International Mathematical Forum, 4, 2009, no. 8, 363 - 366

Reduction Technique for $n \times n$ Complex Matrix Systems

N. M. A. Nik Long, Z. K. Eshkuvatov and H. S. Sapar

Department of Mathematics and
Institute for Mathematical Research
University Putra Malaysia
43400 Serdang, Selangor, Malaysia
nmasri@math.upm.edu.my

Abstract

In this short paper, we describe a technique of reducing the system of complex matrix equations obtained from solving the system of hypersingular integral equation, into the real system of algebraic equations, in which it can be solved numerically. The example of using of the technique is shown.

Mathematics Subject Classification: 15A24

Keywords: complex matrix

1 Introduction

The formulation of the multiple curved cracks problem using complex potential method has attracted many researchers [1, 2, 3, 4, 5]. In [3], the hypersingular integral equation for n cracks has the form

$$\begin{aligned} &\frac{1}{\pi} \oint_{-a_k}^{a_k} \frac{g_k(t)dt}{(t-s_k)^2} + \\ &\quad \frac{1}{\pi} \sum_{j=1, j \neq k}^{n} \int_{-a_j}^{a_j} \left[M_{jk}(s_j, s_k) g_j(s_j) + L_{jk}(s_j, s_k) \overline{g_j(s_j)} \right] ds \\ &\qquad = N_k(s_k) + iT_k(s_k),\ |s_k| < a_k,\ k = 1, 2, \cdots, n. \end{aligned} \tag{1}$$

where M_{jk} and L_{jk} are known regular kernels, $N_k(s_k) + iT_k(s_k)$ are the right hand term of the equations and $g_k(t)$ are to be determined. The equal sign in the first integral denotes the hypersingular integral, and must be interpreted in the sense of Hadamart finite part integral. By letting

$$g_k(t_k) = \sqrt{a_k^2 - s_k^2} H_k(s_k)$$

and using the following quadrature formulas for hypersingular and regular integrals [1], respectively,

$$\frac{1}{\pi}\,\mathop{\int\!\!\!\!\!\!=}\nolimits_{-a_k}^{a_k} \frac{\sqrt{a_K^2 - s_k^2}H_k(s_k)ds}{(s_k - s_{k0})^2} = \sum_{j=1}^{M+1} W_j(s_{k0})H_k(s_{k0})) \tag{2}$$

and

$$\frac{1}{\pi}\int_{-a_k}^{a_k} \sqrt{a_k^2 - s_k^2}H_k(s_k)ds = \frac{1}{M+2}\sum_{j=1}^{M+1} (a_k^2 - s_j^2)G(s_j) \tag{3}$$

where $W_j(s_{k0})$ are the weight functions, and collocate at n points, one arrives at the following system

$$\begin{array}{rcl} A_{11}H_1 + A_{12}H_2 + \cdots + A_{1n}H_n &=& C_1 \\ A_{21}H_1 + A_{22}H_2 + \cdots + A_{2n}H_n &=& C_2 \\ &\vdots& \\ A_{n1}H_1 + A_{n2}H_2 + \cdots + A_{nn}H_n &=& C_n \end{array} \tag{4}$$

where A_{jk} are $n \times n$ complex matrices with elements $z_{lm}^{jk} = x_{lm}^{jk} + iy_{lm}^{jk}$, $H_k = h_{k1} + ih_{k2}$ are unknown complex coefficients and $C_j = c_{j1} + ic_{j2}$ are known complex vectors. The system (4) can be written in a compact form as

$$A_{jk}H_k = C_j \tag{5}$$

where the repeating subscript denotes the summation notation. Solving this complex system can be complicated. It is the aim of this paper to present a simple and effective technique of reducing the above system of complex matrices into the real system of linear algebraic equations.

2 Solution Technique

Choose $H_k = C_j = 1 + i$ and multiply (4) by $1 + i$ gives

$$\mathbf{A}_{jk}h_{k1,2} = \mathbf{C}_j \tag{6}$$

where

$$\mathbf{A}_{jk} = \begin{pmatrix} A'_{11} & A'_{12} & \cdots & A'_{1n} \\ A'_{21} & A'_{22} & \cdots & A'_{2n} \\ \vdots & & & \\ A'_{n1} & A'_{n2} & \cdots & A'_{nn} \end{pmatrix}. \tag{7}$$

The odd and even rows of matrix (7) are defined as $A'_{(2j-1)k} = Re(1+i)A_{jk}$ and $A'_{(2j)k} = Im(1+i)A_{jk}$,

$$h_{k1,2} = \begin{pmatrix} h_{11} & h_{12} & h_{21} & h_{22} & \cdots & h_{(2k)1} & h_{(2k)2}, \end{pmatrix}^T$$

and $\mathbf{C}_{2j-1} = Re((1+i)C_j)$ and $\mathbf{C}_{2j} = Im((1+i)C_j)$, $j,k = 1,2,3,\cdots,n$. Each of the matrix A'_{jk} has the following elements

$$A'_{jk} = \begin{pmatrix} x_{11}^{jk} & -y_{11}^{jk} & x_{12}^{jk} & -y_{12}^{jk} & \cdots & x_{1n}^{jk} & -y_{1n}^{jk} \\ y_{11}^{jk} & x_{11}^{jk} & y_{12}^{jk} & x_{12}^{jk} & \cdots & y_{1n}^{jk} & x_{1n}^{jk} \\ x_{21}^{jk} & -y_{21}^{jk} & x_{22}^{jk} & -y_{22}^{jk} & \cdots & x_{2n}^{jk} & -y_{2n}^{jk} \\ y_{21}^{jk} & x_{21}^{jk} & & & & & \\ \vdots & \vdots & & \ddots & & \vdots & \vdots \\ x_{n1}^{jk} & -y_{n1}^{jk} & \cdots & & & x_{nn}^{jk} & -y_{nn}^{jk} \\ y_{n1}^{jk} & x_{n1}^{jk} & \cdots & & & y_{nn}^{jk} & x_{nn}^{jk} \end{pmatrix}.$$

Note that $Diag(A'_{jk}) = Re(A_{jk})$ and has the following order

$$Diag(A'_{jk}) = \begin{pmatrix} x_{11} & x_{11} & x_{22} & x_{22} & \cdots & x_{nn} & x_{nn} \end{pmatrix}$$

and all terms in Equation (6) are now real. It can be solved numerically without difficulty using any well known method of solving $n \times n$ system of linear algebraic equations, for example Gauss elimination method.

3 Example

Consider the system

$$\begin{pmatrix} 2+3i & 1-i \\ -1-2i & i \end{pmatrix}\begin{pmatrix} h_{11}+ih_{12} \\ h_{21}+ih_{22} \end{pmatrix}+\begin{pmatrix} 2-i & i \\ 1-3i & 4+2i \end{pmatrix}\begin{pmatrix} g_{11}+ig_{12} \\ g_{21}+ig_{22} \end{pmatrix}=\begin{pmatrix} 1 \\ i \end{pmatrix}$$

and

$$\begin{pmatrix} -1-i & 2-3i \\ 1-i & 4+i \end{pmatrix}\begin{pmatrix} h_{11}+ih_{12} \\ h_{21}+ih_{22} \end{pmatrix}+\begin{pmatrix} 1-2i & 2+i \\ -1-i & -1+i \end{pmatrix}\begin{pmatrix} g_{11}+ig_{12} \\ g_{21}+ig_{22} \end{pmatrix}=\begin{pmatrix} 1-i \\ -i \end{pmatrix}.$$

Using Equation (6), we can write the above system as

$$\begin{pmatrix} 2 & -3 & 1 & 1 & 2 & 1 & 0 & -1 \\ 3 & 2 & -1 & 2 & -1 & 2 & 1 & 0 \\ -1 & 2 & 0 & -1 & 1 & 3 & 4 & -2 \\ -2 & -1 & 1 & 0 & -3 & 1 & 2 & 4 \\ -1 & 1 & 2 & 3 & 1 & 2 & 2 & -1 \\ -1 & -1 & -3 & 2 & -2 & 1 & 1 & 2 \\ 1 & 1 & 4 & -1 & -1 & 1 & -1 & -1 \\ -1 & 1 & 1 & 4 & -1 & -1 & 1 & -1 \end{pmatrix}\begin{pmatrix} h_{11} \\ h_{12} \\ h_{21} \\ h_{22} \\ g_{11} \\ g_{12} \\ g_{21} \\ g_{22} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \\ 2 \\ 0 \\ 1 \\ -1 \end{pmatrix}.$$

This system can easily be solved for $h_{k1,2}$ and $g_{k1,2}$ using any existing well known method.

4 Conclusion

The above procedure provides a good and simple technique for solving the system of complex matrix equations.

ACKNOWLEDGEMENTS. The first author would like to thank the Ministry of Science, Technology and Inovation (MOSTI) through Science Fund, Vot No. 5450207.

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Received: May 9, 2008