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Interval Valued Intuitionistic $(S,T)-$Fuzzy Substructures in Semirings

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Abstract

In this paper, the notion of an interval valued intuitionistic fuzzy subsemiring (ideal) of a semiring with respect to $t-$norm T and $s-$norm S is given and the characteristic properties are described. The homomorphic image and inverse image are investigated. In particular, by the help of the congruence relations on semirings, new interval valued intuitionistic $(S,T)-$fuzzy subsemirings (ideals) are constructed.

Keywords: (sub)semiring, interval valued intuitionistic $(S,T)-$fuzzy subsemiring (ideal)

1 Introduction

After the introduction of fuzzy sets by Zadeh [12], there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov [2] is one among them. For more details on intuitionistic fuzzy sets, we refer the reader to [2, 3, 4]. In 1975, Zadeh [13] introduced the concept of interval valued fuzzy subsets, where the values of the membership functions are intervals of numbers instead of the numbers. Such fuzzy sets have some applications in the technological scheme of the functioning of a silo-farm with pneumatic transportation, in a plastic products company and in medicine (see the book [4]).

The fuzzy algebraic structures play a prominent role in mathematics with wide applications in many other branches such as theoretical physics, computer sciences, control engineering, information sciences, coding theory, topological spaces, logic, set theory, group theory, groupoids, real analysis, measure theory etc. Also the notion of fuzzy ideals in rings and semirings(in different views) have seriously studied by many mathematicians. Recently, some researchers are trying to present new views of fuzzy algebraic structures as intuitionistic fuzzy algebraic structures ([9], [10], [14], [15]). In algebra, we notice that the ideals of semirings play a crucial role in the structure theory, but they do not in general coincide with the usual ring ideals, for this reason, their usage is somewhat limited when we try to obtain some analogous ring theorems for semirings. Indeed, many results in rings apparently have no analogous in semirings by using only ideals. In this paper we introduce the notion of interval valued intuitionistic fuzzy subsemiring (ideal) of a semiring with respect to $t-$norm T and $s-$norm S. Then we characterize all of them base on special kind of levels $\mathfrak{U}(\widetilde{M}_{\mathcal{A}};[t,s])$ and $\mathfrak{L}(\widetilde{N}_{\mathcal{A}};[t,s])$, which is a generalization of classic level subsets. At the following the behaviour of these structures under homomorphisms is investigated. In particular, by the help of the congruence relations on semirings, we construct new interval valued intuitionistic $(S,T)-$fuzzy subsemiring (ideal) on semiring of quotient.

2 Preliminaries and notations

By a *semiring* we mean an algebraic system $(\mathcal{SR},+,.)$ consisting of a nonempty set $\mathcal{SR}$ together with binary operations on $\mathcal{SR}$ called add and multiplication such that $(\mathcal{SR},+)$ and $(\mathcal{SR},.)$ are semigroups and for all $x,y,z\in\mathcal{SR}$, we have $x.(y+z)=x.y+x.z$ which is called distributive. By a *subsemring* of $\mathcal{SR}$ we mean a non-empty subset $\mathcal{S}$ of $\mathcal{SR}$ such that for all $x,y\in\mathcal{S}$, we have $x.y\in\mathcal{S}$ and $x+y\in\mathcal{S}$. By a *left (right) ideal* of $\mathcal{SR}$ we mean a subsemiring $\mathcal{I}$ of $\mathcal{SR}$ such that for all $w\in\mathcal{SR}$ and $x\in\mathcal{I}$ we have $w.x\in\mathcal{I}$ $(x.w\in\mathcal{I})$. By an *ideal*, we mean a subsemiring of $\mathcal{SR}$ which both a left and a right ideal of $\mathcal{SR}$ (see [6]).

An equivalence relation θ on a semiring $\mathcal{SR}$ is said to be a *congruence relation*, if for all $x,y,z\in\mathcal{SR}$, $x\theta y$ implies $(x+z)\theta(y+z)$ and $(xz)\theta(yz)$, where by $x\theta y$ we mean $(x,y)\in\theta$. Also by $\mathcal{SR}/\theta$ we mean the set of all equivalence classes with respect to θ, or $\mathcal{SR}/\theta=\{\theta(x)|\ x\in\mathcal{SR}\}$ (see [6]).

By an *interval number* $\widetilde{a}$ we mean ([13]) an interval $[a^-,a^+]$, where $0\le a^-\le a^+\le 1$. The set of all interval number is denoted by $D[0,1]$. The interval $[a,a]$ is identified with the number $a\in[0,1]$. For interval numbers $\widetilde{a}_i=[a_i^-,a_i^+]\in D[0,1], i\in I$, we define

$$inf\widetilde{a}_i=[\bigwedge_{i\in I}a_i^-,\bigwedge_{i\in I}a_i^+],\qquad sup\widetilde{a}_i=[\bigvee_{i\in I}a_i^-,\bigvee_{i\in I}a_i^+]$$

and put

(1) $\widetilde{a}_1\le\widetilde{a}_2\Longleftrightarrow a_1^-\le a_2^-$ and $a_1^+\le a_2^+$,
(2) $\widetilde{a}_1=\widetilde{a}_2\Longleftrightarrow a_1^-=a_2^-$ and $a_1^+=a_2^+$,
(3) $\widetilde{a}_1<\widetilde{a}_2\Longleftrightarrow\widetilde{a}_1\le\widetilde{a}_2$ and $\widetilde{a}_1\ne\widetilde{a}_2$,
(4) $k\tilde{a}=[ka^-,ka^+]$, whenever $0\le k\le 1$.

It is clear that $(D[0,1],\leq,\bigvee,\bigwedge)$ is a complete lattice with $0=[0,0]$ as the least element and $1=[1,1]$ as the greatest element.

By an *interval number fuzzy set* F on X we mean ([11]) the set

$$F=\{(x,[\mu_F^-(x),\mu_F^+(x)])|\ x\in X\},$$

where μ_F^- and μ_F^+ are two fuzzy subset of X such that $\mu_F^-(x)\leq\mu_F^+(x)$ for all $x\in X$. Putting $\mu_F(x)=[\mu_F^-,\mu_F^+]$, we see that $F=\{(x,\mu_F(x))\mid x\in X\}$, where $\mu_F(x):X\longrightarrow D[0,1]$.

As it is well-known, the function $\delta:[0,1]\times[0,1]\longrightarrow[0,1]$ is called a *$t-$norm* (resp. *$s-$norm*) if δ satisfied the condition: (i) $\delta(x,1)=x$ (resp. $\delta(x,0)=x$), (ii) $\delta(x,y)=\delta(y,x)$, (iii) $\delta(\delta(x,y),z)=\delta(x,\delta(y,z)$, $(iv)\delta(x,u)\leq\delta(x,w)$, for all $x,y,z,u,w\in[0,1]$, where $u\leq w$. A $t-$norm (resp. $s-$norm) δ is called an *idempotent $t-$norm* if $\delta(x,x)=x$, for all $x\in[0,1]$, (see [13]). If δ is an idempotent $t-$norm ($s-$norm), then the mapping $\Delta:D[0,1]\times D[0,1]\longrightarrow D[0,1]$ defined by $\Delta(\widetilde{a}_1,\widetilde{a}_2)=[\delta(a_1^-,a_2^-),\delta(a_1^+,a_2^+)]$ is, as it is not difficult to verify, an idempotent $t-$norm ($s-$norm, respectively) and is called an *idempotent interval $t-$norm ($s-$norm,* respectively).

According to Atanassov ([2], [3], [4]) *an interval valued intuitionistic fuzzy* set on X is defined as an object of the form $\mathcal{A}=\{(x,\widetilde{M}_{\mathcal{A}}(x),\widetilde{N}_{\mathcal{A}}(x))\mid x\in X\}$, where $\widetilde{M}_{\mathcal{A}}(x)$ and $\widetilde{N}_{\mathcal{A}}(x)$ are interval valued fuzzy sets on X such that $0\leq sup\widetilde{M}_{\mathcal{A}}(x)+sup\widetilde{N}_{\mathcal{A}}(x)\leq 1$ for all $x\in X$. For the sake of simplicity, in the following such interval valued intuitionistic fuzzy sets will be denoted by $\mathcal{A}=(\widetilde{M}_{\mathcal{A}},\widetilde{N}_{\mathcal{A}})$.

3 Interval valued intuitionistic $(S,T)-$fuzzy substructures of semirings

In what follows, let $\mathcal{SR}$ denote a semiring unless otherwise specified.

Definition 3.1. An interval valued intuitionistic fuzzy set $\mathcal{A}=(\widetilde{M}_{\mathcal{A}},\widetilde{N}_{\mathcal{A}})$ of $\mathcal{SR}$ is called an:

(i) *interval valued intuitionistic $(S,T)-$fuzzy subsemring* if for all $x,y\in\mathcal{SR}$:

(1) $\widetilde{M}_{\mathcal{A}}(x+y)\geq T(\widetilde{M}_{\mathcal{A}}(x),\widetilde{M}_{\mathcal{A}}(y))$, $\widetilde{N}_{\mathcal{A}}(x+y)\leq S(\widetilde{N}_{\mathcal{A}}(x),\widetilde{N}_{\mathcal{A}}(y))$,

(2) $\widetilde{M}_{\mathcal{A}}(xy)\geq T(\widetilde{M}_{\mathcal{A}}(x),\widetilde{M}_{\mathcal{A}}(y))$, $\widetilde{N}_{\mathcal{A}}(xy)\leq S(\widetilde{N}_{\mathcal{A}}(x),\widetilde{N}_{\mathcal{A}}(y))$.

(ii) *interval valued intuitionistic $(S,T)-$fuzzy left ideal* if for all $x,y\in\mathcal{SR}$:

(1) $\widetilde{M}_{\mathcal{A}}(x+y)\geq T(\widetilde{M}_{\mathcal{A}}(x),\widetilde{M}_{\mathcal{A}}(y))$, $\widetilde{N}_{\mathcal{A}}(x+y)\leq S(\widetilde{N}_{\mathcal{A}}(x),\widetilde{N}_{\mathcal{A}}(y))$,

(2) $\widetilde{M}_{\mathcal{A}}(xy)\geq\widetilde{M}_{\mathcal{A}}(y)$, $\widetilde{N}_{\mathcal{A}}(xy)\leq\widetilde{N}_{\mathcal{A}}(y)$.

Similarly an *interval valued intuitionistic $(S,T)-$fuzzy right ideal* is defined. An *interval valued intuitionistic $(S,T)-$fuzzy ideal* is an interval valued intuitionistic $(S,T)-$fuzzy right and left ideal. Clearly if $\mathcal{A}=(\widetilde{M}_{\mathcal{A}},\widetilde{N}_{\mathcal{A}})$ is an interval valued intuitionistic $(S,T)-$fuzzy ideal of $\mathcal{SR}$ then $\widetilde{M}_{\mathcal{A}}(xy)\geq S(\widetilde{M}_{\mathcal{A}}(x),\widetilde{M}_{\mathcal{A}}(y))$ and $\widetilde{N}_{\mathcal{A}}(xy)\leq T(\widetilde{N}_{\mathcal{A}}(x),\widetilde{N}_{\mathcal{A}}(y))$.

Example. On a four element semiring $(\mathcal{SR},+,.)$ defined by the following two tables:

$+$	0	a	b	c
0	0	a	b	c
a	a	a	b	c
b	b	b	b	c
c	c	c	c	b

$.$	0	a	b	c
0	0	0	0	0
a	0	a	a	a
b	0	a	a	a
c	0	a	a	a

consider interval valued fuzzy set $\mathcal{A}=(\widetilde{M}_{\mathcal{A}},\widetilde{N}_{\mathcal{A}})$ as follow

$$\widetilde{M}_{\mathcal{A}}(x)=\begin{cases}[0.4,0.5], & \text{if } x=0\\ [0.2,0.3], & \text{if } x\neq 0\end{cases}\qquad \widetilde{N}_{\mathcal{A}}(x)=\begin{cases}[0.2,0.3], & \text{if } x=0\\ [0.7,0.8], & \text{if } x\neq 0\end{cases}$$

It is easy to verify that $\mathcal{A}=(\widetilde{M}_{\mathcal{A}},\widetilde{N}_{\mathcal{A}})$ is not only an interval valued intuitionistic $(S,T)-$fuzzy subsemiring but also an interval valued intuitionistic $(S,T)-$fuzzy ideal.

Example. Let $\mathbb{N}$ be the set of integers and let

$$\widetilde{M}_{\mathcal{A}}(x)=\begin{cases}[0.9,1], & \text{if } x\in<4>\\ [0.4,0.5], & \text{if } x\in<2>-<4>\\ [0,0.1], & \text{otherwise}\end{cases}$$

$$\widetilde{N}_{\mathcal{A}}(x)=\begin{cases}[0,0.1], & \text{if } x\in<4>\\ [0.4,0.5], & \text{if } x\in<2>-<4>\\ [0.9,1], & \text{otherwise}\end{cases}$$

where $<n>$ denotes the set of all integers divide by n. It is routine to calculate that $\mathcal{A}=(\widetilde{M}_{\mathcal{A}},\widetilde{N}_{\mathcal{A}})$ is an interval valued intuitionistic $(S,T)-$fuzzy subsemiring and ideal of semiring $(\mathbb{N},+,.)$.

With any interval valued intuitionistic fuzzy set $\mathcal{A}=(\widetilde{M}_{\mathcal{A}},\widetilde{N}_{\mathcal{A}})$ of $\mathcal{SR}$ there are connected two levels:

$$\mathfrak{U}(\widetilde{M}_{\mathcal{A}};[t,s])=\{x\in\mathcal{SR}\mid \widetilde{M}_{\mathcal{A}}(x)\geq[t,s]\},$$

$$\mathfrak{L}(\widetilde{N}_{\mathcal{A}};[t,s])=\{x\in\mathcal{SR}\mid \widetilde{N}_{\mathcal{A}}(x)\leq[t,s]\}.$$

Theorem 3.2. *Let T and S be idempotent intervals $t-$norm and $s-$norm respectively. Then $\mathcal{A}=(\widetilde{M}_{\mathcal{A}},\widetilde{N}_{\mathcal{A}})$ is an interval valued intuitionistic $(S,T)-$fuzzy subsemiring (ideal) of $\mathcal{SR}$ if and only if for all $t,s\in[0,1],\ t\leq s,\ \mathfrak{U}(\widetilde{M}_{\mathcal{A}};[t,s])$ and $\mathfrak{L}(\widetilde{N}_{\mathcal{A}};[t,s])$ are subsemirings (ideals) of $\mathcal{SR}$.*

Proof. Let $\mathcal{A}=(\widetilde{M}_{\mathcal{A}},\widetilde{N}_{\mathcal{A}})$ be an interval valued intuitionistic $(S,T)-$fuzzy ideal of $\mathcal{SR}$. Then for every $x,y\in\mathfrak{U}(\widetilde{M}_{\mathcal{A}};[t,s])$ and $r\in\mathcal{SR}$ we have $\widetilde{M}_{\mathcal{A}}(x)\geq[t,s]$ and $\widetilde{M}_{\mathcal{A}}(y)\geq[t,s]$. Hence $T(\widetilde{M}_{\mathcal{A}}(x),\widetilde{M}_{\mathcal{A}}(y))\geq T([t,s],[t,s])=[t,s]$, and so $\widetilde{M}_{\mathcal{A}}(x+y)\geq T(\widetilde{M}_{\mathcal{A}}(x),\ \widetilde{M}_{\mathcal{A}}(y))\geq[t,s]$. This means $x+y\in\mathfrak{U}(\widetilde{M}_{\mathcal{A}};[t,s])$. Also $\widetilde{M}_{\mathcal{A}}(xr)\geq\widetilde{M}_{\mathcal{A}}(x)\geq[t,s]$. This means $xr\in\mathfrak{U}(\widetilde{M}_{\mathcal{A}};[t,s])$. Similarly we can show that $rx\in\mathfrak{U}(\widetilde{M}_{\mathcal{A}};[t,s])$. Therefore $\mathfrak{U}(\widetilde{M}_{\mathcal{A}};[t,s])$ is an ideal of $\mathcal{SR}$. The proof of $\mathfrak{L}(\widetilde{N}_{\mathcal{A}};[t,s])$ is a subsemiring of $\mathcal{SR}$ is similar.

Conversely, assume that for every $[t,s]\in D[0,1]$ any non-empty $\mathfrak{U}(\widetilde{M}_{\mathcal{A}};[t,s])$ is an ideal of $\mathcal{SR}$. If $[t_0,s_0]=T(\widetilde{M}_{\mathcal{A}}(x),\widetilde{M}_{\mathcal{A}}(y))$ for some $x,y\in\mathcal{SR}$, then $x,y\in\mathfrak{U}(\widetilde{M}_{\mathcal{A}};[t_0,s_0])$ and so $x+y\in\mathfrak{U}(\widetilde{M}_{\mathcal{A}};[t_0,s_0])$. Therefore $\widetilde{M}_{\mathcal{A}}(x+y)\geq[t_0,s_0]=T(\widetilde{M}_{\mathcal{A}}(x),\widetilde{M}_{\mathcal{A}}(y))$. Also if $[t'_0,s'_0]=\widetilde{M}_{\mathcal{A}}(x)$, then $x\in\mathfrak{U}(\widetilde{M}_{\mathcal{A}};[t'_0,s'_0])$, and so $rx\in\mathfrak{U}(\widetilde{M}_{\mathcal{A}};[t'_0,s'_0])$ for every $r\in\mathcal{SR}$. Therefore $\widetilde{M}_{\mathcal{A}}(rx)\geq[t'_0,s'_0]=\widetilde{M}_{\mathcal{A}}(x)$. This proves that $\widetilde{M}_{\mathcal{A}}$ is an interval valued $T-$fuzzy left ideal of $\mathcal{SR}$. Analogously we prove $\widetilde{M}_{\mathcal{A}}$ is an interval valued $T-$fuzzy right ideal of $\mathcal{SR}$. Similarly, we can show that $\widetilde{N}_{\mathcal{A}}$ is an interval valued $S-$fuzzy ideal of $\mathcal{SR}$. Therefore $\mathcal{A}=(\widetilde{M}_{\mathcal{A}},\widetilde{N}_{\mathcal{A}})$ is an interval valued intuitionistic $(S,T)-$fuzzy ideal of $\mathcal{SR}$. For interval valued intuitionistic $(S,T)-$fuzzy subsemiring the proof is similar. □

Let $\mathcal{A}=(\widetilde{M}_{\mathcal{A}},\widetilde{N}_{\mathcal{A}})$ be an interval valued intuitionistic fuzzy set of $\mathcal{SR}$ and let $t,s,t',s'\in[0,1]$ such that $t\leq s$ and $t'\leq s'$. Put

$$\mathcal{R}^{[t,s]}_{[t',s']}=\{x\in\mathcal{SR}\mid\widetilde{M}_{\mathcal{A}}(x)\geq[t,s],\ \widetilde{N}_{\mathcal{A}}(x)\leq[t',s']\}.$$

Clearly $\mathcal{R}^{[t,s]}_{[t',s']}=\mathfrak{U}(\widetilde{M}_{\mathcal{A}};[t,s])\bigcap\mathfrak{L}(\widetilde{M}_{\mathcal{A}};[t',s'])$.

Corollary 3.3. *Let T and S be idempotent intervals t-norm and s-norm respectively. Then $\mathcal{A}=(\widetilde{M}_{\mathcal{A}},\widetilde{N}_{\mathcal{A}})$ is an interval valued intuitionistic $(S,T)-$fuzzy subsemiring (ideal) of $\mathcal{SR}$ if and only if for all $t,s,t',s'\in[0,1],t\leq s,\ t'\leq s'$, $\mathcal{R}^{[t,s]}_{[t',s']}$ is a subsemiring (ideal) of $\mathcal{SR}$.*

Proof. It is immediately followed by Theorem 3.2. □

Definition 3.4. Let $f:X\longrightarrow Y$ be a mapping and $\mathcal{A}=(\widetilde{M}_{\mathcal{A}},\widetilde{N}_{\mathcal{A}})$ and $\mathcal{B}=(\widetilde{M}_{\mathcal{B}},\widetilde{N}_{\mathcal{B}})$ interval valued intuitionistic sets X and Y, respectively. Then the image of $f[\mathcal{A}]=(f(\widetilde{M}_{\mathcal{A}}),f(\widetilde{N}_{\mathcal{A}}))$ of $\mathcal{A}$ is the interval valued intuitionistic fuzzy set of Y defined by

$$f(\widetilde{M}_{\mathcal{A}})(y)=\begin{cases}\sup\limits_{z\in f^{-1}(y)}\widetilde{M}_{\mathcal{A}}(z) & \text{if } f^{-1}(y)\neq\emptyset,\\ [0,0] & \text{otherwise}\end{cases}$$

$$f(\widetilde{N}_{\mathcal{A}})(y)=\begin{cases}\inf\limits_{z\in f^{-1}(y)}\widetilde{N}_{\mathcal{A}}(z) & \text{if } f^{-1}(y)\neq\emptyset,\\ [1,1] & \text{otherwise}\end{cases}$$

for all $y \in Y$. The inverse image $f^{-1}(\mathcal{B}) = (f^{-1}(\widetilde{M}_{\mathcal{B}}), f^{-1}(\widetilde{N}_{\mathcal{B}}))$ of $\mathcal{B}$ is an interval valued intuitionistic fuzzy set defined by $f^{-1}(\widetilde{M}_{\mathcal{B}})(x) = \widetilde{M}_{f^{-1}(\mathcal{B})}(x) = \widetilde{M}_{\mathcal{B}}(f(x))$, $f^{-1}(\widetilde{N}_{\mathcal{B}})(x) = \widetilde{N}_{f^{-1}(\mathcal{B})}(x) = \widetilde{N}_{\mathcal{B}}(f(x))$ for all $x \in X$.

Definition 3.5. Let $\mathcal{SR}_1$ and $\mathcal{SR}_2$ be two semirings. A mapping $f : \mathcal{SR}_1 \longrightarrow \mathcal{SR}_2$ is called a *homomorphism* if for all $x, y \in \mathcal{SR}_1$ we have $f(x+y) = f(x) + f(y)$. and $f(x.y) = f(x).f(y)$.

Lemma 3.6. *Let $\mathcal{SR}_1$ and $\mathcal{SR}_2$ be two semirings and $f : \mathcal{SR}_1 \longrightarrow \mathcal{SR}_2$ an epimorphism.*

(i) If $\mathcal{I}$ is a subsemiring (ideal) of $\mathcal{SR}_1$, then $f(\mathcal{I})$ is a subsemiring (ideal) of $\mathcal{SR}_2$.

(ii) If $\mathcal{J}$ is a subsemiring (ideal) of $\mathcal{SR}_2$, then $f^{-1}(\mathcal{J})$ is a subsemiring (ideal) of $\mathcal{SR}_1$.

Proof. Straightforward. □

Theorem 3.7. *Let $\mathcal{SR}_1$ and $\mathcal{SR}_2$ be two semirings and $f : \mathcal{SR}_1 \longrightarrow \mathcal{SR}_2$ an epimorphism and T and S idempotent intervals $t-$norm and $s-$norm respectively.*

(i) If $\mathcal{A} = (\widetilde{M}_{\mathcal{A}}, \widetilde{N}_{\mathcal{A}})$ is an interval valued intuitionistic $(S,T)-$fuzzy subsemiring (ideal) of $\mathcal{SR}_1$, then the image $f[\mathcal{A}] = (f(\widetilde{M}_{\mathcal{A}}), f(\widetilde{N}_{\mathcal{A}}))$ of $\mathcal{A}$ is an interval valued intuitionistic $(S,T)-$fuzzy subsemiring (ideal) of $\mathcal{SR}_2$.

(ii) If $\mathcal{B} = (\widetilde{M}_{\mathcal{B}}, \widetilde{N}_{\mathcal{B}})$ is an interval valued intuitionistic $(S,T)-$fuzzy subsemiring (ideal) of $\mathcal{SR}_2$, then the inverse image $f^{-1}(\mathcal{B}) = (f^{-1}(\widetilde{M}_{\mathcal{B}}), f^{-1}(\widetilde{M}_{\mathcal{B}}))$ is an interval valued intuitionistic $(S,T)-$fuzzy subsemring (ideal) of $\mathcal{SR}_1$.

Proof. (i) We prove this part for interval valued intuitionistic $(S,T)-$fuzzy subsemirings. For interval valued intuitionistic $(S,T)-$fuzzy ideal the proof is similar. Let $\mathcal{A} = (\widetilde{M}_{\mathcal{A}}, \widetilde{N}_{\mathcal{A}})$ be an interval valued intuitionistic $(S,T)-$fuzzy subsemiring of $\mathcal{SR}_1$. By Theorem 3.2, $\mathfrak{U}(\widetilde{M}_{\mathcal{A}}; [t,s])$ and $\mathfrak{L}(\widetilde{N}_{\mathcal{A}}; [t,s])$ are subsemirings of $\mathcal{SR}_1$ for every $[t,s] \in D[0,1]$. Therefore, by Lemma 3.6, $f(\mathfrak{U}(\widetilde{M}_{\mathcal{A}}; [t,s]))$ and $f(\mathfrak{L}(\widetilde{N}_{\mathcal{A}}; [t,s]))$ are subsemirings of $\mathcal{SR}_2$. But $\mathfrak{U}(f(\widetilde{M}_{\mathcal{A}}); [t,s]) = f(\mathfrak{U}(\widetilde{M}_{\mathcal{A}}; [t,s]))$ and $\mathfrak{L}(f(\widetilde{N}_{\mathcal{A}}); [t,s]) = f(\mathfrak{L}(\widetilde{N}_{\mathcal{A}}; [t,s]))$, so $\mathfrak{U}(f(\widetilde{M}_{\mathcal{A}}); [t,s])$ and $\mathfrak{L}(f(\widetilde{N}_{\mathcal{A}}); [t,s])$ are subsemirings of $\mathcal{SR}_2$. Therefore $f[\mathcal{A}]$ is an interval valued intuitionistic $(S,T)-$fuzzy subsemiring of $\mathcal{SR}_2$.

(ii) We prove this part for interval valued intuitionistic $(S,T)-$fuzzy ideals. For interval valued intuitionistic $(S,T)-$fuzzy subsemiring the proof is similar.

For any $x, y \in \mathcal{SR}_1$, we have

$$\widetilde{M}_{f^{-1}(\mathcal{B})}(x+y) = \widetilde{M}_{\mathcal{B}}(f(x+y)) \geq T(\widetilde{M}_{\mathcal{B}}(f(x)), \widetilde{M}_{\mathcal{B}}(f(y))) =$$

$$T(\widetilde{M}_{f^{-1}(\mathcal{B})}(x), \widetilde{M}_{f^{-1}(\mathcal{B})}(y)).$$

Also we have

$$\widetilde{M}_{f^{-1}(\mathcal{B})}(xy) = \widetilde{M}_{\mathcal{B}}(f(xy)) \geq S(\widetilde{M}_{\mathcal{B}}(f(x)), \widetilde{M}_{\mathcal{B}}(f(y))) =$$

$$S(\widetilde{M}_{f^{-1}(\mathcal{B})}(x), \widetilde{M}_{f^{-1}(\mathcal{B})}(y)).$$

This completes the proof that $\widetilde{M}_{f^{-1}(\mathcal{B})}$ is an interval valued $T-$fuzzy ideal of $\mathcal{SR}_1$. Similarly we can prove $\widetilde{N}_{f^{-1}(\mathcal{B})}$ is an interval valued $S-$fuzzy ideal of $\mathcal{SR}_1$. Therefore $f^{-1}(\mathcal{B}) = (f^{-1}(\widetilde{M}_{f^{-1}(\mathcal{B})}), f^{-1}(\widetilde{N}_{f^{-1}(\mathcal{B})}))$ is an interval valued intuitionistic $(S,T)-$fuzzy ideal of $\mathcal{SR}_1$. $\square$

Let θ be a congruence relation on a semiring $\mathcal{SR}$. It is easy to check that $(\mathcal{SR}/\theta, \oplus, \odot)$ is a semiring, where $\theta(x)\odot\theta(y) = \theta(xy)$ and $\theta(x)\oplus\theta(y) = \theta(x+y)$ for every $\theta(x), \theta(y) \in \mathcal{SR}/\theta$.

Definition 3.8. Let $\mathcal{A} = (\widetilde{M}_{\mathcal{A}}, \widetilde{N}_{\mathcal{A}})$ be an interval valued intuitionistic fuzzy set. The intuitionistic fuzzy set $\mathcal{A}/\theta = (\widetilde{M}_{\mathcal{A}/\theta}, \widetilde{N}_{\mathcal{A}/\theta})$ is defined as a pair of maps

$$\begin{cases} \widetilde{M}_{\mathcal{A}/\theta} : \mathcal{SR}/\theta \longrightarrow D[0,1] \\ \widetilde{N}_{\mathcal{A}/\theta} : \mathcal{SR}/\theta \longrightarrow D[0,1] \end{cases}$$

Such that $\widetilde{M}_{\mathcal{A}/\theta}(\theta(x)) = \sup\limits_{a\in\theta(x)} \widetilde{M}_{\mathcal{A}}(a)$ and $\widetilde{N}_{\mathcal{A}/\theta}(\theta(x)) = \inf\limits_{a\in\theta(x)} \widetilde{N}_{\mathcal{A}}(a)$.

Theorem 3.9. *Let $\mathcal{SR}$ be a semiring. If $\mathcal{A} = (\widetilde{M}_{\mathcal{A}}, \widetilde{N}_{\mathcal{A}})$ is an interval valued intuitionistic $(S,T)-$fuzzy subsemiring (ideal) of $\mathcal{SR}$, then $\mathcal{A}/\theta = (\widetilde{M}_{\mathcal{A}/\theta}, \widetilde{N}_{\mathcal{A}/\theta})$ is an interval valued intuitionistic $(S,T)-$fuzzy subsemiring (ideal) of $\mathcal{SR}/\theta$.*

Proof. Let $\theta(x), \theta(y) \in \mathcal{SR}/\theta$. We have

$$T(\widetilde{M}_{\mathcal{A}/\theta}(\theta(x)), \widetilde{M}_{\mathcal{A}/\theta}(\theta(y))) = T(\sup_{a\in\theta(x)} \widetilde{M}_{\mathcal{A}}(a), \sup_{b\in\theta(y)} \widetilde{M}_{\mathcal{A}}(b)) =$$

$$\sup_{a\in\theta(x),\ b\in\theta(y)} T(\widetilde{M}_{\mathcal{A}}(a), \widetilde{M}_{\mathcal{A}}(b)) \leq \sup_{a\in\theta(x),\ b\in\theta(y)} \widetilde{M}_{\mathcal{A}}(a+b) \leq$$

$$\sup_{a\in\theta(x),\ b\in\theta(y)} (\sup_{t\in\theta(a+b)} \widetilde{M}_{\mathcal{A}}(t)) = \sup_{a\in\theta(x),\ b\in\theta(y)} \widetilde{M}_{\mathcal{A}/\theta}(\theta(a+b)) = \widetilde{M}_{\mathcal{A}/\theta}(\theta(a+b)),$$

for all $a \in \theta(x), b \in \theta(y)$. On the other hand we have

$$\widetilde{M}_{\mathcal{A}/\theta}(\theta(a+b)) = \widetilde{M}_{\mathcal{A}/\theta}(\theta(a) \oplus \theta(b)) = \widetilde{M}_{\mathcal{A}/\theta}(\theta(x) \oplus \theta(y)) = \widetilde{M}_{\mathcal{A}/\theta}(\theta(x+y)).$$

So,

$$T(\widetilde{M}_{\mathcal{A}/\theta}(\theta(x)), \widetilde{M}_{\mathcal{A}/\theta}(\theta(y))) \leq \widetilde{M}_{\mathcal{A}/\theta}(\theta(x) \oplus \theta(y)).$$

Similarly we can show that

$$T(\widetilde{M}_{\mathcal{A}/\theta}(\theta(x)), \widetilde{M}_{\mathcal{A}/\theta}(\theta(y))) \leq \widetilde{M}_{\mathcal{A}/\theta}(\theta(x) \odot \theta(y)).$$

The proof of the inequalities

$$S(\widetilde{N}_{\mathcal{A}/\theta}(\theta(x)), \widetilde{N}_{\mathcal{A}/\theta}(\theta(y))) \geq \widetilde{N}_{\mathcal{A}/\theta}(\theta(x) \oplus \theta(y)),$$

$$S(\widetilde{N}_{\mathcal{A}/\theta}(\theta(x)), \widetilde{N}_{\mathcal{A}/\theta}(\theta(y))) \geq \widetilde{N}_{\mathcal{A}/\theta}(\theta(x) \odot \theta(y)),$$

are similar. Therefore $\mathcal{A}/\theta = (\widetilde{M}_{\mathcal{A}/\theta}, \widetilde{N}_{\mathcal{A}/\theta})$ is an interval valued intuitionistic $(S, T)-$fuzzy subsemiring of $\mathcal{SR}/\theta$.

Also we have

$$\widetilde{M}_{\mathcal{A}/\theta}(\theta(x)) \leq S(\widetilde{M}_{\mathcal{A}/\theta}(\theta(x)), \widetilde{M}_{\mathcal{A}/\theta}(\theta(y))) =$$

$$S(\sup_{a\in\theta(x)} \widetilde{M}_{\mathcal{A}}(a), \sup_{b\in\theta(y)} \widetilde{M}_{\mathcal{A}}(b)) = \sup_{a\in\theta(x),\ b\in\theta(y)} S(\widetilde{M}_{\mathcal{A}}(a), \widetilde{M}_{\mathcal{A}}(b)) \leq$$

$$\sup_{a\in\theta(x),\ b\in\theta(y)} \widetilde{M}_{\mathcal{A}}(ab) \leq \sup_{a\in\theta(x),\ b\in\theta(y)} (\sup_{t\in\theta(ab)} \widetilde{M}_{\mathcal{A}}(t)) = \widetilde{M}_{\mathcal{A}/\theta}(\theta(ab)).$$

On the other hand

$$\widetilde{M}_{\mathcal{A}/\theta}(\theta(ab)) = \widetilde{M}_{\mathcal{A}/\theta}(\theta(a) \odot \theta(b)) = \widetilde{M}_{\mathcal{A}/\theta}(\theta(x) \odot \theta(y)) = \widetilde{M}_{\mathcal{A}/\theta}(\theta(xy)).$$

Therefore $\widetilde{M}_{\mathcal{A}/\theta}(\theta(x)) \leq \widetilde{M}_{\mathcal{A}/\theta}(\theta(x) \odot \theta(y))$. Similarly we can show that $\widetilde{N}_{\mathcal{A}/\theta}(\theta(x)) \geq \widetilde{N}_{\mathcal{A}/\theta}(\theta(x) \odot \theta(y))$. This means $A/\theta = (\widetilde{M}_{\mathcal{A}/\theta}, \widetilde{N}_{\mathcal{A}/\theta})$ is an interval valued intuitionistic $(S, T)-$fuzzy right ideal of $\mathcal{SR}/\theta$. Analogously we can prove $A/\theta = (\widetilde{M}_{\mathcal{A}/\theta}, \widetilde{N}_{\mathcal{A}/\theta})$ is an interval valued intuitionistic ideal $(S, T)-$fuzzy left ideal of $\mathcal{SR}/\theta$. Therefore $A/\theta = (\widetilde{M}_{\mathcal{A}/\theta}, \widetilde{N}_{\mathcal{A}/\theta})$ is an interval valued intuitionistic $(S, T)-$fuzzy ideal of $\mathcal{SR}/\theta$. This completes the proof. □

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