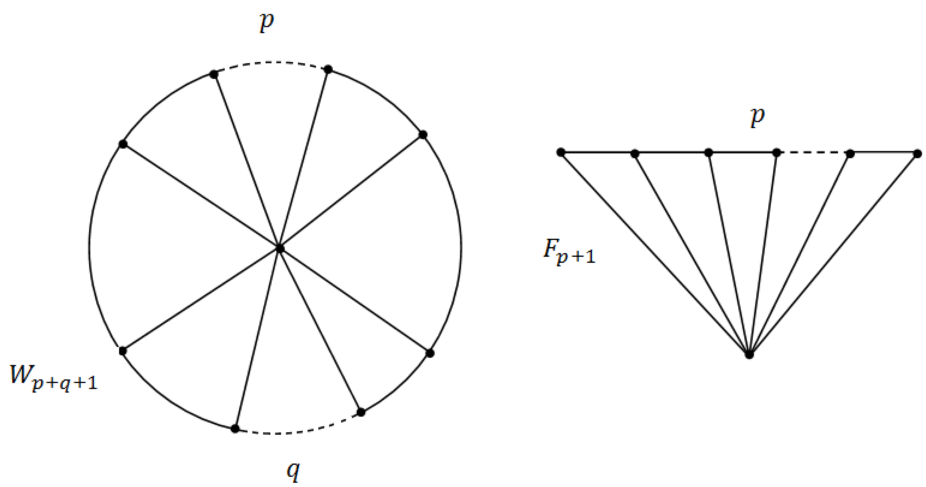


Figure 6: Wheel and Fan graphs

5 Spanning Tree Algorithm

For the results above, it was necessary to use the spanning tree recursion to find the complexity of a graph G, however, for the family of the chained graphs, the formula that gives the number of spanning tree is complex. In this section, we give an algorithm that enumerates the number of spanning trees for some families of chained graphs.

Definition 5.1 *Let C_i be a planar graph, we call $\mathcal{G}_n$ as a chained graph if it is a succession of n split graphs of type C_i, we denote $\mathcal{G}_n = C_i \ddagger C_i \ddagger C_i \ddagger ... \ddagger C_i$ such that $\ddagger$ is a simple path that contains k edges (See figure 7).*

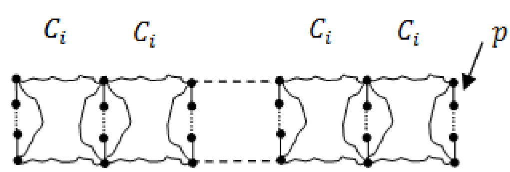


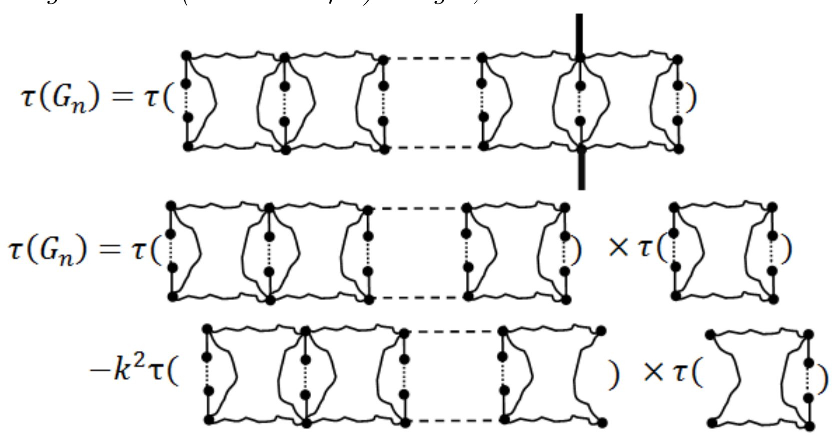
Figure 7: Chained planar graph $\mathcal{G}_n$

Theorem 5.2 *Let $\mathcal{G}_n$ be a planar chained graph of type $\mathcal{G}_n = C_i \ddagger C_i \ddagger C_i \ddagger ... \ddagger C_i$ such that $\ddagger$ is a simple path that contains $k+1$ vertices such that $deg(v_j) = 2$ for $j = 2, 3, ..., k$ and k edges, the number of spanning tree in the graph $\mathcal{G}_n$ is given by the following system*

$$\begin{cases} \tau(\mathcal{G}_n) = & \tau(\mathcal{G}_{n-1}) \times \tau(\mathcal{G}_1) - k^2\tau(\mathcal{G}'_{n-1}) \times \tau(\mathcal{G}'_1) \\ \tau(\mathcal{G}'_n) = & \tau(\mathcal{G}_{n-1}) \times \tau(\mathcal{G}'_1) - k^2\tau(\mathcal{G}'_{n-1}) \times \tau(\mathcal{G}''_1), \quad n \geq 2 \end{cases}$$

Where $\mathcal{G}'_n$ is the planar chained graph obtained by deleting the last path p and $\mathcal{G}''_n$ is the planar chained graph obtained by deleting the first and last path p (See figure 7).

Proof 5.3 *Let $\mathcal{G}_n$ be the planar chained graph (See figure 7), by using the splitting method (Theorem 4.3) we get,*



therefore, $\tau(\mathcal{G}_n) = \tau(\mathcal{G}_{n-1}) \times \tau(\mathcal{G}_1) - k^2\tau(\mathcal{G}'_{n-1}) \times \tau(\mathcal{G}'_1)$, the same proof goes for $\mathcal{G}'_n$, we use the splitting method then we get the second equation of the system.

Corollary 5.4 *Let $\mathcal{G}_n$ be a planar chained graph of type $\mathcal{G}_n = C_i \ddagger C_i \ddagger ... \ddagger C_i$ such that $\ddagger$ is a simple path that contains $k+1$ vertices such that $deg(v_j) = 2$ for $j = 2, 3, ..., k$ and k edges, and $\mathcal{G}'_n$ is the planar graph obtained after deleting the last path p from the graph $\mathcal{G}_n$, we denote $g_n = \tau(\mathcal{G}_n)$ and $g'_n = \tau(\mathcal{G}'_n)$, the number of spanning tree in the graphs $\mathcal{G}_n$ and $\mathcal{G}'_n$ is given by the following equation*

$$\begin{pmatrix} g_n \\ g'_n \end{pmatrix} = M^{n-1} \begin{pmatrix} g_1 \\ g'_1 \end{pmatrix}, \quad where \quad M = \begin{pmatrix} g_1 & -k^2 g'_1 \\ g'_1 & -k^2 g''_1 \end{pmatrix}$$

Example: Let $\mathcal{R}_n$ be the grid chained graph, it contains $2n+2$ vertices (See Figure 8).

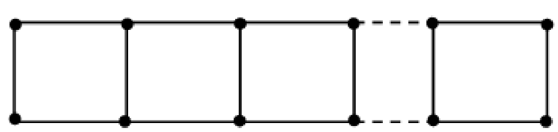


Figure 8: Grid Chained graph $\mathcal{R}_n$

The complexity of $\mathcal{R}_n$ is given by the following equation:

$$\tau(\mathcal{R}_n) = \frac{1}{2\sqrt{3}}((2+\sqrt{3})^{n+1} - (2-\sqrt{3})^{n+1}) \quad n \geq 1 \quad [4], [9], [10].$$

Since the complexity can be easy to compute by using the previous formula, we can also use the Theorem 5.2 then we get the following system:

$$\begin{cases} \tau(\mathcal{R}_n) = 4\tau(\mathcal{R}_{n-1}) - \tau(\mathcal{R}'_{n-1}) \\ \tau(\mathcal{R}'_n) = 4\tau(\mathcal{R}_{n-1}), \qquad\qquad n \geq 2 \end{cases}$$

The complexity matrix is defined by the initials conditions r_1, r'_1 and r''_1,

$$M = \begin{pmatrix} 4 & -1 \\ 4 & 0 \end{pmatrix}$$

then using the previous theorem, the complexity of the grid chained graph is based on the power of the complexity matrix and the initial conditions r_1 and r'_1. To obtain the complexity of the grid chained graph, we run the following algorithm:

1. Let $n = 1$.
2. Let $\mathcal{R}_1$ and $\mathcal{R}'_1$ be the initial conditions of the grid chained graph.
3. Let V the vector formed by the initial conditions $V =< r_1, r'_1 >$.
4. Let M the complexity matrix formed by r_1, r'_1 and r''_1.
5. For a given n, Using the square and multiply method, we calculate M^n.
6. Calculate the product $M^n \times V$.

6 Applications

6.1 Complexity of the chained wheel graph

Let $\mathcal{E}_n$ be the chained wheel graph, it is formed by a chain of wheels that contains $p + q + 1$ vertices, and let $\mathcal{E}'_n$ be the chained Fan graph obtained by deleting the last edge of each wheel from the chained wheel graph (See Figure 9).

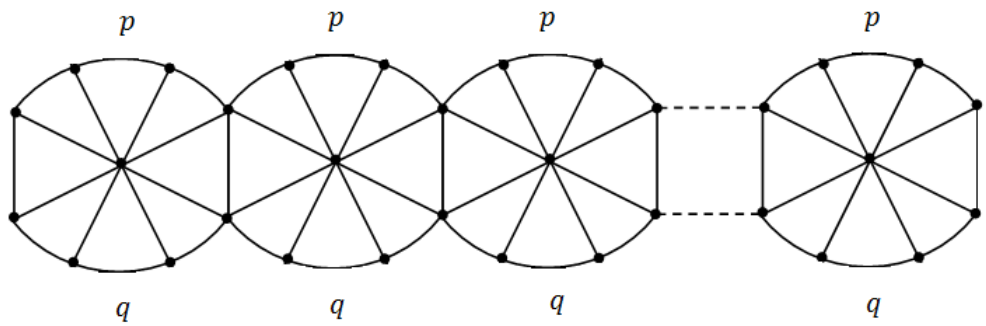


Figure 9: The chained Wheel graph $\mathcal{E}_n$

Theorem 6.1 *Let $\mathcal{E}_n$ be the chained Wheel graph and $\mathcal{E}_n^{'}$ be the chained Fan graph, the complexity of $\mathcal{E}_n$ and $\mathcal{E}_n^{'}$ are given by the following system:*

$$\begin{cases} \tau(\mathcal{E}_n) & = \tau(\mathcal{E}_{n-1}) \times \tau(W_{p+q+1}) - \tau(\mathcal{E}_{n-1}^{'}) \times \tau(F_{p+q+1}) \\ \tau(\mathcal{E}_n^{'}) & = \tau(\mathcal{E}_{n-1}) \times \tau(F_{p+q+1}) - \tau(\mathcal{E}_{n-1}^{'}) \times \tau(F_{p+1}) \times \tau(F_{q+1}) \end{cases}$$

$$\textit{with} \quad \tau(\mathcal{E}_1) = \tau(W_{p+q+1}) \quad \textit{and} \quad \tau(\mathcal{E}_1^{'}) = \tau(F_{p+q+1}), p, q \geq 1, n \geq 1.$$

Proof 6.2 *The graphs $\mathcal{E}_n$ and $\mathcal{E}_n^{'}$ belong to the family of chained graph $\mathcal{G}_n$, then the complexities $\tau(\mathcal{E}_n)$ and $\tau(\mathcal{E}_n^{'})$ can be found by using the Theorem 5.2, where the simple path p is the path that connects the last wheel with the chained wheel $\mathcal{E}_{n-1}$.*

The spanning trees recursions of the graph $\mathcal{E}_n$ and $\mathcal{E}_n^{'}$ can be found by using the result in the Theorem 6.1, the downside of this method is that it can produce a huge function which is not easy to compute, the algorithm given earlier can formulates the complexity system by the following matrix equation: Let $e_n, e_n^{'}, w_{p+q+1}, f_{p+q+1}, f_{p+1}$ and f_{q+1} be respectively the complexities of the graphs $\mathcal{E}_n$, $\mathcal{E}_n^{'}$, W_{p+q+1}, F_{p+q+1}, F_{p+1} and F_{q+1} .

$$\begin{pmatrix} e_n \\ e_n^{'} \end{pmatrix} = M \begin{pmatrix} e_{n-1} \\ e_{n-1}^{'} \end{pmatrix}, \quad \textit{where} \quad M = \begin{pmatrix} \mathrm{w}_{p+q+1} & -\mathrm{f}_{p+q+1} \\ \mathrm{f}_{p+q+1} & -\mathrm{f}_{p+1} \times f_{q+1} \end{pmatrix}$$

$$\begin{pmatrix} e_n \\ e_n^{'} \end{pmatrix} = M \begin{pmatrix} e_{n-1} \\ e_{n-1}^{'} \end{pmatrix} = ... = M^{n-1} \begin{pmatrix} e_1 \\ e_1^{'} \end{pmatrix}$$

For a given n, we calculate the power n of the complexity matrix and the multiplication with the vector formed by the initial conditions gives the complexity of the wheel chained graph. Let $\mathcal{E}_n$ be the chained wheel graph, it is formed by n wheels of type W_{p+q+1} such that $p = 3$ and $q = 3$ (See figure 9), the complexity of $\mathcal{E}_n$ in this case is given by the following equation:

$$\begin{pmatrix} e_n \\ e_n^{'} \end{pmatrix} = M^{n-1} \begin{pmatrix} 320 \\ 144 \end{pmatrix}, \quad where \quad M = \begin{pmatrix} 320 & -144 \\ 144 & -64 \end{pmatrix}$$

The following table gives some values of the number of spanning trees in the chained wheel graph $\mathcal{E}_n$.

n	$\tau(\mathcal{E}_n)$
1	320
2	81664
3	20824064
4	5310054400
5	1354042966016
6	345275625373696
7	88043925096366080
8	22450854264574050304
9	5724879446906287161344
10	1459821719716278556426240

Table 1: Some values of the complexity $\tau(\mathcal{E}_n)$.

Our algorithm does not give only the complexity of the chained wheel graph, it gives also the number of spanning trees in the chained fan graph $\mathcal{E}_n^{'}$.

Another example of a chained graph is given in the following section, therefore, we can keep the same basic method used to calculate the number of spanning trees in the chained wheel graph. Note that we use the Lemma 4.2, 4.4 and 4.5 to calculate the number of spanning trees in the corn graphs which are the initial conditions of our algorithm.

6.2 Complexity of the chained Corn graph

The chained corn graph $\mathcal{O}_n$ is the juxtaposition of n corns, each one contains $p + 2$ vertices such that p is the number of petals (Triangles) as illustrated in figure 10.

Theorem 6.3 *Let $\tau(\mathcal{O}_n)$ be the complexity of the chained corn graph of type N_p and $\tau(\mathcal{O}_n^{'})$ be the complexity of the chained corn graph of type $N_p^{'}$, we have the following system:*

$$\begin{cases} \tau(\mathcal{O}_n) & = \tau(\mathcal{O}_{n-1}) \times \tau(\mathcal{O}_1) - \tau(\mathcal{O}_{n-1}^{'}) \times \tau(\mathcal{O}_1^{'}) \\ \tau(\mathcal{O}_n^{'}) & = \tau(\mathcal{O}_{n-1}) \times \tau(\mathcal{O}_1^{'}) - \tau(\mathcal{O}_{n-1}^{'}) \times \tau(\mathcal{O}_1^{''}) \end{cases}$$

$$\tau(\mathcal{O}_1) = \tau(N_p), \quad \tau(\mathcal{O}_1^{'}) = \tau(N_p^{'}) \quad and \quad \tau(\mathcal{O}_1^{''}) = \tau(N_p^{''}), n \geq 1, p \geq 2.$$

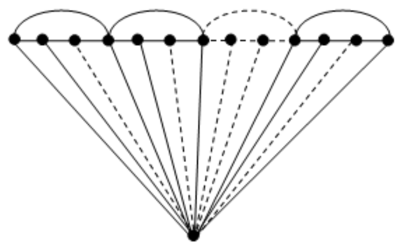


Figure 10: The chained Corn graph $\mathcal{O}_n$

Proof 6.4 *From the Theorem 5.2, we obtain the system that calculates the complexity of the chained corn graph.*

The number of spanning trees in the chained corn graph can be found by using the Theorem 6.3 or the algorithm given above. Let $o_n, o_n^{'}, n_p, n_p^{'}$ and $n_p^{''}$ be respectively the complexities of the graphs $\mathcal{O}_n$, $\mathcal{O}_n^{'}$, N_p, $N_p^{'}$ and $N_p^{''}$.

$$\begin{pmatrix} o_n \\ o_n^{'} \end{pmatrix} = M \begin{pmatrix} o_{n-1} \\ o_{n-1}^{'} \end{pmatrix}, \quad where \quad M = \begin{pmatrix} n_p^{'} & -n_p^{'} \\ n_p & -n_p^{''} \end{pmatrix}$$

$$\begin{pmatrix} o_n \\ o_n^{'} \end{pmatrix} = M \begin{pmatrix} o_{n-1} \\ o_{n-1}^{'} \end{pmatrix} = ... = M^{n-1} \begin{pmatrix} o_1 \\ o_1^{'} \end{pmatrix}$$

Let $\mathcal{O}_n$ be the chained corn graph, it is formed by n corns of type N_3, each one has 3 petals (see Figure 10), the complexity of $\mathcal{O}_n$ in this case is given by the following equation:

$$\begin{pmatrix} o_n \\ o_n^{'} \end{pmatrix} = M^{n-1} \begin{pmatrix} 34 \\ 21 \end{pmatrix}, \quad where \quad M = \begin{pmatrix} 34 & -21 \\ 21 & -11 \end{pmatrix}$$

The following table gives some values of the number of spanning trees in the chained corn graph $\mathcal{O}_n$ of type N_3.

n	1	2	3	4	5	6	7
$\tau(\mathcal{O}_n)$	34	715	14167	277936	5443339	106575085	2086523242

Table 2: Some values of the complexity $\tau(\mathcal{O}_n)$.

7 Conclusion

Many applications in computer science such as data structure, algorithms, compilers and expression evaluation require a tree representation, one of the huge targets of these applications is to find a recursive function that calculates the number of spanning trees in graph G, however, for some families of graph, the spanning tree recursions can be complex. In this paper, we survey the several methods that can derive spanning tree recursions such as the deletion and contraction method and the splitting method, therefore, we gave the complexity of the families of corn graph, moreover, we gave an algorithm that counts the number of spanning trees in some families of graph without using recursive functions, finally, as applications we gave the complexity of the chained wheel graph and the chained corn graph. Our future purpose takes two different ways, the first one is the generalization of the splitting method, such that the decomposition of the planar graph can not be necessarily through a simple path and as consequence we will give a general method that can derive recursive functions for another families of planar graphs such as the grid graph $G_{n,m}$. The second direction is the generalization of the spanning tree algorithm given above such that it will count the complexity of any planar graph.

References

[1] Z. R. Bogdanowicz, *Formulas for the Number of Spanning Trees in a Fan*, Applied Mathematical Sciences, Vol. 2, N.16 (2008), 781-786.

[2] B. Bolläs, *Extremal Graph Theory*, Academic Press Inc., Lodon, (1978).

[3] A. Cayley, *A theorem on trees*, Quart. J. Math., (1889), 23, 376378.

[4] M. Desjarlais and R. Molina, *Counting spanning trees in grid graphs*, Congressus Numerantium, (2000), 145, 177185.

[5] R. Diestel, *Graph Theory, Electronic Edition 2000*, Springer-Verlag, New York, (2000), pp.2-15.

[6] M.H.S Haghighi and K. Bibak, *Recursive Relations for the Number of Spanning Trees*, Applied Mathematical Sciences, Vol. 3, (2009), N.46, 2263 - 2269.

[7] G. Kirchhoff, *über die Auflösung der Gleichungen auf, welche man bei der Untersuchung der linearen Verteilung galvanischer Ströme geführt wird*, Ann. Phy. Chem., 72, (1847), 497-508.

[8] D. Lotfi, M. EL Marraki and A. Modabish, *Recursive Relation for Couting the Complexity of Butterfly Map*, Journal of Theoretical and Applied Information Technology, Vol.21, (2011), N.1, 43-46.

[9] D. Lotfi, M. EL Marraki and D. Aboutajdine, *Spanning Tree Recursions for Crosses Maps*, Journal of Theoretical and Applied Information Technology, to appear.

[10] B. R. Myers, *Number of Spanning Trees in a Wheel*, IEEE Transactions on Circuit Theory, CT-18 (1971), 280-282.

[11] A. Modabish, D. Lotfi and M. El Marraki, *The Number of Spanning Trees of Planar Maps: theory and applications*, Proceeding of the International Conference on Multimedia Computing and Systems IEEE, Ouarzazate, Morocco, (2011).

[12] A. Modabish and M. El Marraki, *The Number of Spanning Trees of Certain Families of Planar Maps*, Applied Mathematical Sciences, Vol.5, (2011), N. 18, 883 - 898.

[13] T. Nishizeki and Md. Saidur Rahman, *Planar Graph Drawing*, Lecture Note Series on Computing, Vol.12, (2004), World Scientific Publishing Co. Pte. Ltd.5 Toh Tuck Link, Singapore 596224.

[14] J. Sedlacek , *On the Spanning Trees of Finite Graphs*, Cas. Pestovani Mat., 94, (1969), 217-221.

Received: October, 2011