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Some New Types of Open and Closed Sets in Minimal Structures-II[1]

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Abstract. In this article different forms of closed sets in m-spaces are introduced, studied and characterized. We show that the obtained results are a generalization of many of the results obtained by N. Rajesh in [10] and N. Rajesh et al. in [11].

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1. Introduction

In the literature, Aslim et al. in [1], studied different notions of πgs-closed sets; Rajesh in [10], introduced the notions of λ-closed sets and he related it with diffrents notions of $\widetilde{g}$-closed set, g-closed set, $^{\#}gs$-closed set, $\widetilde{g}s$-closed set and he studied its relations on a topological spaces as well as in [11], Rajesh et. al. studied the different notions of continuous functions and irresolute functions, where they used the concepts mentioned previously. In [13], Rosas et.

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al. introduce some new types of open and closed sets in minimal structure and proved that the obtained results are a generalizations of many results obtained in [1]. The fundamental idea of this article is to define and characterize the notions of $\widetilde{g}$-closed set, g-closed set, $^{\#}gs$-closed set, $\widetilde{g}s$-closed set and λ-closed sets on an m-space. Also, we look for conditions on the m-structure in order to generalize the well known results in this matter. Moreover, we find the existent relation between the different notions of continuity and irresoluteness of this sets using m-structure. Finally, we show that the obtained results are a generalization of many of the results obtained by Rajesh in [10], Rajesh et al. in [11] and [12].

2. Minimal Structures

In this section, we introduce the m-structure and define some important subsets associated to the m-structure and the relation between them.

Definition 2.1. *Let X be a nonempty set and let $m_X \subseteq P(X)$, where $P(X)$ denote the power set of X. We say that m_X is an m-structure (or a minimal structure) on X, if $\varnothing$ and X belong to m_X.*

The members of the minimal structure m_X are called m_X-open sets, and the pair (X, m_X) is called an m-space. The complement of m_X-open set is said to be m_X-closed. Given $A \subseteq X$, we define the m_X-interior of A abbreviate $m_X\text{-Int}(A)= \bigcup\{W/W \in m_X, W \subseteq A\}$ and the m_X-closure of A abbreviate $m_X\text{-Cl}(A) = \bigcap\{F/A \subseteq F, X \setminus F \in m_X\}$. An immediate consequence of the above Definition is the following theorem.

Theorem 2.2. *Let (X, m_X) be an m-space and A a subset of X. Then $x \in m_X\text{-Cl}(A)$ if and only if $U \cap A \neq \varnothing$ for every $U \in m_X$ containing x.*

And satisfying the following properties:

(i) $m_X\text{-Cl}(m_X\text{-Cl}(A))=m_X\text{-Cl}(A)$.
(ii) $m_X\text{-Int}(m_X\text{-Int}(A))=m_X\text{-Int}(A)$.
(iii) $m_X\text{-Int}(X \setminus A) = X \setminus m_X\text{-Cl}(A)$.
(iv) $m_X\text{-Cl}(X \setminus A)=X \setminus m_X\text{-Int}(A)$.
(v) *If* $A \subseteq B$ *then* $m_X\text{-Cl}(A) \subseteq m_X\text{-Cl}(B)$.
(vi) $m_X\text{-Cl}(A \cup B) \subseteq m_X\text{-Cl}(A) \cup m_X\text{-Cl}(B)$.
(vii) $A \subseteq m_X\text{-Cl}(A)$ *and* $m_X\text{-Int}(A) \subseteq A$.

Proof. Follows from Definition 2.1. □

Definition 2.3. *Let (X, m_X) be an m-space. We say that $A \subseteq X$ is an m_X-semiopen set if there exists $U \in m_X$ such that $U \subseteq A \subseteq m_X\text{-Cl}(U)$. Also we say that $A \subseteq X$ is an m_X-semiclosed set if $X \setminus A$ is an m_X-semiopen set.*

Definition 2.4. *Let (X, m_X) be an m-space. We say that $A \subseteq X$ is an m_X-pre-open set if $A \subseteq m_X\text{-Int}(m_X\text{-Cl}(A))$. Also we say that $A \subseteq X$ is an m_X-pre closed set if $X \setminus A$ is an m_X-pre open set.*

Definition 2.5. *Let (X, m_X) be an m-space. We say that $A \subseteq X$ is an m_X-nowhere dense if* m_X-Int(m_X-Cl(A))=$\varnothing$.

Theorem 2.6. *Let x be a point in (X, m_X). Then $\{x\}$ is either m_X-pre-open or m_X-nowhere dense.*

Observation 2.7. *We denote by*
X_1=$\{x \in X : \{x\}$ *is m_X-nowhere dense*$\}$
X_2=$\{x \in X : \{x\}$*is m_X-pre open*$\}$

We denote by $SO(X, m_X)$ the collection of all m_X-semiopen sets of X, $SC(X, m_X)$ the collection of all m_X-semiclosed sets of X, $PO(X, m_X)$ the collection of all m_X-preopen sets of X and $PC(X, m_X)$ the collection of all m_X-preclosed sets of X.

Observe that when m_X is a topology on X, the m_X-Cl(A) is exactly the Cl(A).

Definition 2.8. *Let (X, m_X) be an m-space and $B \subseteq X$.*

(i) *The m_X-semiclosure of B denoted by m_X-$s\,$Cl(B) is defined as the intersection of all m_X-semiclosed sets of (X, m_X) containing B.*
(ii) *The m_X-preclosure of B denoted by m_X-$p\,$Cl(B) is defined as the intersection of all m_X-preclosed sets of (X, m_X) containing B.*

We can observe that the m_X-semi closure of a subset B of (X, m_X) satisfy the following properties:

(i) m_X-$s\,$Cl$(\varnothing)$=$\varnothing$.
(ii) m_X-$s\,$Cl(X)=X.
(iii) If $A \subseteq B$ then m_X-$s\,$Cl$(B) \subseteq m_X$-$s\,$Cl(B).
(iv) If $\varnothing \neq B \neq X$. Then m_X-$s\,$Cl(B) is not necessarily an m_X-semiclosed set.
(v) m_X-$s\,$Cl$(X \setminus A)$=$X \setminus (m_X$-$s\,$Int$(A))$.
(vi) m_X-$s\,$Int$(X \setminus A)$=$X \setminus (m_X$-$s\,$Cl$(A))$.

In the same way, the m_X-preclosure of a subset B of X satisfy the following properties:

(i) m_X-$p\,$Cl$(\varnothing)$=$\varnothing$.
(ii) m_X-$p\,$Cl(X)=X.
(iii) If $A \subseteq B$ then m_X-$p\,$Cl$(B) \subseteq m_X$-$p\,$Cl(B).
(iv) If $\varnothing \neq B \neq X$. Then m_X-$p\,$Cl(B) is not necessarily an m_X-pre closed set.
(v) m_X-$p\,$Cl$(X \setminus A)$=$X \setminus (m_X$-$p\,$Int$(A))$.
(vi) m_X-$p\,$Int$(X \setminus A)$=$X \setminus (m_X$-$p\,$Cl$(A))$.

At this point there are a natural question. There exist any condition on the set X or in the m-structure of X in order to guarantee that the m_X-$s\,$Cl(B) is an m_X-semi closed set. In order to do that, we introduce the following property.

Definition 2.9. *Let (X, m_X) be an m-space. We say that m_X to have the property of Maki, if the union of any family of elements of m_X belongs to m_X.*

Observe that any collection $\varnothing \neq \mathcal{J} \subseteq P(X)$, always is contained in an m-structure that have the property of Maki, as we know, $m_X(\mathcal{J})=\{\varnothing, X\} \cup \{\bigcup_{M\in J} M : \varnothing \neq J \subseteq \mathcal{J}\}$. In particular, when $\mathcal{J} = m_X$, we denote by $m_X^{'} = m_X(\mathcal{J})$. Clearly $m_X = m_X^{'}$, if m_X have the property of Maki. Note that if m_X is an m-structure and $Y \subseteq X$, then $\{M \cap Y : M \in m_X\}$ is an m-structure on Y, and is denoted by $m_{X|Y}$, and the pair $(Y, m_{X|Y})$ is called an m-subspace of (X, m_X).

Theorem 2.10. *Let (X, m_X) be an m-space and m_X satisfying the property of Maki. For a subset A of X, the following properties hold:*

(i) *$A \in m_X$ if and only if m_X-Int(A)=A.*
(ii) *A is m_X-closed if and only if m_X-$Cl(A)$=A.*
(iii) *m_X-Int$(A) \in m_X$ and m_X-Cl(A) is m_X-closed.*

Proof. Follows from the definition of m_X-closed, m_X-$Interior$ and the property of Maki. □

In general the m_X-open sets and the m_X-semiopen sets are not stable for the union. Nevertheless, for certain m_X-structure, the class of m_X-semiopen sets are stable under union of sets, like it is demonstrated in the following lemma.

Lemma 2.11. *Let m_X be an m-structure which satisfy the property of Maki. If $A_i \in SO(X, m_X)$ for each $i \in I$, then $\bigcup_{i\in I} A_i \in SO(X, m_X)$.*

Proof. Suppose that m_X has the property of Maki, and $A_i \in SO(X, m_X)$ for each $i \in I$. For each $i \in I$, there exists $U_i \in m_X$ such that $U_i \subseteq A_i \subseteq U_i$, in consequence, $\bigcup_{i\in I} U_i \subseteq \bigcup_{i\in I} A_i \subseteq \bigcup_{i\in I} m_X\text{-Cl}(U_i)$. Since m_X-Cl is a monotone operator, then $\bigcup_{i\in I} m_X\text{-Cl}(U_i) \subseteq m_X\text{-Cl}(\bigcup_{i\in I} U_i)$; and $\bigcup_{i\in I} U_i \in m_X$, because m_X has the property of Maki. In consequence, $\bigcup_{i\in I} U_i \in m_X$ and $\bigcup_{i\in I} U_i \subseteq \bigcup_{i\in I} A_i \subseteq m_X\text{-Cl}(\bigcup_{i\in I} U_i)$, therefore $\bigcup_{i\in I} A_i \in SO(X, m_X)$. □

As a consequence of the definition of the m_X-semi closure, we have the following.

Theorem 2.12. *Let m_X be an m-structure on X then:*

(i) $x \in m_X$-$s\,$Cl(A) if and only if $U \cap A \neq \varnothing$ for every m_X-semi open set U such that $x \in U$. In the case that m_X satisfy the property of Maki, then
(ii) A is an m_X-semiclosed set if and only if A=m_X-$s\,$Cl(A).

Theorem 2.13. *Let (X, m_X) be an m-space and $A \subseteq X$. If m_X satisfy the property of Maki. Then m_X-$s\,$Cl(A)=$A \cup m_X$-Int$(m_X$-Cl$(A))$.*

Proof. Since m_X satisfy the property of Maki, then m_X-$s\,$Cl(A) is an m_X-semi closed set, using Definition 2.3, we obtain that
$m_X\text{-Int}(m_X\text{-Cl}(m_X\text{-}s\,\text{Cl}(A))) \subseteq m_X\text{-}s\,\text{Cl}(A)$.

Therefore,
m_X-$Int(m_X$-Cl$(A)) \subseteq m_X$-$s\,$Cl(A) and follows that $A \cup m_X$-Int$(m_X$-Cl$(A)) \subseteq m_X$-sCl(A).
The opposite inclusion, we observe that
m_X-Int$(m_X$-Cl$(A \cup m_X$-Int$(m_X$-Cl$(A))))$ $= m_X$-Int$(m_X$-Cl$(A)) \cup m_X$-Cl$(m_X$-Int$(m_X$-Cl$(m_X$-Cl$(A))))\subseteq (m_X$-Cl$(A)) \cup m_X$-Int$(m_X$-Cl$(m_X$-Int$(m_X$-Cl$(A)))) = m_X$-Cl$(A) \cup m_X$-Int$(m_X$-Cl$(A)) = m_X$-Cl(A).
Thus,
m_X-Int$(m_X$-Cl$(A \cup m_X$-Int$(m_X$-Cl$(A))))\subseteq m_X$-Int$(m_X$-Cl$(A)) \subseteq A \cup m_X$-Int$(m_X$-Cl$(A))$.
Follows that,
m_X-Int$(m_X$-Cl$(A \cup m_X$-Int$(m_X$-Cl$(A))))\subseteq A \cup m_X$-Int$(m_X$-Cl$(A))$.
In consequence, by Definition 2.2, $A \cup m_X$-Int$(m_X$-Cl$(A))$ is an m_X-semiclosed set and so m_X-$s\,$Cl$(A) \subseteq A \cup m_X$-Int$(m_X$-Cl$(A))$. □

The following example shows that, if the property of Maki is removed in the previous theorem the equality is not necessarily true.

Example 2.14. *Let $X = \mathbb{N}$. Define the m-structure on X as follows:* $m_X = \{\varnothing, \mathbb{N}, P(\{2n : n \in \mathbb{N}\}), \{1\}\}$. *Then the set of all m_X- closed sets is* $\{\varnothing, \mathbb{N}, P(\{2n : n \in \mathbb{N}\})^c, \mathbb{N} - \{1\}\}$. *Also,* $SO(X, m_X) = \{\varnothing, \mathbb{N}, P(\{2n : n \in \mathbb{N}), \{1\}, F\}$, *where* $F \cap \{2n : n \in \mathbb{N}\} \neq \varnothing$. *If we take* $A = \{3\}$, *then* m_X-$s\,$Cl$(A) = \{3\}$, *the* m_X-Cl$(A) = \{2n + 1 : n \in \mathbb{N}\}$ *and the* m_X-Int$(\{2n + 1 : n \in \mathbb{N}\}) = \{1\}$. $A \cup m_X$-Int$(m_X$-Cl$(A)) = \{1, 3\}$. *In consequence,* m_X-$s\,$Cl$(A) \subset A \cup m_X$-$Int(m_X$-Cl$(A))$.

Theorem 2.15. *Let (X, m_X) be an m-space and $A \subseteq X$. If m_X satisfy the property of Maki. Then*

(i) m_X-$s\,$Int$(A) = A \cap m_X$-Cl$(m_X$-Int$(A))$.
(ii) m_X-$p\,$Cl$(A) = A \cup m_X$-Cl$(m_X$-Int$(A))$.
(iii) m_X-$p\,$Int$(A) = A \cap m_X$-Int$(m_X$-Cl$(A))$.

Proof. (i). Follows from Theorems 2.10 and 2.12.
(ii). The proof is similar to the proof of Theorem 2.12.
(iii). Follows from (ii). □

3. New generalized closed sets under minimal conditions

Definition 3.1. *Let (X, m_X) be an m-space. We say that $A \subseteq X$ is an:*

(i) *m_X-regular open set if $A = m_X$-Int$(m_X$-Cl$(A))$. Also we say that $A \subseteq X$ is an m_X-regular closed set if $X \setminus A$ is an m_X-regular open set.*
(ii) *m_X-$\widehat{g}$-closed set if the m_X-Cl$(A) \subseteq U$ whenever $A \subseteq U$ and $U \in SO(X, m_X)$. The complement of m_X-$\widehat{g}$-closed set is called m_X-$\widehat{g}$-open.*
(iii) *m_X-g-closed set if the m_X-Cl$(A) \subseteq U$ whenever $A \subseteq U$ and U is m_X-$\widehat{g}$-open. The complement of m_X-g-closed set is called m_X-g-open.*

(iv) *m_X-$^{\#}gs$-closed set if the m_X-$s\,\mathrm{Cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is m_X-g-open. The complement of m_X-$^{\#}gs$-closed set is called m_X-$^{\#}gs$-open.*

(v) *m_X-$\widetilde{g}s$-closed set if the m_X-$s\,\mathrm{Cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is m_X-$^{\#}gs$-open. The complement of m_X-$\widetilde{g}s$-closed set is called m_X-$\widetilde{g}s$-open.*

(vi) *m_X-λ-closed set if the m_X-$\mathrm{Cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is m_X-$\widetilde{g}s$-open. The complement of m_X-λ-closed set is called m_X-λ-open. The family of all m_X-λ-open sets of (X, m_X) is denoted by $\lambda O(X, m_X)$.*

Definition 3.2. *Let (X, m_X) be an m-space and A be a subset of X. The intersection of all m_X-λ-closed sets containing A is called the m_X-λ-closure of A and is denoted by m_X-λ-$\mathrm{Cl}(A)$.*

Example 3.3. *Let $X = \{a, b, c, d\}$. Define the m-structure on X as follows: $m_X = \{\varnothing, X, \{a\}, \{b\}, \{a.c\}\}$. Then $SO(X, m_X) = \{\varnothing, X, \{a\}, \{b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{a, c, d\}\}$. $PO(X, m_X) = \{\varnothing, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{a, b, c\}\}$, $RO(X, m_X) = \{\varnothing, X, \{b\}, \{a, c\}\}$, and $\lambda O(X, m_X) = \{\varnothing, X, \{d\}, \{b, d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$.*

Example 3.4. *Let $X = \{a, b, c\}$. Define the m-structure on X as follows $m_X =$ $\{\varnothing, X, \{a\}, \{b\}\}$. Then, we obtain that: $SO(X, m_X) = \{\varnothing, X, \{a\}, \{b\}, \{b, c\}, \{a, c\}\}$, $PO(X, m_X) = \{\varnothing, X, \{a\}, \{b\}, \{a, b\}\}$, $RO(X, m_X) = \{\varnothing, X, \{a\}, \{b\}\}$, and $\lambda O(X, m_X) = \{\varnothing, X, \{c\}, \{a, c\}, \{b, c\}\}$.*

Example 3.5. *Let $X = \{a, b, c, d\}$. Define the m_X structure on X as follows $m_X = \{\varnothing, X, \{a\}, \{b\}, \{a, b, d\}, \{a, b, c\}\}$. Then, we obtain that: $SO(X, m_X) = \{\varnothing, X, \{a\}, \{b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$, $PO(X, m_X) = \{\varnothing, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$, $RO(X, m_X) = \{\varnothing, X, \{a\}, \{b\}\}$, $\lambda O(X, m_X) = \{\varnothing, X, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$.*

Example 3.6. *Let $X = \{a, b, c, d\}$. Define the m_X structure on X as follows $m_X = \{\varnothing, X, \{a\}, \{b\}, \{a, b\}, \{a, b, d\}, \{a, b, c\}\}$. Then, we obtain that: $SO(X, m_X) = \{\varnothing, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c\}, \{a, b, d\}\}$, $PO(X, m_X) = \{\varnothing, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$, $RO(X, m_X) = \{\varnothing, X, \{a\}, \{b\}\}$, $\lambda O(X, m_X) = \{\varnothing, X, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$.*

Example 3.7. *Let $X = \{a, b, c, d\}$. Define the m_X structure on X as follows $m_X = \{\varnothing, X, \{a, b, c\}, \{c, d\}\}$. Then, we obtain that: $SO(X, m_X) =$ $\{\varnothing, X, \{c, d\},$ $\{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$, $PO(X, m_X) = \{\varnothing, X, \{c\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, \{a, b, d\}\}$, $RO(X, m_X) = \{\varnothing, X\}$, $\lambda O(X, m_X) = \{\varnothing, X, \{d\}, \{a, b\}, \{a, b, d\}\}$.*

Example 3.8. *Let $X = \{a, b, c, d\}$. Define the m_X structure on X as follows $m_X = \{\varnothing, X, \{a, b\}, \{a, c\}, \{a\}, \{b\}, \{c\}\}$. Then, we obtain that: $SO(X, m_X) = \{\varnothing, X, \{a, b\}, \{a, c\}, \{a\}, \{b\}, \{c\}\}$, $PO(X, m_X) = \{\varnothing, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$, $RO(X, m_X) = \{\varnothing, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$, $\lambda O(X, m_X) = \{\varnothing, X, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$.*

Theorem 3.9. *Let (X, m_X) be any m-space with m_X satisfied the property of Maki and A be any subset of X. Then*

(i) *If A is m_X-$\widetilde{g}$-closed, then A is m_X-g-closed.*
(ii) *If A is m_X-$^{\#}gs$closed then A is m_X-$\widetilde{g}s$ closed.*

Observation 3.10. *If the property of Maki is dropped in Theorem 3.9, the result is not necessarily true as we can see as follows:*

(i) *Take $A = \{a, b, c\}$ in Example 3.3.*
(ii) *Take $A = \{a, b, d\}$ in Example 3.4.*

Definition 3.11. *Let (X, m_X) be an m-space.*

(i) *We say that (X, m_X) is an m_X-$T_{1/2}$ if every m_X-g-closed set is m_X-closed.*
(ii) *We say that (X, m_X) is an m_X-$sT_{1/2}$ if every m_X-$^{\#}gs$-closed set is m_X-semiclosed.*

Example 3.12. *The Example 3.6, shows that (X, m_X) is an m_X-$T_{1/2}$ space.*

The following theorem, characterize the m_X-$\widetilde{g}$-closed, m_X-g closed, m_X-$^{\#}gs$-closed, m_X-$\widetilde{g}s$-closed, m_X-λ-closed sets where m_X is any structure on X.

Theorem 3.13. *Let m_X be an m-structure on X.*

(i) *$A \subseteq X$ is an m_X-$\widehat{g}$-closed set if and only if m_X-$\mathrm{Cl}(A) \subseteq m_X$-$sKer(A)$.*
(ii) *$A \subseteq X$ is an m_X-g-closed set if and only if m_X-$\mathrm{Cl}(A) \subseteq m_X$-$\widehat{g}Ker(A)$.*
(iii) *$A \subseteq X$ is an m_X-$^{\sharp}gs$-closed set if and only if m_X-$sCl(A) \subseteq m_X$-$gKer(A)$.*
(iv) *$A \subseteq X$ is an m_X-$\widetilde{g}s$-closed set if and only if m_X-$sCl(A) \subseteq m_X$-$^{\#}gsKer(A)$.*
(v) *$A \subseteq X$ is an m_X-λ-closed set if and only if m_X-$\mathrm{Cl}(A) \subseteq m_X$-$\widetilde{g}sKer(A)$.*

Where m_X-$sKer(A)$ is defined as the intersection of all m_X-semiopen sets containing A and $\widehat{g}Ker(A)$, $gKer(A)$, $^{\#}gsKer(A)$, $\widetilde{g}sKer(A)$ are similarly defined.

Proof. (i).Let $D = \{S : S \subseteq X, A \subseteq S, S \in SO(X, m_X)\}$. Follows m_X-$sKer(A)$=
$\bigcap_{S \in D} S$. Observe that $S \in D$ implies that $A \subseteq S$ follows m_X-$\mathrm{Cl}(A) \subseteq S$ for all $S \in D$. In consequence, m_X-$\mathrm{Cl}(A) \subseteq m_X$-$sKer(A)$. Conversely, if m_X-$\mathrm{Cl}(A) \subseteq m_X$-sKer$(A)$, take $S \in SO(X, m_X)$ such that $A \subseteq S$, then by hypothesis m_X-$\mathrm{Cl}(A) \subseteq m_X$-Ker$(A) \subseteq S$. Follows that A is m_X-$\widehat{g}$-closed.
The other proofs are similar. □

The following theorem, characterize the m_X-$\widehat{g}$-closed, m_X-g-closed, m_X-$^{\#}gs$-closed,, m_X-$\widetilde{g}s$-closed and the m_X-λ-closed, when the m-structure on X satisfying the property of Maki.

Theorem 3.14. *Let m_X be an m-structure on X.*

(i) *The set $A \subseteq X$ is an m_X-$\widehat{g}$-closed set if and only if there does not exist an m_X-semiclosed set F such that $F \neq \varnothing$ and $F \subseteq m_X$-$\mathrm{Cl}(A) \setminus A$.*
(ii) *The set $A \subseteq X$ is an m_X-g-closed set if and only if there does not exist an m_X-$\widehat{g}$-closed set F such that $F \neq \varnothing$ and $F \subseteq m_X$-$\mathrm{Cl}(A) \setminus A$.*

(iii) *The set* $A \subseteq X$ *is an* m_X-${}^{\#}gs$*-closed set if and only if there does not exist an* m_X-g*-closed set* F *such that* $F \neq \varnothing$ *and* $F \subseteq m_X$-$sCl(A) \setminus A$.

(iv) *The set* $A \subseteq X$ *is an* m_X-$\widetilde{g}s$*-closed set if and only if there does not exist an* m_X-${}^{\#}gs$*-closed set* F *such that* $F \neq \varnothing$ *and* $F \subseteq m_X$-$sCl(A) \setminus A$.

(v) *The set* $A \subseteq X$ *is an* m_X-λ*-closed set if and only if there does not exist an* m_X-$\widetilde{g}s$*-closed set* F *such that* $F \neq \varnothing$ *and* $F \subseteq m_X$-$\mathrm{Cl}(A) \setminus A$.

Proof. (i) Suppose that A is an m_X-$\widehat{g}$-closed and let $F \subseteq X$ be an m_X-semiclosed set such that $F \subseteq m_X$-$\mathrm{Cl}(A) \setminus A$. It follows that, $A \subseteq X \setminus F$ and $X \setminus F$ is an m_X-semiopen set, since A is an m_X-$\widehat{g}$-closed, we have that m_X-$\mathrm{Cl}(A) \subseteq X \setminus F$ and $F \subseteq X \setminus m_X$-$\mathrm{Cl}(A)$. Follows that, $F \subseteq m_X$-$\mathrm{Cl}(A) \cap (X \setminus m_X$-$\mathrm{Cl}(A)) = \varnothing$, implying that $F = \varnothing$. Reciprocally, if $A \subseteq U$ and U is an m_X-semiopen set, then m_X-$\mathrm{Cl}(A) \cap (X \setminus U) \subseteq m_X$-$\mathrm{Cl}(A) \cap (X \setminus A) = m_X$-$\mathrm{Cl}(A) \setminus A$. Since m_X-$\mathrm{Cl}(A) \setminus A$ does not contain subsets m_X-semiclosed different from the empty set, we obtain that m_X-$\mathrm{Cl}(A) \cap (X \setminus U) = \varnothing$, and this implies that m_X-$\mathrm{Cl}(A) \subseteq U$ in consequence A is an m_X-$\widehat{g}$-closed.
The other proofs are similar. □

Theorem 3.15. *Let* (X, m_X) *be an m-space and* A, B *subsets of* X. *The following properties hold:*

(i) *If* A *is* m_X-$\widehat{g}$*-closed and* $A \subset B \subset m_X$-$\mathrm{Cl}(A)$. *Then* B *is* m_X-$\widehat{g}$*-closed.*

(ii) *If* A *is* m_X-g *closed and* $A \subset B \subset m_X$-$\mathrm{Cl}(A)$. *Then* B *is* m_X-g*-closed.*

(iii) *If* A *is* m_X-${}^{\#}gs$*-closed and* $A \subset B \subset m_X$-$sCl(A)$. *Then* B *is* m_X-${}^{\#}gs$*-closed.*

(iv) *If* A *is* m_X-$\widetilde{g}s$*-closed and* $A \subset B \subset m_X$-$sCl(A)$. *Then* B *is* m_X-$\widetilde{g}s$*-closed.*

(v) *If* A *is* m_X-λ*-closed and* $A \subset B \subset m_X$-$\mathrm{Cl}(A)$. *Then* B *is* m_X-λ*-closed.*

Proof. (i) Let A be an m_X-$\widehat{g}$-closed set, $B \subset U$ and U m_X-semiopen. Since $A \subset B$ then m_X-$\mathrm{Cl}(A) \subset m_X$-$\mathrm{Cl}(B)$. Follows that m_X-$\mathrm{Cl}(A) \subset U$ and $B \subset m_X$-$\mathrm{Cl}(A)$ implies that m_X-$\mathrm{Cl}(B) \subset U$. In consequence B is m_X-$\widehat{g}$-closed.
The other proofs are similar. □

Theorem 3.16. *Let* (X, m_X) *be an m-space and* $A \subseteq X$. *The following properties hold:*

(i) A *is* m_X-$\widehat{g}$*-open if and only if* $F \subset m_X$-$\mathrm{Int}(A)$ *whenever* F *is* m_X*-semiclosed and* $F \subset A$.

(ii) A *is* m_X-g*-open if and only if* $F \subset m_X$-$\mathrm{Int}(A)$ *whenever* F *is* m_X-$\widehat{g}$*-closed and* $F \subset A$.

(iii) A *is* m_X-${}^{\#}gs$*-open if and only if* $F \subset m_X$-$s\,\mathrm{Int}(A)$ *whenever* F *is* m_X-g*-closed and* $F \subset A$.

(iv) A *is* m_X-$\widetilde{g}s$*-open if and only if* $F \subset m_X$-$\mathrm{Int}(A)$ *whenever* F *is* m_X-${}^{\#}gs$*-closed and* $F \subset A$.

(v) A *is* m_X-λ*-open if and only if* $F \subset m_X$-$\mathrm{Int}(A)$ *whenever* F *is* m_X-$\widetilde{g}s$*-closed and* $F \subset A$.

Proof. (i) Let A be m_X-$\widehat{g}$-open, F be m_X-semiopen such that $F \subset A$. Then $X - A \subset X - F$, but $X - F$ is m_X-semiclosed and $X - A$ is m_X-$\widehat{g}$-closed implies that

m_X-Cl(X–A) $\subset X$–F. Follows that X–m_X-Int(A) $\subset X$–F. In consequence $F \subset m_X$-Int(A). Conversely, if F is m_X-semiclosed, $F \subset A$ and $F \subset m_X$-Int(A). Let X–$A \subset U$ where U is m_X-semi open, then X–$U \subset A$ and X–U is m_X-semiclosed. By hypothesis X–$U \subset m_X$-Int(A). Follows X–m_X-Int(A) $\subset U$ but it is equivalent to m_X-Cl(X–A) $\subset U$. Therefore, X–A is m_X-$\widehat{g}$-closed and conclude that A is m_X-$\widehat{g}$-open.
In a similar form, we can prove (ii), (iii) and (iv). □

Lemma 3.17. *Let* (X, m_X), *for any subset* A *of* X, $X_2 \cap m_X$-Cl$(A) \subseteq m_X$-$\widetilde{g}s$-$Ker(A)$, *where* X_2 *is defined in Observation 2.7.*

Proof. Let $x \in X_2 \cap m_X$-Cl(A) and suppose that x does not belong m_X-$\widetilde{g}s$-$Ker(A)$. Then there is an m_X-$\widetilde{g}s$-open U containing A such that x does not belong U. If $F = X - U$ then F is m_X-$\widetilde{g}s$-closed, since m_X-$cl(\{x\}) \subseteq m_X$-Cl(A), then
m_X-Int(m_X-Cl($\{x\}$)) $\subseteq A \cup m_X$-Int(m_X-Cl(A)). Since $x \in X_2$, we have that x does not in X_1, and so m_X-Int(m_X-Cl(A)) $= \varnothing$. Therefore, there has to be some point $y \in m_X$-Int(m_X-Cl($\{x\}$)) and hence $y \in F \cap A$ contradiction. □

Lemma 3.18. *A subset* A *of* (X, m_X), *is* m_X-λ-*closed if and only if* $X_1 \cap m_X$-Cl$(A) \subseteq A$, *where* X_1 *is defined in Observation 2.7.*

Proof. Suppose that A is m_X-λ-closed. Let $x \in X_1 \cap m_X$-Cl(A), then $x \in X_1$ and $x \in m_X$-Cl(A). Since $x \in X_1$, m_X-Int(m_X-Cl($\{x\}$))$=\varnothing$. Therefore $\{x\}$ is m_X-semiclosed, since m_X-Int(m_X-Cl($\{x\}$)) $\subseteq \{x\}$. Since every m_X-semiclosed set is m_X-$\widetilde{g}s$-closed, $\{x\}$ is m_X-$\widetilde{g}s$-closed. If x does not in A and $U = X$-$\{x\}$. Then U is m_X-$\widetilde{g}s$-open and contain A and so m_X-Cl$(A) \subseteq U$, a contradiction. Converesely. Suppose that $X_1 \cap m_X$-Cl$(A) \subseteq A$, then $X_1 \cap m_X$-Cl$(A) \subseteq m_X$ $\widetilde{g}s$-$Ker(A)$. Since m_X-Cl(A)=$X \cap m_X$-Cl(A)=$(X_1 \cup X_2) \cap m_X$-Cl$(A) \subseteq m_X$-$\widetilde{g}s$-$Ker(A)$. It follows that A is m_X-λ-closed. □

Theorem 3.19. *An arbitrary intersection of* m_X-λ-*closed sets is* m_X-λ-*closed.*

Proof. Let $F = \{A_i : i \in I\}$ be a family of m_X-λ-closed sets and let $A = \cap A_i$. Since $A \subseteq A_i$ for all $i \in I$, $X_1 \cap m_X$-Cl$(A) \subseteq X_1 \cap m_X$-Cl$(A_i) \subseteq A_i$ for all $i \in I$. It follows that $X_1 \cap m_X$-Cl$(A) \subseteq X_1 \cap m_X$-Cl$(A_i) \subseteq A$ and this implies that A is m_X-λ-closed. □

4. (m_X, m_Y)-continuous maps, (m_X, m_Y)-λ-Irresolute maps and (m_X, m_Y)-Contra λ-Continuous maps

In this section, we define different forms of continuity, irresoluteness and contra continuity on m-structures where the notions of $\widehat{g}$-closed set, g-closed set, $^{\#}gs$-closed set, $\widetilde{g}s$-closed set, and λ-closed set are involucrate.

Definition 4.1. *A map* $f : (X, m_X) \to (Y, m_Y)$ *is called:*

(i) (m_X, m_Y)-*continuous if* $f^{-1}(O)$ *is* m_X-*closed in* X *for all* m_Y-*closed set* $O \in Y$.

(ii) (m_X, m_Y)-$\widehat{g}$-*continuous if* $f^{-1}(O)$ *is* m_X-$\widehat{g}$-*closed in* X *for all* m_Y-*closed set* $O \in Y$.
(iii) (m_X, m_Y)-g-*continuous if* $f^{-1}(O)$ *is* m_X-g-*closed in* X *for all* m_Y-*closed set* $O \in Y$.
(iv) (m_X, m_Y)-$^{\#}gs$-*continuous if* $f^{-1}(O)$ *is* m_X-$^{\#}gs$-*closed in* X *for all* m_Y-*closed set* $O \in Y$.
(v) (m_X, m_Y)-$\widetilde{g}s$-*continuous if* $f^{-1}(O)$ *is* m_X-$\widetilde{g}s$-*closed in* X *for all* m_Y-*closed set* $O \in Y$.
(vi) $(m_X, m_Y) - \lambda$ *continuous if,* $f^{-1}(O)$ *is* m_X-λ-*closed in* X *for all* m_Y-*closed set* $O \in Y$.

Example 4.2. *In the Example 3.4, take* $X = Y = \{a, b, c\}, m_X = m_Y = \{\varnothing, X, \{a\}, \{b\}\}$ *and* $f : X \mapsto Y$ *defined as:* $f(a) = f(c) = c$ *and* $f(b) = a$. *Then* f *satisfies all different notions of continuity described in Definition 4.1.*

From the above definition, easily we have the following theorem

Theorem 4.3. *Let* $f : (X, m_X) \to (Y, m_Y)$, *then:*

(i) *If* f *is* (m_X, m_Y)-*continuous, then* f *is* (m_X, m_Y)-$\widehat{g}$-*continuous.*
(ii) *If* f *is* (m_X, m_Y)-*continuous, then* f *is* (m_X, m_Y)-g- *continuous.*
(iii) *If* f *is* (m_X, m_Y)-*continuous, then* f *is* (m_X, m_Y)-$^{\#}gs$-*continuous.*
(iv) *If* f *is* (m_X, m_Y)-*continuous, then* f *is* (m_X, m_Y)-$\widetilde{g}s$-*continuous.*
(v) *If* f *is* (m_X, m_Y)-*continuous, then* f *is* (m_X, m_Y)-λ-*continuous.*

and none of them are reversible.

Proof. The proof follows from Definitions 3.1 and 4.1. □

In the Example 3.4, take $X = Y = \{a, b, c\}, m_X = m_Y = \{\varnothing, X, \{a\}, \{b\}\}$ and $f : X \mapsto Y$ defined as: $f(a) = f(b) = a$ and $f(c) = c$. Then f is (m_X, m_Y)-$\widehat{g}$-continuous, (m_X, m_Y)-g-continuous, (m_X, m_Y)-$^{\#}gs$-continuous, (m_X, m_Y)-$\widetilde{g}s$-continuous and (m_X, m_Y)-λ-continuous but not $(m_X, m_Y)-$continuous.

In the case that, the $f : (X, m_X) \to (Y, m_Y)$ is a map, where m_X satisfy the condition of Maki, we have the following Theorem.

Theorem 4.4. *Let* $f : (X, m_X) \to (Y, m_Y)$, *where* m_X *satisfies the condition of Maki then:*

(i) *If* f *is* (m_X, m_Y)-$\widehat{g}$-*continuous, then it is* (m_X, m_Y)-g-*continuous.*
(ii) *If* f *is* (m_X, m_Y)-$^{\#}gs$-*continuous, then it is* (m_X, m_Y)-$\widetilde{g}s$-*continuous.*

and none of them are reversible.

Proof. The proof follows from Theorem 3.9. □

In the following example, we shows that if the condition of Maki on m_X is omitted, then the Theorem 4.4 can be false.

Example 4.5. *In the Example 3.4, take* $X = Y = \{a, b, c\}$, $m_X = m_Y = \{\varnothing, X, \{a\}, \{b\}\}$ *and* $f : X \mapsto Y$ *defined as:* $f(a) = c, f(b) = a$ *and* $f(c) = b$. *Then:*

(i) *f is (m_X, m_Y)-$\widetilde{g}$-continuous but not (m_X, m_Y)-g-continuous.*
(ii) *f is (m_X, m_Y)-$\widetilde{g}$-continuous but not (m_X, m_Y)-λ-continuous.*

Example 4.6. *In the Example 3.3, take $X = \{a, b, c, d\}, m_X = \{\varnothing, X, \{a\}, \{b\}, \{a, c\}\}$ and $Y = \{x, y\}$, and $m_Y = \{\varnothing, Y, \{x\}\}$, $f : X \mapsto Y$ defined as: $f(a) = f(b) = f(d) = y$ and $f(c) = x$. Then f is (m_X, m_Y)-$^{\#}gs$-continuous but not (m_X, m_Y)-$\widetilde{g}s$-continuous and (m_X, m_Y)-λ-continuous.*

Example 4.7. *In the Example 3.3, take $X = \{a, b, c, d\}, m_X = \{\varnothing, X, \{a\}, \{b\}, \{a, c\}\}$ and $Y = \{x, y\}$, and $m_Y = \{\varnothing, Y, \{x\}\}$. Define a function $f : X \mapsto Y$ as: $f(a) = f(b) = f(c) = y$ and $f(d) = y$. Then f is (m_X, m_Y)-$\widehat{g}$-continuous, but not (m_X, m_Y)-g-continuous, (m_X, m_Y)-$^{\#}gs$-continuous, (m_X, m_Y)-$\widetilde{g}s$-continuous, (m_X, m_Y)-λ-continuous.*

Definition 4.8. *A map $f : (X, m_X) \to (Y, m_Y)$ is called:*

(i) *(m_X, m_Y)-irresolute if $f^{-1}(O)$ is m_X-semiclosed in X for every m_Y-$\widehat{g}$-semiclosed set $O \in Y$.*
(ii) *(m_X, m_Y)-$\widehat{g}$-irresolute if $f^{-1}(O)$ is m_X-$\widehat{g}$-closed in X for every m_Y-$\widehat{g}$-closed set $O \in Y$.*
(iii) *(m_X, m_Y)-g-irresolute if $f^{-1}(O)$ is m_X-g- closed in X for every m_Y-g-closed set $O \in Y$.*
(iv) *(m_X, m_Y)-$^{\#}gs$-irresolute if $f^{-1}(O)$ is m_X-$^{\#}gs$-closed in X for every m_Y-$^{\#}gs$-closed set $O \in Y$.*
(v) *(m_X, m_Y)-$\widetilde{g}s$-irresolute if $f^{-1}(O)$ is m_X-$\widetilde{g}s$-closed in X for every m_Y-$\widetilde{g}s$-closed set $O \in Y$.*
(vi) *(m_X, m_Y)-λ-irresolute if $f^{-1}(O)$ is m_X-λ-closed in X for every m_Y-λ-closed set $O \in Y$.*

Example 4.9. *In the Example 3.8, take $X = \{a, b, c, d\}, m_X = \{\varnothing, X, \{a, b\}, \{a, c\}, \{a\}, \{b\}, \{c\}\}$. $Y = \{x, y\}$, and $m_Y = \{\varnothing, Y, \{x\}\}$, and a function $f : X \mapsto Y$ defined as: $f(b) = f(c) = f(d) = x$ and $f(a) = y$. Then f is (m_X, m_Y)-irresolute but not (m_X, m_Y)−-$\widehat{g}$-irresolute, (m_X, m_Y)-g-irresolute, (m_X, m_Y)-$^{\#}gs$-irresolute, (m_X, m_Y)-$\widetilde{g}s$-irresolute, (m_X, m_Y)-λ-irresolute.*

Example 4.10. *In the Example 3.5, take $X = \{a, b, c, d\}, m_X = \{\varnothing, X, \{a, b, d\}, \{a, b, c\}, \{a\}, \{b\}\}$. $Y = \{x, y\}$, and $m_Y = \{\varnothing, Y, \{x\}\}$, $f : X \mapsto Y$ defined as: $f(b) = f(c) = f(d) = x$ and $f(a) = y$. Then f is (m_X, m_Y)-$^{\#}gs$-irresolute but not (m_X, m_Y)-$\widehat{g}$ irresolute, (m_X, m_Y)-g-irresolute, (m_X, m_Y)-λ-irresolute.*

Example 4.11. *In the Example 3.3, take $X = \{a, b, c, d\}, m_X = \{\varnothing, X, \{a\}, \{b\}, \{a, c\}\}$. $Y = \{x, y\}$, and $m_Y = \{\varnothing, Y, \{x\}\}$, $f : X \mapsto Y$ defined as: $f(a) = f(b) = f(c) = y$ and $f(d) = x$. Then f is (m_X, m_Y)-$\widehat{g}$ irresolute but not (m_X, m_Y)-irresolute, (m_X, m_Y)-g-irresolute, (m_X, m_Y)-$^{\#}gs$-irresolute, (m_X, m_Y)-$\widetilde{g}s$-irresolute, (m_X, m_Y)-λ-irresolute.*

Definition 4.12. *A map $f : (X, m_X) \to (Y, m_Y)$ is called:*

(i) (m_X, m_Y)-*contra continuous if* $f^{-1}(O)$ *is* m_X-*closed in* X *for every* m_Y-*open set* $O \in Y$.
(ii) (m_X, m_Y)-*contra* $\widehat{g}$-*continuous if* $f^{-1}(O)$ *is* m_X-$\widehat{g}$-*closed in* X *for every* m_Y-*open set* $O \in Y$.
(iii) (m_X, m_Y)-*contra* g-*continuous if* $f^{-1}(O)$ *is* m_X-g-*closed in* X *for every* m_Y-*open set* $O \in Y$.
(iv) (m_X, m_Y)-*contra* $^{\#}gs$-*continuous if* $f^{-1}(O)$ *is* m_X-$^{\#}gs$-*closed in* X *for every* m_Y-*open set* $O \in Y$.
(v) (m_X, m_Y)-*contra* $\widetilde{g}$-*continuous if* $f^{-1}(O)$ *is* m_X-$\widetilde{g}s$-*closed in* X *for every* m_Y-*open set* $O \in Y$.
(vi) (m_X, m_Y)-*contra*-λ-*continuous if,* $f^{-1}(O)$ *is* m_X-λ-*closed in* X *for every* m_Y-*open set* $O \in Y$.

The following lemma generalize the Theorem 3.5 given in [11].

Lemma 4.13. *Let* (X, m_X) *and* (Y, m_Y) *be two m-spaces where* m_X *satisfies the property of Maki. The following conditions are equivalent:*
(i) $f : (X, m_X) \to (Y, m_Y)$ *is* (m_X, m_Y)-*contra* λ-*continuous function.*
(ii) *For each* m_X-*closed set* $A \subseteq Y$, $f^{-1}(A)$ *is* m_X-λ-*open.*
(iii) *For each* $x \in X$ *and each* m_Y-*closed set* $A \subseteq Y$ *with* $f(x) \in A$, *there exists an* m_X-λ-*open* $U \in X$ *such that* $f(U) \subset A$.
(iv) *For all* $A \subseteq X$, $f(m_X\text{-}\lambda\text{-}\mathrm{Cl}(A)) \subseteq m_Y\text{-}Ker(f(A))$.
(v) *For all* $B \subseteq Y$, $m_X\text{-}\lambda\text{-}\mathrm{Cl}(f^{-1}(B)) \subseteq f^{-1}(m_Y\text{-}Ker(B))$.

Proof. The implications $(i) \to (ii)$ and $(ii) \to (iii)$ are obvious.
$(iii) \to (ii)$. Let A be any m_Y-closed set of Y and $x \in f^{-1}(A)$. Then $f(x) \in A$ and there exists an m_X-λ-open set U_x of X such that $x \in A$ and $f(U_x) \subseteq A$. Therefore using Theorem $f^{-1}(A)=\bigcup\{U_x / x \in f^{-1}(A)\}$ is an m_X-λ-open.
$(ii) \to (iv)$. Let A be any subset of X. Suppose that $y \notin \mathrm{Ker}(f(A)))$. Then there exists an m_X-closed set F in Y such that $y \in F$ and $f(A) \cap F = \varnothing$. Thus $A \cap f^{-1}(F) = \varnothing$ and $m_X\text{-}\lambda\text{-}\mathrm{Cl}(A) \cap f^{-1}(F) = \varnothing$. Therefore, we obtain that $f(m_X\text{-}\lambda\text{-}\mathrm{Cl}(A)) \cap F = \varnothing$ and $y \notin f(m_X\text{-}\lambda\text{-}\mathrm{Cl}(A))$. This implies that $f(m_X\text{-}\lambda\text{-}\mathrm{Cl}(A)) \subseteq m_Y\text{-}Ker(f(A))$.
$(iv) \to (v)$. Let B any subset of Y. Then $f(m_X\text{-}\lambda\text{-}\mathrm{Cl}(f^{-1}(B)) \subseteq m_Y\text{-}Ker(f(f^{-1}(B))) \subseteq m_Y\text{-}Ker(B)$. In consequence $m_X\text{-}\lambda\text{-}\mathrm{Cl}(f^{-1}(B)) \subseteq f^{-1}(m_Y\text{-}Ker(B))$.
$(v) \to (i)$. Let V any m_X-open set in Y. Then $m_X\text{-}\lambda\text{-}\mathrm{Cl}(f^{-1}(V)) \subseteq f^{-1}(m_Y\text{-}Ker(V))=f^{-1}(V)$, in consequence $f^{-1}(V)$ is m_X-λ-closed. □

Definition 4.14. *A map* $f : (X, m_X) \to (Y, m_Y)$ *is called:*
(i) (m_X, m_Y)-*pre semiclosed if* $f(O)$ *is* m_Y-*semiclosed in* Y *for all* m_X-*semiclosed set* O *of* X.
(ii) (m_X, m_Y)-*pre semiopen if* $f(O)$ *is* m_Y-*semiopen in* Y *for all* m_X *semiopen set* O *of* X.
(iii) (m_X, m_Y)-*regular open if* $f(O)$ *is* m_Y-*regular open in* Y *for every* m_X-*open set* O *of* X.

Observation 4.15. *The following Lemma generalize the Theorem 4.2, given in* [1].

Lemma 4.16. *Let* (X, m_X) *and* (Y, m_Y) *be two m-spaces where* m_X *satisfies the property of Maki. If* $f : (X, m_X) \to (Y, m_Y)$ *is* π-(m_X, m_Y)*-irresolute function and* $(m_X.m_Y)$*-pre semiclosed then* $f(A)$ *is* m_Y-πgs*-closed for every* m_X-πgs*-closed set* A *in* X.

Proof. Let A be any m_X-πgs-closed set in X, U an m_Y-π closed set in y such that $f(A) \subseteq U$. By hypothesis $f^{-1}(U)$ is m_X-open set in X and $A \subseteq f^{-1}(U)$. Follows that m_X-$s\,\mathrm{Cl}(A) \subseteq f^{-1}(U)$, in consequence $f(m_X\text{-}s\,\mathrm{Cl}(A)) \subseteq U$. Since $A \subseteq m_X$-$s\,\mathrm{Cl}(A)$ then $f(A) \subseteq f(m_X\text{-}s\,\mathrm{Cl}(A))$, in consequence, m_Y-$s\,\mathrm{Cl}(f(A)) \subseteq m_Y$-$s\,\mathrm{Cl}(f(m_X\text{-}s\,\mathrm{Cl}(A)))$. Since f is (m_X, m_Y)-pre semiclosed, m_Y-$s\,\mathrm{Cl}(f(m_X\text{-}s\,\mathrm{Cl}(A)))$=$f(m_X\text{-}s\,\mathrm{Cl}(A))$. Follows that m_Y-$s\,\mathrm{Cl}((A)) \subseteq f(m_X$-$s\,\mathrm{Cl}(A) \subseteq U)$. In consequence $f(A)$ is m_Y-πgs closed set in Y. □

Observation 4.17. *The following Lemma generalize the Theorem 3.6, given in* [12].

Lemma 4.18. *Let* (X, m_X) *and* (Y, m_Y) *be two m-spaces and* $f : (X, m_X) \to (Y, m_Y)$ *be a function. Then the following statements are equivalent:*

(i) f *is* (m_X, m_Y)-λ*-continuous.*

(ii) *For each point* x *in* X *and each* m_Y*-open set* V *with* $f(x) \in V$*, there is* m_X-λ *open set* U_x *such that* $x \in U_x$*, and* $f(U_x) \subseteq V$.

Proof. $(i) \to (ii)$ Let $f(x) \in V$. Since f is (m_X, m_Y)-λ-continuous, we have $x \in f^{-1}(V)$ and $f^{-1}(V)$ is m_X-λ-open. Let $U = f^{-1}(V)$. Follows that $x \in U$ and $f(U) \subseteq V$.
$(ii) \to (i)$ Let V an m_Y-open set and let $x \in f^{-1}(V)$. Then $f(x) \in V$ follows that there exists a m_X-λ open set U_x such that $x \in U_x$, $f(U_x) \subseteq V$. Follows $x \in U_x$, and $f^{-1}(V) = \cup_{x \in f^{-1}(V)} U_x$. Now using Theorem 2.12, $f^{-1}(V)$ is m_X-λ-open. □

Lemma 4.19. *Let* (X, m_X) *and* (Y, m_Y) *be two m-spaces where* m_Y *satisfies the property of Maki. The following conditions are equivalent:*

(i) $f : (X, m_X) \to (Y, m_Y)$ *is* (m_X, m_Y) *irresolute function.*

(ii) *For each subset* $A \subseteq X$, $f(m_X\text{-}s\,\mathrm{Cl}(A)) \subseteq m_Y\text{-}s\,\mathrm{Cl}(f(A))$.

(iii) *For each* m_Y*-semiclosed subset,* $V \subseteq Y$*, the inverse image* $f^{-1}(V)$ *is an,* m_X*-semiclosed in* X.

(iv) *For all* $B \subseteq Y$, $m_X\text{-}s\,\mathrm{Cl}(f^{-1}(B)) \subseteq f^{-1}(m_Y\text{-}s\,\mathrm{Cl}(B))$.

Proof. $(iii) \Rightarrow (ii)$. Let A be a subset of X and suppose that $y \notin m_Y$-$s\,\mathrm{Cl}(f(A))$, then there exists an m_Y-semi open set G in Y, such that $y \in G$ and $f(A) \cap G = \varnothing$, therefore, $f^{-1}(f(A) \cap G) = \varnothing$, it says that $A \cap f^{-1}(G) = \varnothing$. In consequence, m_X-$s\,\mathrm{Cl}(A) \subset (f^{-1}(G))^c$, follows that $f(m_X\text{-}s\,\mathrm{Cl}(A)) \cap G = \varnothing$; and therefore, $y \notin f(m_X\text{-}s\,\mathrm{Cl}(A))$. But it is said that $f(m_X\text{-}s\,\mathrm{Cl}(A)) \subset m_Y$-$s\,\mathrm{Cl}(f(A))$ for all subset A of X.

$(ii) \Rightarrow (iii)$. Let V any m_Y-semiclosed subset in Y, then $f^{-1}(V) \subseteq X$. By hypothesis $f(m_X\text{-}s\,\mathrm{Cl}(f^{-1}(V))) \subset m_Y\text{-}s\,\mathrm{Cl}(f(f^{-1}(V)))$, follows that $f(m_X\text{-}s\,\mathrm{Cl}(f^{-1}(V)))$
$\subset m_Y\text{-}s\,\mathrm{Cl}(V)$. In consequence, $f(m_X\text{-}s\,\mathrm{Cl}(f^{-1}(V))) \subset V$, follows that $m_X\text{-}s\,\mathrm{Cl}(f^{-1}(V)) \subset f^{-1}(V)$. Therefore $f^{-1}(V)$ is an m_X-semiclosed set.
$(ii) \Rightarrow (iv)$. Let B be a subset of Y, then $f^{-1}(B) \subseteq X$. Using the hypothesis, that $f(m_X\text{-}s\,\mathrm{Cl}(f^{-1}(B))) \subseteq m_Y\text{-}s\,\mathrm{Cl}(f(f^{-1}(B))) \subseteq m_Y\text{-}s\,\mathrm{Cl}(B)$, therefore, $m_X\text{-}s\,\mathrm{Cl}(f^{-1}(B)) \subseteq f^{-1}(m_Y\text{-}s\,\mathrm{Cl}(B))$.
$(iv) \Rightarrow (iii)$. Suppose that V is any m_Y-semiclosed set in Y. Then $f^{-1}(V) \subseteq X$, by hypothesis, we obtain that $m_X\text{-}s\,\mathrm{Cl}(f^{-1}(V)) \subseteq f^{-1}(m_Y\text{-}s\,\mathrm{Cl}(V))$. But V is a m_Y-semiclosed set, then $m_Y\text{-}s\,\mathrm{Cl}(V){=}V$. In consequence, $m_X\text{-}s\,\mathrm{Cl}(f^{-1}(V)) \subseteq f^{-1}(V)$. But this says that $f^{-1}(V)$ is an m_X-semiclosed set in X.
The others implications $((i) \Rightarrow (iii))$ and $((iii) \Rightarrow (i))$, follow from the definition of (m_X, m_Y)-irresolute function and the complement of set. □

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