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$( )^{*p}$ and ψ_p Operator

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Abstract. This paper deals with a space in which topology is replaced by its generalized open sets. Two operators have been discussed in the space in aspect of defining a new topology.

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1. Introduction

The study of ideal in topological space was first introduced by K. Kuratowski[13] in 1930. After that Vaidyanathaswamy[24] has introduced local function with the help of ideal. This field has been enriched by so many mathematicians like Vaidyanathaswamy [23],[24], Hashimoto[8], Hatir and Noiri[9], Hayashi[10], Dontchev[5] and many other. Actually, using this concept, a new field has been opened in topological space. The mathematicians, Hamlett and Jankovic[12],[7], Modak and Bandyopadhyay[18] were able to define a closure operator with the help of local function, and hence defined a new topology. Interior and closure are related by a nice relation[13] in a topological space and in this respect, Hamlett and Jankovic have defined an interior operator[7]. This interior operator and the closure operator(defined by local function) are related by the similar relation of interior and closure operator in the topological space.

The study of generalized open sets in topological space is going on by the mathematicians like Levine[14], Andrijevic[1], Bhattacharyya and Lahiri[3], Cao et al [4], Ganster[6], Hatir and Noiri[9], Njastad[20], Reilly and Vamanamurty[22]. In this regards Mashhour et al in [15] ,[17] and [16] have introduced preopen set in literature and have discussed its properties. Further Baker and Ekici[2], Noiri[11], [21], have studied this field. Again Ganster in [6] has given a remarkable representation of preopen set and has shown that, in a particular topological space, this collection forms a topology.
In this paper we shall introduced a new type of space with the help of ideal on a topological space and preopen sets of this topological space. Further in this space we shall introduce prelocal function. With help of prelocal function we shall define an operator which is related to the local function by the similar relation of closure and interior in topological space. Again we shall try to find out a closure operator and an interior operator with the help of prelocal function.

2. Preliminaries

Let (X,τ) be a topological space and $A \subset X$. We denote closure of A and interior of A by clA and intA respectively.

Definition2.1.[17]. A set A in a topological space (X,τ) is called preopen if $A\subset$ intclA.

The set of all preopen sets in a topological space (X,τ) is denoted as $PO(X,\tau)$. The complement of preopen is called preclosed. The intersection of all preclosed sets containing A is called the preclosure of A[4] and it is denoted as pclA. The preinterior[1] of A, denoted by pintA, is defined to be the union of all preopen sets contained in A.

A set A in a topological space (X,τ), $A\subset pclA\subset clA$ is hold for $\tau \subset PO(X,\tau)$.

Definition2.2. A set M in a topological space (X,τ) is said to be a preneighbourhood of a point $x\in X$ if for some preopen set U in (X,τ), $x \in U \subset M$.

Here we have discussed two results related to preclosure and preinterior:

Result 2.1.[1]. Let A be a subset of a topological space (X,τ). Then

(i). $pclA = A\cup cl(intA)$

(ii). $pintA = A\cap int(clA)$.

Result 2.2. Let (X,τ) be a topological space.Then for any $A\subset X$, $pintA = X - pcl(X-A)$.

Proof. $X - pintA = X - A\cap int(clA) = (X - A)\cup(X - int(clA)) = (X - A)\cup cl(X - clA)$
$=(X - A)\cup clint(X - A)$ [13] $= pcl(X - A)$.Therefore $pintA = X - pcl(X - A)$.

Definition2.3.[14]. A set A in a topological space (X,τ) is called semi-open if there is an open set O such that $O\subset A\subset clO$.
Its equivalent definition is:
Definition2.4.[14]. A set A in a topological space (X,τ) is called semi-open if $A\subset clintA$.
The set of all semi-open sets in a topological space (X,τ) is denoted as $SO(X,\tau)$.
Definition2.5.[20]. A set A in a topological space (X,τ) is called α-set if $A\subset intclintA$.
The set of all α-sets in a topological space (X,τ) is denoted as τ^{α}.
The definitions of ideal on the topological space and ideal topological space are following:
Definition2.6.[13]. A nonempty collection **I** of subsets of a given set X is said to be an ideal on X if (i). $A\in\mathbf{I}$ and $B\subset A$ implies $B\in\mathbf{I}$ (heredity) and (ii). $A\in\mathbf{I}$ and $B\in\mathbf{I}$ implies $A\cup B\in\mathbf{I}$ (finite additivity).
If (X,τ) is a topological space and **I** is an ideal on X, then $(X, \tau, \mathbf{I})$ is called ideal topological space[5].
Let $\wp(X)$ denote the power set of X.
Definition2.7.[24]. Let $(X, \tau, \mathbf{I})$ be an ideal topological space. A set operator $(\ )^* : \wp(X) \to \wp(X)$, is called the local function of **I** on X with respect to τ, is defined as: $(A)^*(\mathbf{I}, \tau) = \{x \in X: O_x \cap A \notin \mathbf{I}$, for every open set O_x containing $x\}$, for every $A\in \wp(X)$. This is simply called local function and simply denoted as A^*.
Definition2.8.[19]. Let $(X, \tau, \mathbf{I})$ be an ideal topological space. An operator $\psi : \wp(X) \to \tau$ is defined as: $\psi(A) = \{x \in X$: there exists an open set O_x containing x such that $O_x - A \in \mathbf{I}\}$, for every $A\in \wp(X)$.
Its equivalent definition is; $\psi(A) = X - (X - A)^*$.

3. ( )^{*p} – Operator

In this section we shall introduce preideal space and $(\ )^{*p}$ operator and discuss various properties of this operator.
Let (X,τ) be a topological space and **I** be an ideal on X, then $(X, PO(X,\tau), \mathbf{I})$ is called preideal space. Now we shall define the operator $(\ )^{*p}$.
Definition3.1. Let $(X, PO(X,\tau), \mathbf{I})$ be a preideal space. A set operator $(\ )^{*p} : \wp(X) \to \wp(X)$, is called the prelocal function of **I** on X with respect to $PO(X,\tau)$, is defined as: $(A)^{*p}(\mathbf{I}, PO(X,\tau)) = \{x \in X: U_x \cap A \notin \mathbf{I}$, for every preopen set U_x containing $x\}$, for every $A\in \wp(X)$.
This is simply called prelocal function and simply denoted as A^{*p}.
We have discussed the properties of prelocal function in following theorem:

Theorem3.1. Let $(X, PO(X,\tau), \mathbf{I})$ be a preideal space, and let A, B, A_1, A_2, ----- A_i,----- be subsets of X. Then

(i). $\phi^{*s} = \phi$.
(ii). $A \subseteq B$ implies $A^{*p} \subseteq B^{*p}$.
(iii). for another ideal $\mathbf{J} \supseteq \mathbf{I}$ on X, $A^{*p}(\mathbf{J}) \subset A^{*p}(\mathbf{I})$.
(iv). $A^{*p} \subset A^*$.
(v). $A^{*p} \subset$ pclA.
(vi). $A^{*p} \subset A^* \subset$ clA.
(vii). $A^{*p} \subset$ pclA $\subset$ clA.
(viii). $(A^*)^{*p} \subset A^*$.
(ix). $(A^{*p})^* \subset A^*$.
(x). $(A^{*p})^{*p} \subset A^*$.
(xi). A^{*p} is a preclosed set.
(xii). $(A^{*p})^{*p} \subset A^{*p}$.
(xiii). $A^{*p} \cup B^{*p} \subseteq (A \cup B)^{*p}$
(xiv). $\cup_i A_i^{*p} \subseteq (\cup_i A_i)^{*p}$.
(xv). $(A \cap B)^{*p} \subset A^{*p} \cap B^{*p}$.
(xvi). for α - set V, $V \cap A^{*p} = V \cap (V \cap A)^{*p} \subset (V \cap A)^{*p}$.
(xvii). for open set O, $O \cap A^{*p} = O \cap (O \cap A)^{*p} \subset (O \cap A)^{*p}$.
(xviii). for preopen set U, $U \cap (U \cap A)^{*p} \subset U \cap A^{*p}$.
(xix). for $I \in \mathbf{I}$, $(A \cup I)^{*p} = A^{*p} = (A - I)^{*p}$.

Proof. (i). Proof is obvious from definition of prelocal function.
(ii). Let $x \in A^{*p}$. Then for every preopen set U_x containing x, $U_x \cap A \notin \mathbf{I}$. Since $U_x \cap A \subseteq U_x \cap B$, then $U_x \cap B \notin \mathbf{I}$. This implies that $x \in B^{*p}$.
(iii). Let $x \in A^{*p}(\mathbf{J})$. Then for every preopen set U_x (containing x), $U_x \cap A \notin \mathbf{J}$. This implies that $U_x \cap A \notin \mathbf{I}$, so $x \in A^{*p}(\mathbf{I})$. Hence $A^{*p}(\mathbf{J}) \subset A^{*p}(\mathbf{I})$.
(iv). Let $x \in A^{*p}$. We claim that $x \in A^*$. If not, then there is an open set O_x (containing x), $O_x \cap A \in \mathbf{I}$. Let U_x be the preopen set containing x. Now $O_x \cap U_x \cap A \subset O_x \cap A \in \mathbf{I}$, a contraction to the fact that $O_x \cap U_x$ is a preopen set containing x[15]. So $x \in A^*$ and hence $A^{*p} \subset A^*$.
(v). Let $x \in A^{*p}$. Then for every preopen set U_x containing x, $U_x \cap A \notin \mathbf{I}$. This implies that $U_x \cap A \neq \phi$. Hence $x \in$ pclA.
(vi). We know $A^* \subset$ clA[12], and from result (iv), $A^{*p} \subset A^* \subset$ clA holds.
(vii). We know pclA $\subset$ clA, and from result (v), $A^{*p} \subset$ pclA $\subset$ clA holds.
(viii). From (iv), we get $(A^*)^{*p} \subset (A^*)^*$, and we know that $(A^*)^* \subset A^*$[12]. Hence the result.
(ix). From (iv), $A^{*p} \subset A^*$. Then $(A^{*p})^* \subset (A^*)^* \subset A^*$[12].
(x). Proof is obvious from (iv) and (viii).

(xi). From definition of preneighbourhood, each preneighbourhood M of x contains an preopen set U_x containing x . Now if $A\cap M\in \mathbf{I}$ then for $A\cap U_x \subset A\cap M$, $A\cap U_x \in \mathbf{I}$. It follows that $X - A^{*p}$ is the union of preopen sets. We know that arbitrary union of preopen sets is a preopen set[15]. So $X - A^{*p}$ is a preopen set and hence A^{*p} is a preclosed set[4].

(xii). From (v), $(A^{*p})^{*p} \subset pclA^{*p} = A^{*p}$ [4], since A^{*p} is a preclosed set.

(xiii). We know that $A \subseteq (A\cup B)$ and $B \subseteq (A\cup B)$. Then from (ii), $A^{*p}\subseteq (A\cup B)^{*p}$ and $B^{*p} \subseteq (A\cup B)^{*p}$. Hence $A^{*p}\cup B^{*p} \subseteq (A\cup B)^{*p}$.

(xiv). Proof is obvious from (xiii).

(xv). We know that $A\cap B\subset A$ and $A\cap B\subset B$, then from (ii), $(A\cap B)^{*p}\subset A^{*p}$ and $(A\cap B)^{*p}\subset B^{*p}$. Hence $(A\cap B)^{*p}\subset A^{*p}\cap B^{*p}$.

(xvi). Let $x\in V\cap A^{*p}$. Let U_x be a preopen set containing x, then $U_x\cap V\cap A \notin \mathbf{I}$, since $x\in A^{*p}$ and $U_x\cap V$ is a preopen set containing x[6]. Hence $x\in (V\cap A)^{*p}$. So $V\cap A^{*p} \subset (V\cap A)^{*p}$. Therefore

$$V\cap A^{*p}\subset V\cap (V\cap A)^{*p} \text{---------(i).}$$

Again for $V\cap A \subset A$, $(V\cap A)^{*p} \subset A^{*p}$. So

$$V\cap (V\cap A)^{*p} \subset V\cap A^{*p} \text{---------(ii).}$$

From (i) and (ii), we have $V\cap A^{*p} = V\cap (V\cap A)^{*p}$.
Hence the result.

(xvii). Proof is obvious from the result $\tau \subset PO(X,\tau) = PO(X,\tau^{\alpha})$ [6].

(xviii). Since $U\cap A \subset A$, then $(U\cap A)^{*p} \subset A^{*p}$. So $U\cap (U\cap A)^{*p} \subset U\cap A^{*p}$.

Remark3.1. The reverse inclusion of the above result does not hold because the intersection of two preopen sets may not be a preopen set in general.

Proof of the Theorem3.1.(xix). Since $A \subset (A\cup I)$, then

$$A^{*p} \subset (A\cup I)^{*p} \text{ ----------------(i).}$$

Let $x\in (A\cup I)^{*p}$, then for every preopen set U_x containing x, $U_x\cap (A\cup I) \notin \mathbf{I}$. This implies that $U_x\cap A\notin \mathbf{I}$ (If possible suppose that $U_x\cap A\in \mathbf{I}$. Again $U_x\cap I \subset I$ implies $U_x\cap I\in \mathbf{I}$ and hence $U_x\cap (A\cup I) \in \mathbf{I}$, a contradiction). Hence $x\in A^{*p}$ and

$$(A\cup I)^{*p} \subset A^{*p} \text{ -------------------(ii).}$$

From (i) and (ii) we have

$$(A\cup I)^{*p} = A^{*p} \text{ -------------------(iii).}$$

Since $(A - I) \subset A$, then

$$(A - I)^{*p}\subset A^{*p} \text{-------------------(iv).}$$

For reverse inclusion, let $x\in A^{*p}$. We claim that $x\in (A - I)^{*p}$, if not, then there is a preopen set U_x containing x, $U_x\cap (A - I)\in \mathbf{I}$. Given that $I\in \mathbf{I}$, then $I\cup (U_x\cap (A - I)\in \mathbf{I}$. This implies that $I\cup (U_x\cap A)\in \mathbf{I}$. So, $U_x\cap A\in \mathbf{I}$, a contradiction to the fact that $x\in A^{*p}$. Hence

$$A^{*p}\subset (A - I)^{*p} \text{ --------------------(v).}$$

From (iv) and (v) we have

$$A^{*p} = (A - I)^{*p} \text{ -------------------(vi).}$$

Again from (iii) and (vi), we get $(A\cup I)^{*p} = A^{*p} = (A - I)^{*p}$.
Following example shows that $A^{*p} \cup B^{*p} = (A \cup B)^{*p}$ does not hold in general.
Example3.1. Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{c,d\}, \{b,c,d\}, \{a,c,d\}\}$, $\mathbf{I} = \{\phi, \{a\}\}$. Then $C(\tau)$ (closed sets) $= \{\phi, X, \{a,b\}, \{a\}, \{b\}\}$ and $PO(X,\tau) = \{\phi, X, \{c\}, \{d\}, \{c,d\}, \{a,c\}, \{b,c\}, \{a,d\}, \{b,d\}, \{a,b,c\}, \{b,c,d\}, \{a,c,d\}, \{a,b,d\}\}$. preopen sets containing 'a' are: X,{a,c},{a,d}, {a,b,c}, {a,c,d}, {a,b,d}; preopen sets containing 'b' are: X,{b,c},{b,d}, {a,b,c}, {b,c,d}, {a,b,d}; preopen sets containing 'c' are: X,{c},{c,d}, {a,c}, {b,c},{a,b,c}, {b,c,d}, {a,c,d}; preopen sets containing 'd' are: X,{d},{c,d}, {a,d},{b,d},{b,c,d},{a,c,d}, {a,b,d}. Consider $A = \{a,c\}$ and $B = \{a,d\}$, then $A^{*p} = \{c\}$ and $B^{*p} = \{d\}$. Now $(A \cup B)^{*p} = \{a,c,d\}^{*p} = \{a,b,c,d\}$. Hence $A^{*p} \cup B^{*p} \neq (A \cup B)^{*p}$.
In [12], Jankovic and Hamlett have shown that $A\cup A^{*}$ is a closure (Kuratowski Closure) operator. But we are not able to define a closure operator with the help of prelocal function $((\ )^{*p}$ operator) because $A^{*p} \cup B^{*p} \neq (A \cup B)^{*p}$. Hence we are unable to define a new topology.

4. ψ_p – operator

In topological space $clA = X - int(X - A)$[13] is remarkable result. Many useful result have been proved with the help of this result. This relation is the motivation of defining the operator ψ_p .
Definition4.1. Let $(X, PO(X,\tau), \mathbf{I})$ be a preideal space. An operator $\psi_p : \wp(X) \to PO(X,\tau)$ is defined as: $\psi_p(A) = \{x \in X$: there exists a preopen set U_x containing x such that $U_x - A \in \mathbf{I}\}$, for every $A \in \wp(X)$.
We observe that $\psi_p(A) = X - (X - A)^{*p}$.
Behaviors of the operator ψ_p:
Theorem4.1. Let $(X, PO(X,\tau), \mathbf{I})$ be a preideal space.

(i). If $A\subseteq X$, then $\psi_p(A) \supset pintA$.
(ii). If $A\subseteq X$, then $\psi_p(A)$ is preopen.
(iii). If $A\subseteq B$, then $\psi_p(A) \subseteq \psi_p(B)$.
(iv). If $A, B\in \wp(X)$, then $\psi_p(A) \cup \psi_p(B) \subset \psi_p(A\cup B)$.
(v). If $A, B\in \wp(X)$, then $\psi_p(A\cap B) \subset \psi_p(A) \cap \psi_p(B)$.
(vi). If $U\in PO(X,\tau)$, then $U \subset \psi_p(U)$.
(vii). If $O\in\tau$, then $O \subset \psi_p(O)$.
(viii). If $V \in \tau^{\alpha}$, $V \subset \psi_p(V)$.
(ix). If $A\subseteq X$, then $\psi_p(A) \supset \psi(A)$.
(x). If $A\subseteq X$, then $\psi_p(A) \subset \psi_p(\psi_p(A))$.
(xi). If $A\subseteq X$, $I\in\mathbf{I}$, then $\psi_p(A - I) = \psi_p(A)$.
(xii). If $A\subseteq X$, $I\in\mathbf{I}$, then $\psi_p(A\cup I) = \psi_p(A)$.

(xiii). If $(A – B) \cup (B – A) \in \mathbf{I}$, then $\psi_p(A) = \psi_p(B)$.

Proof.(i). From definition of ψ_p operator, $\psi_p(A) = X – (X – A)^{*p}$. Then $\psi_p(A) = X – (X – A)^{*p} \supset X - pcl(X – A)$, from Theorem3.1(v). Hence $\psi_p(A) \supset pintA$ (using Result2.2.).

(ii). Since $(X – A)^{*p}$ is a preclosed set (from Theorem3.1(xi)), then $X - (X – A)^{*p}$ is a preopen set[4]. Hence $\psi_p(A)$ is preopen.

(iii). Given that $A \subseteq B$, then $(X – A) \supseteq (X – B)$. Then from Theorem3.1(ii), $(X – A)^{*p} \supseteq (X – B)^{*p}$ and hence $\psi_p(A) \subseteq \psi_p(B)$.

(iv). Proof is obvious from above property.

(v). Since $A \cap B \subset A$ and $A \cap B \subset B$, then from (iii), $\psi_p(A \cap B) \subset \psi_p(A) \cap \psi_p(B)$.

(vi). Let $U \in PO(X,\tau)$. Then $(X – U)$ is a preclosed set and hence $pcl(X – U) = (X – U)$[4]. This implies that $(X – U)^{*p} \subset pcl(X – U) = (X – U)$. Hence $U \subset X - (X – U)^{*p}$, so $U \subset \psi_p(U)$.

(vii). Proof is obvious from the relation $\tau \subset PO(X,\tau)$.

(viii). Proof is obvious from the relation $\tau^{\alpha} = SO(X,\tau) \cap PO(X,\tau)$ [22].

(ix). From Theorem3.1.(iv), we have that $(X – A)^{*p} \subset (X – A)^{*}$. This implies that $X - (X – A)^{*p} \supset X - (X – A)^{*}$ and hence $\psi_p(A) \supset \psi(A)$.

(x). From (ii), $\psi_p(A) \in PO(X,\tau)$. Again from (vi), $\psi_p(A) \subset \psi_p(\psi_p(A))$.

(xi). We know that $X- (X – (A – I))^{*p} = X – ((X – A) \cup I)^{*p} = X – (X – A)^{*p}$ (from Theorem3.1.(xix)). So $\psi_p(A - I) = \psi_p(A)$.

(xii). We know that $X – (X - A \cup I)^{*p} = X – ((X – A) – I)^{*p} = X – (X – A)^{*p}$ (using the Theorem3.1.(xix)). Thus $\psi_p(A \cup I) = \psi_p(A)$.

(xiii). Given that $(A – B) \cup (B – A) \in \mathbf{I}$, and let $A – B = I_1$, $B – A = I_2$. We observe that I_1 and $I_2 \in \mathbf{I}$ by heredity. Also observe that $B = (A – I_1) \cup I_2$. Thus $\psi_p(A) = \psi_p(A – I_1) = \psi_p((A – I_1) \cup I_2) = \psi_p(B)$.

In following example we shall show that $\psi_p(A \cap B) = \psi_p(A) \cap \psi_p(B)$ does not hold in general.

Example4.1. Consider Example3.1.Here we consider $A = \{b,d\}$ and $B = \{b,c\}$, then $\psi_p(A) = X – \{a,c\}^{*p} = X – \{c\} = \{a,b,d\}$ and $\psi_p(B) = X – \{a,d\}^{*p} = X – \{d\} = \{a,b,c\}$. Now $\psi_p(\{b\}) = X – \{a,c,d\}^{*p} = X – \{a,b,c,d\} = \phi$.

In [7], Hamlett and Jankovic have shown that $A \cap \psi(A)$ is an interior operator. Here we are not able to define an interior operator with the help of ψ_p operator because $\psi_p(A \cap B) \neq \psi_p(A) \cap \psi_p(B)$ in general. That is we can not define a new topology with the help of ψ_p operator.

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