International Mathematical Forum, 4, 2009, no. 44, 2169 - 2184

Some New Types of Open and Closed Sets in Minimal Structures-I[1]

Ennis Rosas

Department of Mathematics, Universidad De Oriente
Nucleo De Sucre Cumana, Venezuela
erosas@sucre.udo.edu.ve

Neelamegarajan Rajesh

Department of Mathematics, Kongu Engineering College
Perundurai, Erode-638 052, Tamilnadu, India
nrajesh_topology@yahoo.co.in

Carlos Carpintero

Department of Mathematics, Universidad de Oriente
Nucleo De Sucre Cumana, Venezuela
ccarpi@sucre.udo.edu.ve

Abstract. In this article different forms of closed sets in m-spaces are introduced and studied. We show that the obtained results are a generalization of many of the results obtained by G. Aslim et al. in [1].

Mathematics Subject Classification: 54A05, 54A20, 54C08, 54D10

Keywords: Minimal Structure, m_X-πgs closed set, m_X-πgp closed set, m_X-sg closed set

1. Introduction

In the literature, the notions of g-closed set, gs-closed set, sg-closed set, π-closed set, πg-closed set, πgs-closed set, gp-closed set, πgp-closed set, and their relationships, are studied in a topological space as well as the different notions of continuous functions and irresolute functions, where they use the concepts mentioned previously. The fundamental idea of this article is to define

[1]Research Partially Supported by Consejo de Investigación UDO.

the above notions on a m-space [8]. Also, we look for conditions on the m-structure in order to generalize the well known results in this matter. Moreover, we find the existent relation between the different notions of continuity and irresoluteness and Also we find conditions under what the direct image of any m_X-sg-open set in X is m_Y- sg-open in Y and the inverse image of any m_Y-sg-open set in Y is m_X- sg-open in X. Finally, we show that the obtained results are a generalization of many of the results obtained by G. Aslim et al. in [1].

2. **Minimal Structures**

In this section, we introduce the m-structure and define some important subsets associated to the m-structure and the relation between them.

Definition 2.1. Let X be a nonempty set and let $m_X \subseteq P(X)$, where $P(X)$ denote the set of power of X. We say that m_X is an m-structure (or a minimal structure) on X, if $\emptyset$ and X belong to m_X.

The members of the minimal structure m_X are called m_X-open sets, and the pair (X, m_X) is called an m-space. The complement of an m_X-open set is called m_X-closed. Given $A \subseteq X$, we define m_X-interior of A abbreviate m_X-Int(A) as $\bigcup\{W|W \in m_X, W \subseteq A\}$ and the m_X-closure of A abbreviate m_X-Cl(A) as $\bigcap\{F|A \subseteq F, X \setminus F \in m_X\}$. An immediate consequence of the above Definition is the following theorem.

Theorem 2.1. *Let (X, m_X) be an m-space and A a subset of X. Then $x \in m_X$-Cl(A) if and only if $U \cap A \neq \emptyset$ for every $U \in m_X$ containing x.*

And satisfying the following properties:

(i) m_X-Cl(m_X-Cl(A))=m_X-Cl(A).
(ii) m_X-Int(m_X-Int(A))=m_X-Int(A).
(iii) m_X-Int$(X \setminus A)$=$X \setminus m_X-$Cl(A).
(iv) m_X-Cl$(X \setminus A)$=$X \setminus m_X-$Int(A).
(v) If $A \subseteq B$ then m_X-Cl$(A) \subseteq m_X-$Cl(B)
(vi) m_X-Cl$(A \cup B) \subseteq m_X$-Cl$(A) \cup m_X$-Cl(B).
(vii) $A \subseteq m_X$-Cl(A) and m_X-Int$(A) \subseteq A$.

Proof. Follows from Definition 2.1. □

Definition 2.2. Let (X, m_X) be an m-space. We say that $A \subseteq X$ is an m_X-semiopen set if there exists $U \in m_X$ such that $U \subseteq A \subseteq m_X-Cl(U)$. Also we say that $A \subseteq X$ is m_X-semiclosed if $X \setminus A$ is m_X-semiopen.

Definition 2.3. Let (X, m_X) be an m-space. We say that $A \subseteq X$ is m_X-preopen set if $A \subseteq m_X$-Int(m_X-Cl(A)). Also we say that $A \subseteq X$ is m_X-preclosed if $X \setminus A$ is m_X-preopen.

We denote by $SO(X, m_X)$ (resp. $SC(X, m_X)$, $PO(X, m_X)$, $PC(X, m_X)$) the collection of all m_X-semiopen (resp. m_X-semiclosed, m_X-preopen, m_X-preclosed) sets of (X, m_X).

Observe that when m_X is a topology on X, then m_X-Cl(A) = Cl(A) for every subset A of X.

Definition 2.4. Let (X, m_X) be an m-space and $B \subseteq X$.

(i) *The m_X-semiclosure of B denoted by m_X-sCl(B) is defined to be the intersection of all m_X-semiclosed sets of (X, m_X) containing B.*
(ii) *The m_X-preclosure of B denoted by m_X-pCl(B) is defined to be the intersection of all m_X-preclosed sets of (X, m_X) containing B.*

We can observe that the m_X-semiclosure of a subset B of X satisfies the following properties:

(i) m_X-sCl$(\emptyset)$=$\emptyset$.
(ii) m_X-sCl(X)=X.
(iii) If $A \subseteq B$ then m_X-sCl$(B) \subseteq m_X-$sCl$(B.)$
(iv) If $\emptyset \neq B \neq X$. Then m_X-sCl(B) is not necessarily an m_X-semiclosed set.
(v) m_X-sCl$(X \setminus A)$=$X \setminus m_X-$sInt(A).
(vi) m_X-sInt$(X \setminus A)$=$X \setminus m_X-$sCl(A).

In the same way, the m_X-preclosure of a subset B of X satisfies the following properties:

(i) m_X-pCl$(\emptyset)$=$\emptyset$.
(ii) m_X-pCl(X)=X.
(iii) If $A \subseteq B$ then m_X-pCl$(B) \subseteq m_X-$pCl$(B.)$
(iv) If $\emptyset \neq B \neq X$. Then m_X-pCl(B) is not necessarily an m_X-pre closed set.
(v) m_X-pCl$(X \setminus A)$=$X \setminus m_X-$pInt(A).
(vi) m_X-pInt$(X \setminus A)$=$X \setminus m_X-$pCl(A).

At this point there is a natural question. There exist any conditions on the m-structure of X in order to guarantee that the m_X-sCl(B) is an m_X-semiclosed set. At this point we introduce the following property.

Definition 2.5. Let (X, m_X) be an m-space. We say that m_X to have the property of Maki, if the union of any family of elements of m_X is in m_X.

Observe that any collection $\emptyset \neq \mathcal{J} \subseteq P(X)$, always is contained in an m-structure that have the property of Maki, as we know, $m_X(\mathcal{J}) = \{\emptyset, X\} \cup \{\bigcup_{M \in J} M : \emptyset \neq J \subseteq \mathcal{J}\}$. In particular, when $\mathcal{J} = m_X$, we denote by $m_X' = m_X(\mathcal{J})$. Clearly $m_X = m_X'$, if m_X have the property of Maki. Note that if m_X is an m-structure and $Y \subseteq X$, then $\{M \cap Y : M \in m_X\}$ is an m-structure on Y, and is denoted by $m_{X|Y}$, and the pair $(Y, m_{X|Y})$ is called an m-subspace of (X, m_X).

Theorem 2.2 (8)**.** *Let (X, m_X) be an m-space and m_X satisfying the property of Maki. For a subset A of X, the following properties hold:*

(i) *$A \in m_X$ if and only if m_X-Int$(A) = A$.*
(ii) *A is m_X-closed if and only if m_X-Cl$(A) = A$.*

(iii) *m_X-Int$(A) \in m_X$ and m_X-Cl(A) is m_X-closed.*

Proof. Follows from the definition of m_X-closed, m_X-Interior and the property of Maki. □

In general the m_X-open sets and the m_X-semiopen sets are not stable for the union. Nevertheless, for certain m_X-structure, the class of m_X-semiopen sets are stable under union of sets, like it is demonstrated in the following lemma.

Lemma 2.1. *Let m_X be an m-structure on X which satisfy the property of Maki. If $A_i \in SO(X, m_X)$ for each $i \in I$, then $\bigcup_{i \in I} A_i \in SO(X, m_X)$.*

Proof. Suppose that m_X has the property of Maki, and $A_i \in SO(X, m_X)$ for each $i \in I$. For each $i \in I$, there exists a set $U_i \in m_X$ such that $U_i \subseteq A_i \subseteq U_i$, in consequence, $\bigcup_{i \in I} U_i \subseteq \bigcup_{i \in I} A_i \subseteq \bigcup_{i \in I} m_X\text{-Cl}(U_i)$. Since m_X-Cl is a monotone operator, then $\bigcup_{i \in I} m_X\text{-Cl}(U_i) \subseteq m_X\text{-Cl}(\bigcup_{i \in I} U_i)$; and $\bigcup_{i \in I} U_i \in m_X$, because m_X has the property of Maki. In consequence, $\bigcup_{i \in I} U_i \in m_X$ and $\bigcup_{i \in I} U_i \subseteq \bigcup_{i \in I} A_i \subseteq m_X\text{-Cl}(\bigcup_{i \in I} U_i)$ and consequently $\bigcup_{i \in I} A_i \in SO(X, m_X)$. □

As a consequence of the definition of the m_X-semiclosure, we have the following.

Theorem 2.3. *Let m_X be an m-structure on X. Then:*

(i) *$x \in m_X$-sCl(A) if and only if $U \cap A \neq \emptyset$ for every m_X-semi open set U containing x. In the case that m_X satisfy the property of Maki, then*

(ii) *A is an m_X-semiclosed set if and only if $A = m_X$-sCl(A).*

Theorem 2.4. *Let (X, m_X) be an m-space and $A \subseteq X$. If m_X satisfy the property of Maki. Then m_X-sCl$(A) = A \cup m_X$-Int$(m_X$-Cl$(A))$.*

Proof. Since m_X satisfies the property of Maki, then m_X-sCl(A) is an m_X-semiclosed set, using Definition 2.2, we obtain that m_X-Int(m_X-Cl(m_X-sClA)) $\subseteq m_X$-sCl(A). Therefore m_X-Int(m_X-Cl(A)) $\subseteq m_X$-sCl(A) and follows that $A \cup m_X$-Int(m_X-Cl(A)) $\subseteq m_X$ -sCl(A).
The opposite inclusion, we observe that m_X-Int(m_X-Cl($A \cup m_X$-Int (m_X-Cl(A)))) $=$ m_X-Int(m_X-Cl$(A) \cup$ m_X-Cl (m_X-Int(m_X-Cl(m_X-ClA)))) $\subseteq$ (m_X-Cl(A)) $\cup$ m_X-Int(m_X-Cl(m_X-Int(m_X-Cl(A)))) $=$ m_X-Cl$(A) \cup$ m_X-Int(m_X-Cl(A))$=m_X$-Cl(A). Thus m_X-Int (m_X-Cl ($A \cup m_X$-Int (m_X-Cl(A)))) $\subseteq m_X$-Int (m_X-Cl(A)) $\subseteq A \cup m_X$-Int(m_X-Cl(A)). Follows that m_X-Int(m_X-Cl($A \cup m_X$-Int(m_X-Cl(A)))) $\subseteq A \cup m_X$-Int (m_X-Cl(A)). In consequence, by Definition 2.2, $A \cup m_X$-Int (m_X-Cl(A)) is an m_X-semiclosed set and so m_X-sCl$(A) \subseteq A \cup m_X$-Int(m_X-Cl(A)). □

The following example shows that if the maki condition is removed in the previous theorem the equality is not necessarily true.

Example 2.1. *Let $X = \mathbb{N}$. Also, define the m-structure on X as follows: $m_X = \{\emptyset, \mathbb{N}, P(\{2n : n \in \mathbb{N}\}), \{1\}\}$. Then, the m_X- closed sets $\emptyset$, $\mathbb{N}$,$P(\{2n :$*

$n \in \mathbb{N}\})^c$ and $\mathbb{N} - \{1\}$. Also, $SO(X, m_X)$=$\{\emptyset, \mathbb{N}, P(\{2n : n \in \mathbb{N}), \{1\}, F\}$, where $F \cap \{2n : n \in \mathbb{N}\} \neq \emptyset$. If we take $A = \{3\}$, then m_X-$sCl(A)$=$\{3\}$, m_X-$Cl(A)$=$\{2n+1 : n \in \mathbb{N}\}$ and m_X-$Int(\{2n+1 : n \in \mathbb{N}\}) = \{1\}$. It is clear that $A \cup m_X$-$Int(m_X$-$Cl(A))$=$\{1, 3\} \supset \{3\} = m_X$-$sCl(A)$. In consequence, m_X-$sCl(A) \subset A \cup m_X$-$Int(m_X$-$Cl(A))$.

Theorem 2.5. *Let (X, m_X) be an m-space and $A \subseteq X$. If m_X satisfy the property of Maki. Then*

(i) m_X-$sInt(A) = A \cap m_X$-$Cl(m_X$-$Int(A))$.
(ii) m_X-$pCl(A) = A \cup m_X$-$Cl(m_X$-$Int(A))$.
(iii) m_X-$pInt(A) = A \cap m_X$-$Int(m_X$-$Cl(A))$.

Proof. (i) Follows from Theorems 2.3 and 2.4.
(ii) The proof is similar to the proof of Theorem 2.4.
(iii) Follows from (ii). □

Definition 2.6. Let (X, m_X) be an m-space. *We say that $A \subseteq X$ is an:*

(i) *m_X-regular open set if $A = m_X$-$Int(m_X$-$Cl(A))$. Also we say that $A \subseteq X$ is an m_X-regular closed set if $X \setminus A$ is an m_X-regular open set.*
(ii) *m_X-π-open set if A is the finite union of m_X-regular open sets.*
(iii) *m_X-g-closed set if the m_X-$Cl(A) \subseteq U$ whenever $A \subseteq U$ and U belong to m_X.*
(iv) *m_X-πg-closed set if the m_X-$Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is m_X-π-open in X.*
(v) *m_X-gp-closed set if the m_X-$pCl(A) \subseteq U$ whenever $A \subseteq U$ and U is belong to m_X.*
(vi) *m_X-πgp-closed set if the m_X-$pCl(A) \subseteq U$ whenever $A \subseteq U$ and U is m_X-π-open in X.*
(vii) *m_X-gs-closed set if the m_X-$sCl(A) \subseteq U$ whenever $A \subseteq U$ and U belong to m_X.*
(viii) *m_X-sg-closed set if the m_X-$sCl(A) \subseteq U$ whenever $A \subseteq U$ and U is an m_X-semiopen set in X.*
(ix) *m_X-πgs-closed set if the m_X-$sCl(A) \subseteq U$ whenever $A \subseteq U$ and U is m_X-π-open in X.*

The class of all m_X-regular open (resp. m_X-X-open) sets of an m-space (X, m_X) is denoted by $RO(X, m_X)$ (resp. $PIO(X, m_X)$).

Example 2.2. *Let $X = \{a, b, c, d\}$. Define the m-structure on X as follows: $m_X = \{\emptyset, X, \{a\}, \{b\}, \{a, c\}\}$. We obtain that $SO(X, m_X) = \{\emptyset, X, \{a\}, \{b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{a, c, d\}\}$, $PO(X, m_X) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{a, b, c\}\}$, $RO(X, m_X) = \{\emptyset, X, \{b\}, \{a, c\}\}$ and $PIO(X, m_X)$ $\{\emptyset, X, \{b\}, \{a, c\}, \{a, b, c\}\}$.*

Example 2.3. *Let $X = \{a, b, c\}$. Define the m-structure on X as follows: $m_X = \{\emptyset, X, \{a\}, \{b\}\}$. Then, we obtain that: $SO(X, m_X)$=$\{\emptyset, X, \{a\}, \{b\},$*

$\{b,c\}$, $\{a,c\}\}$, $PO(X,m_X) = \{\emptyset$, X, $\{a\}$,$\{b\}$, $\{a,b\}\}$, $RO(X,m_X) = \{\emptyset$, X ,$\{a\}$, $\{b\}\}$.

Example 2.4. *Let* $X = \{a,b,c,d\}$. *Define the* m*-structure on* X *as follows* $m_X = \{\emptyset, X, \{a\}, \{b\}, \{a,b,d\}, \{a,b,c\}\}$. *Then, we obtain that:* $SO(X,m_X) = \{\emptyset, X, \{a\}$, $\{b\}$, $\{a,c\}$, $\{a,d\}$, $\{b,c\}$, $\{b,d\}$, $\{a,b,c\}$, $\{a,b,d\}$, $\{a,c,d\}$, $\{b,c,d\}\}$, $RO(X,m_X)=\{\emptyset$, X, $\{a\}$, $\{b\}\}$, $PIO(X,m_X)$ $\{\emptyset$, X, $\{a\}$, $\{b\}$, $\{a,b\}\}$ *and* $PO(X,m_X) = \{\emptyset$, X, $\{a\}$, $\{b\}$, $\{a,b\}$, $\{a,b,c\}$, $\{a,b,d\}\}$.

Example 2.5. *Let* $X = \{a,b,c,d\}$. *Define the* m *structure on* X *as follows* $m_X = \{\emptyset$, X, $\{a\}$, $\{b\}$, $\{a,b\}$, $\{a,b,d\}$, $\{a,b,c\}\}$. *Then, we obtain that:* $SO(X,m_X) = \{\emptyset$, X, $\{a\}$, $\{b\}$, $\{a,b\}$, $\{a,c\}$, $\{a,d\}$, $\{b,c\}$, $\{b,d\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,c\}$, $\{a,b,d\}\}$, $RO(X,m_X) = \{\emptyset$, X, $\{a\}$, $\{b\}\}$, $PIO(X,m_X) = \{\emptyset$, X, $\{a\}$, $\{b\}$, $\{a,b\}\}$ *and* $PO(X,m_X) = \{\emptyset$, X, $\{a\}$, $\{b\}$, $\{a,b\}$, $\{a,b,c\}$, $\{a,b,d\}\}$.

Example 2.6. *Let* $X = \{a,b,c,d\}$. *Define the* m *structure on* X *as follows* $m_X = \{\emptyset, X, \{a,b,c\}, \{c,d\}\}$. *Then, we obtain that:* $SO(X,m_X) = \{\emptyset, X, \{c,d\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}\}$, $PO(X,m_X) = \{\emptyset$, X, $\{c\}$, $\{a,c\}$, $\{a,d\}$,$\{b,c\}$, $\{b,d\}$, $\{c,d\}$, $\{a,b,c\}$, $\{a,c,d\}$, $\{b,c,d\}$, $\{a,b,d\}\}$, $RO(X,m_X) = PIO(X,m_X) = \{\emptyset, X\}$.

Example 2.7. *Let* $X = \{a,b,c,d\}$. *Define the* m *structure on* X *as follows* $m_X = \{\emptyset, X$, $\{a,b\}$, $\{a,c\}$, $\{a\}$, $\{b\}$, $\{c\}\}$ *Then, we obtain that:* $SO(X,m_X) = \{\emptyset$, X, $\{a,b\}$, $\{a,c\}$, $\{a\}$, $\{b\}$, $\{c\}\}$, $PO(X,m_X) = \{\emptyset$, X, $\{a\}$, $\{b\}$, $\{c\}$, $\{a,b\}$, $\{a,c\}$, $\{b,c\}$, $\{a,b,c\}\}$, $RO(X,m_X) = \{\emptyset$, X, $\{a\}$, $\{b\}$, $\{c\}$, $\{a,b\}$, $\{a,c\}$, $\{b,c\}\}$ *and* $PIO(X,m_X) = \{\emptyset$,X, $\{a\}$, $\{b\}$, $\{c\}$, $\{a,b\}$, $\{a,c\}$, $\{b,c\}$, $\{a,b,c\}\}$.

Observation 2.1. *In general the notions* m_X*-gs-closedness and* m_X*-sg-closedness are independent as we can see:*

(i) *In Example 2.2, the set* $\{a,c\}$ *is* m_X*-gs-closed but not* m_X*-sg-closed.*
(ii) *In Example 2.4, the set* $\{a,b\}$ *is* m_X*-sg-closed but not* m_X*-gs-closed.*

Theorem 2.6. *The following properties are true for a subset* A *of an* m*-space* (X,m_X).

(i) *If* A *is* m_X*-closed, then it is* m_X*-semiclosed.*
(ii) *If* A *is* m_X*-closed, then it is* m_X*-preclosed.*
(iii) *If* A *is* m_X*-semiclosed, then it is* m_X*-gs-closed.*
(iv) *If* A *is* m_X*-g-closed, then it is* m_X*-gs-closed.*
(v) *If* A *is* m_X*-g-closed, then it is* m_X*-gp-closed.*
(vi) *If* A *is* m_X*-*πg*-closed, then it is* m_X*-*πgs*-closed.*
(vii) *If* A *is* m_X*-*πg*-closed, then it is* m_X*-*πgp*-closed.*

Observation 2.2. *The converses in Theorem 2.6 is not necessarily true as we can see.*

(i) *In Example 2.2, the set* $\{b\}$ *is* m_X*-semiclosed but not* m_X*-closed.*

(ii) *In Example 2.6, the set* $\{b\}$ *is* m_X*-preclosed but not* m_X*-closed.*
(iii) *In Example 2.2, the set* $\{c\}$ *is* m_X*-gs-closed but not* m_X*-semiclosed.*
(iv) *In Example 2.2, the set* $\{b\}$ *is* m_X*-gs-closed but not* m_X*-g-closed.*
(v) *In Example 2.2, the set* $\{a, d\}$ *is* m_X*-gp-closed but not* m_X*-g-closed.*
(vi) *In Example 2.2, the set* $\{a\}$ *is* m_X*-*π*gs-closed but not* m_X*-*π*g-closed.*
(vii) *In Example 2.7, the set* $\{a\}$ *is* m_X*-gp-closed but not* m_X*-*π*g-closed.*

The following Theorem improve the Remark 2.1 in [1].

Theorem 2.7. *Let* (X, m_X) *be an m-space where* m_X *satisfy the property of maki and* $A \subseteq X$*. The following properties are hold:*

(i) *If A is* m_X*-gs-closed, then A is* m_X*-*π*gs-closed.*
(ii) *If A is* m_X*-gp-closed, then A is* m_X*-*π*gp-closed.*
(iii) *If A is* m_X*-g-closed, then A is* m_X*-*π*g-closed.*
(iv) *If A is* m_X*-sg-closed, then A is* m_X*-*π*-sg-closed.*

Observation 2.3. *If the condition of Maki is dropped in Theorem 2.7, the result is not necessarily true as we can see as follows:*

(i) *Take the set* $\{a, b\}$ *in Example 2.3.*
(ii) *Take the set* $\{a, b, c\}$ *in Example 2.2.*
(iii) *Take the set* $\{a, b\}$ *in Example 2.3.*
(iv) *Take the set* $\{a, b\}$ *in Example 2.3.*

Observation 2.4. *In Theorem 2.7, none of the implications are reversible.*

(i) *In Example 2.5, the set* $\{a, b, c\}$ *is* m_X*-*π*gs closed but not* m_X*-gs-closed.*
(ii) *In Example 2.6, the set* $\{a, b, c\}$ *is* m_X*-*π*gp-closed but not* m_X*-gp closed.*
(iii) *In Example 2.5, the set* $\{a, c\}$ *is* m_X*-*π*g-closed but not* m_X*-g-closed.*
(iv) *In Example 2.6, the set* $\{c\}$ *is* m_X*-*π*-sg-closed but not* m_X*-sg-closed.*

Observation 2.5. *The notion of* m_X*-gs-closedness* m_X*-gp-closedness are independent.*

(i) *In Example 2.5, the set* $\{a\}$ *is* m_X*-gs-closed but not* m_X*-gp-closed.*
(ii) *In Example 2.6, the set* $\{c\}$ *is* m_X*-gp-closed but not* m_X*-gs-closed.*

Observation 2.6. *The notions* m_X*-*π*gs-closed is different from the notion of* m_X*-*π*gp-closed set as we can see:*

(i) *In Example 2.5, the set* $\{b\}$ *is* m_X*-*π*gs-closed but not* m_X*-*π*gp-closed.*
(ii) *In Example 2.7, the set* $\{a\}$ *is* m_X*-*π*gp closed but not* m_X*-*π*gs-closed.*

Definition 2.7. *An m-space* (X, m_X) *is said to be:*

(i) m_X*-*$T_{1/2}$ *space if every* m_X*-g-closed set is* m_X*-closed in it.*
(ii) m_X*-s*$T_{1/2}$ *space if every* m_X*-gs-closed set is* m_X*-semiclosed in it.*
(iii) m_X*-*$\pi gpT_{1/2}$ *if every* m_X*-*π*gp-closed set is* m_X*-preclosed in it.*

The following Theorem 2.8, characterize the notions m_X-sg-closed, m_X-g-closed, m_X-gs-closed, m_X-gp-closed, m_X-πg-closed, m_X-πgs-closed and m_X-πgp-closed in m-any structure on X.

Theorem 2.8. *The following results are true for a subset A of an m-space (X, m_X) :*

(i) *A is m_X-sg-closed if and only if m_X-Cl$(A) \subseteq m_X$-sKer(A).*
(ii) *A is m_X-g-closed if and only if m_X-Cl$(A) \subseteq m_X$-Ker(A).*
(iii) *A is m_X-gs-closed if and only if m_X-sCl$(A) \subseteq m_X$-sKer(A).*
(iv) *A is m_X-gp-closed if and only if m_X-pCl$(A) \subseteq m_X$-pKer(A).*
(v) *A is m_X-πg-closed if and only if m_X-sCl$(A) \subseteq m_X - \pi$-Ker(A).*
(vi) *A is m_X-πgs-closed if and only if m_X-sCl$(A) \subseteq m_X - \pi$-Ker(A).*
(vii) *A is m_X-πgp-closed if and only if m_X-pCl$(A) \subseteq m_X - \pi$-Ker(A).*

Where m_X-Ker(A) (resp. m_X-sKer(A), m_X-pKer(A), m_X-π-Ker(A)) is defined as the intersection of all m_X-open sets containing A.

Proof. We shall prove the result (i) only , because the proof of all of them are similar. We are going to proof that: $A \subseteq X$ is an m_X-sg-closed set if and only if m_X-sCl$(A) \subseteq m_X$-sKer(A).
Let $D = \{S : S \subseteq X, A \subseteq S, S \in SO(X, m_X)\}$. Then m_X-sKer$(A) = \bigcap_{S \in D} S$. Observe that $S \in D$ implies that $A \subseteq S$ follows m_X-sCl$(A) \subseteq S$ for all $S \in D$. In consequence, m_X-sCl$(A) \subseteq m_X$-sKer(A). If m_X-sCl$(A) \subseteq m_X$-sKer(A), take $S \in SO(X, m_X)$ such that $A \subseteq S$, then by hypothesis m_X-sCl$(A) \subseteq m_X$-sKer$(A) \subseteq S$. This shows that A is m_X-sg-closed. □

It is important to know that the proof of the results of the items of the above Theorem 2.7, only the set D can be changed in an addecuate form as follows: In (ii), (iii) and (iv) take $D = \{S : S \subseteq X, A \subseteq S : S \in m_X\}$. In (v), (vi) and (vii) take $D = \{S : S \subseteq X,\ A \subseteq S$: S is an m_X-π-open$\}$.

The following Theorem 2.8, characterize the m_X-gs-closed sets, m_X-sg-closed sets, m_X-g-open sets, m_X-πg-closed sets, m_X-πgs-closed sets, m_X-gp-closed sets, m_X-πgp-closed sets, where the m-structure on X satisfies the property of Maki. In this form, we generalize the Theorems 2.3 and 2.4 given in [1] .

Theorem 2.9. *Let m_X be an m-structure on X satisfying the property of Maki and $A \subseteq X$. Then:*

(i) *A is m_X-gs-closed if and only if there does not exists a nonempty m_X-closed set F $F \subseteq m_X$-sCl$(A) \setminus A$.*
(ii) *A is an m_X-sg- closed if and only if there does not exists a nonempty m_X-semiclosed set F $F \subseteq m_X$-sCl$(A) \setminus A$.*
(iii) *A is an m_X-g-closed if and only if there does not exists a nonempty m_X-closed set F $F \subseteq m_X$-Cl$(A) \setminus A$.*
(iv) *A is an m_X-πg-closed if and only if there does not exists a nonempty m_X-π-closed set F $F \subseteq m_X$-Cl$(A) \setminus A$.*
(v) *A is an m_X-gp-closed if and only if there does not exists a nonempty m_X-closed set F $F \subseteq m_X$-pCl$(A) \setminus A$.*
(vi) *A is an m_X-πgp-closed if and only if there does not exists a nonempty m_X-π-closed set F $F \subseteq m_X$-pCl$(A) \setminus A$.*

(vii) *A is an m_X-πgs-closed if and only if there does not exists a nonempty m_X-π-closed set F $F \subseteq m_X$-sCl$(A) \setminus A$.*

Proof. (i) Suppose that A is an m_X-gs-closed and let $F \subseteq X$ be an m_X-closed set such that $F \subseteq m_X$-sCl$(A) \setminus A$. It follows that, $A \subseteq X \setminus F$ and $X \setminus F$ is an m_X-open set, since A is an m_X-gs-closed, we have that m_X-sCl$(A) \subseteq X \setminus F$ and $F \subseteq X \setminus m_X$-sCl$(A)$. It follows that,

$$F \subseteq m_X\text{-sCl}(A) \cap (X \setminus m_X\text{-sCl}(A)) = \emptyset,$$

implying that $F = \emptyset$. Reciprocally, if $A \subseteq U$ and U is an m_X- open set, then m_X-sCl$(A) \cap (X \setminus U) \subseteq m_X$-sCl$(A) \cap (X \setminus A) = m_X$-sCl$(A) \setminus A$. Since m_X-sCl$(A) \setminus A$ does not contain any nonempty m_X-closed, we obtain that m_X-sCl$(A) \cap (X \setminus U) = \emptyset$. It follows that m_X-sCl$(A) \subseteq U$ and hence A is m_X-gs-closed in (X, m_X).
(ii) Suppose that A is an m_X-sg-closed and let F be an m_X-semiclosed set of (X, m_X) such that $F \subseteq m_X$-sCl$(A) \setminus A$. It follows that, $A \subseteq X \setminus F$ and $X \setminus F$ is an m_X-semiopen set, since A is an m_X-sg-closed, we have that m_X-sCl$(A) \subseteq X \setminus F$ and $F \subseteq X \setminus m_X$-sCl$(A)$. Follows that,

$$F \subseteq m_X\text{-sCl}(A) \cap (X \setminus m_X\text{-sCl}(A)) = \emptyset,$$

implying that $F = \emptyset$. Reciprocally, if $A \subseteq U$ and U is an m_X-semiopen set, then m_X-sCl$(A) \cap (X \setminus U) \subseteq m_X$-sCl$(A) \cap (X \setminus A) = m_X$-sCl$(A) \setminus A$. Since m_X-sCl$(A) \setminus A$ does not contain any nonempty m_X-semiclosed sets, we obtain that m_X-sCl$(A) \cap (X \setminus U) = \emptyset$. It follows that m_X-sCl$(A) \subseteq U$ and hence A is an m_X-gs-closed.
(iv) Suppose that A is an m_X-πg-closed and let $F \subseteq X$ be an m_X-π-closed set such that $F \subseteq m_X$-Cl$(A) \setminus A$. It follows that, $A \subseteq X \setminus F$ and $X \setminus F$ is an m_X-π- open set, since A is an m_X-πg-closed, we have that m_X-Cl$(A) \subseteq X \setminus F$ and $F \subseteq X \setminus m_X$-Cl$(A)$. Follows that,

$$F \subseteq m_X\text{-Cl}(A) \cap (X \setminus m_X\text{-Cl}(A)) = \emptyset,$$

implying that $F = \emptyset$. Reciprocally, if $A \subseteq U$ and U is an m_X-π- openset, then m_X-Cl$(A) \cap (X \setminus U) \subseteq m_X$-Cl$(A) \cap (X \setminus A) = m_X$-Cl$(A) \setminus A$. Since m_X-Cl$(A) \setminus A$ does not contain any nonempty m_X-π-closed sets, we obtain that m_X-Cl$(A) \cap (X \setminus U) = \emptyset$, and this implies that m_X-Cl$(A) \subseteq U$ in consequence A is an m_X-πg-closed.
The proof of (iii), (v), (vi) and (vii) are similar. □

We can observe that if in Theorem 2.9 the condition of Maki is omitted then the result can be false, as we can see in the following example.

Example 2.8. (i) *In Example 2.4, the set $\{a, b\}$ is sg-closed, and $\{d\}$ is an m_X-semiclosed such that $\{d\} \subseteq (m_X$-sCl$(\{a, b\}) \setminus \{a, b\})$.*

(ii) *In Example 2.4, the set $\{a\}$ is not m_X-g-closed and there not exists m_X-closed set F such that $F \neq \emptyset$ and $F \subseteq m_X$-Cl$(A) \setminus A$.*

(iii) *In Example 2.2, the set* $\{a\}$ *is not* m_X*-gs-closed, and there does not exists* m_X*-closed set* F *such that* $F \neq \emptyset$ *and* $F \subseteq m_X$*-sCl*$(\{a\}) \setminus \{a\}$.

(iv) *In Example 2.7 the set* $\{c\}$ *is not* m_X*-gp-closed, and there does not exists a* m_X*-closed set* F *such that* $F \neq \emptyset$ *and* $F \subseteq m_X$*-pCl*$(\{c\}) \setminus \{c\}$.

Theorem 2.10. *Let* (X, m_X) *be an m-space satisfying the property of Maki and* $A \subseteq X$*. Then the following properties are equivalent:*

(i) A *is* m_X*-*π*-open and* m_X*-*π*-gs-closed.*

(ii) A *is* m_X*-regular open.*

Proof. (i) $\Rightarrow$ (ii): Since A is m_X-π-gs-closed then m_X-sCl$(A) \subseteq A$ because A is m_X-π-open. Using Theorem 2.4, m_X-sCl$(A) = A \cup m_X$-Int$(m_X$-Cl$(A))$. We obtain that m_X-Int$(m_X$-Cl$(A)) \subseteq A$. Using the hypothesis on m_X, we obtain that, A is m_X-open, in consequence A is m_X-preopen. Follows that $A \subseteq m_X$-Int$(m_X$-Cl$(A))$ and therefore m_X-Int$(m_X$-Cl$(A)) \subseteq A \subseteq m_X$-Int$(m_X$-Cl$(A))$. This implies that A is m_X-regular open.
(ii) $\Rightarrow$ (i): Every m_X-regular open set is m_X-π-open and m_X-open therefore m_X-π-open and m_X-semiopen. Follows from Theorem 2.4, m_X-sCl$(A) = A$. In consequence, A is m_X-π-gs-closed. □

Theorem 2.11. *Let* (X, m_X) *be an m-space satisfying the property of Maki and* $A \subseteq X$*. If* A *is* m_X*-*π*-open and* m_X*-*π*-gs-closed then* A *is* m_X*-semiclosed and hence* m_X*-gs-closed.*

Proof. By Theorem 2.9, A is m_X-regular open. Using Theorem 2.4, m_X-sCl$(A) = A$, follows that A is m_X-semiopen in consequence A is m_X-gs-closed. □

It is easy to see in Example 2.2, the set $\{a, b, c\}$ is m_X-π-open and m_X-gs-closed but not m_X-π-gs-closed.

Definition 2.8. A subset A of an m-space (X, m_X) is called m_X-clopen if m_X-Int$(m_X$-Cl$(A)) = m_X$-Int$(m_X$-Cl$(A))$.

Theorem 2.12. *Let* (X, m_X) *be an m-space satisfying the property of Maki and* $A \subseteq X$*. Then the following properties are equivalent:*

(i) A *is* m_X*-*π*-clopen, that is* m_X*-*π*-open and* m_X*-*π*-closed.*

(ii) A *is* m_X*-*π*-open,* m_X*-clopen and* m_X*-*π*-gs-closed.*

Proof. (i) $\Rightarrow$ (ii): If A is m_X-π-clopen, then A is m_X-π-open and m_X-π-closed, follows that A is m_X-open and m_X-closed and m_X-sCl$(A) = A$. In consequence, we obtain that A is m_X-π-open, m_X-clopen and m_X-π-gs-open.
(ii) $\Rightarrow$ (i): Using the fact that A is m_X-π-open and m_X-π-gs-closed implies by Theorem 2.8, A is m_X-regular open. Now if A is m_X-clopen then m_X-Int$(m_X$-Cl$(A)) = m_X$-Int$(m_X$-Cl$(A)) = A$. Follows that A is m_X-π-clopen. □

Theorem 2.13. *Let* (X, m_X) *be an m-space satisfying the property of Maki and* A, B *be subsets of* X*. Then the following properties hold:*

(i) *If A is m_X-g-closed and $A \subset B \subset m_X$-Cl(A), then B is m_X-g-closed.*
(ii) *If A is m_X-π-g-closed and $A \subset B \subset m_X$-Cl(A), then B is m_X-π-g-closed.*
(iii) *If A is m_X-gp-closed and $A \subset B \subset m_X$-pCl(A), then B is m_X-gp-closed.*
(iv) *If A is m_X-π-gp-closed and $A \subset B \subset m_X$-pCl(A), then B is m_X-π-gp-closed.*
(v) *If A is m_X-gs-closed and $A \subset B \subset m_X$-sCl(A), then B is m_X-gp-closed.*
(vi) *If A is m_X-π-gs-closed and $A \subset B \subset m_X$-sCl(A), then B is m_X-π-gs-closed.*

Proof. (i): Let A be an m_X-g-closed subset, $B \subset U$ where U is m_X-open. Since $A \subset B$, then m_X-Cl$(A) \subset m_X$-Cl(B). It follows that m_X-Cl$(A) \subset U$ and $B \subset m_X$-Cl(A) implies that m_X-Cl$(B) \subset U$. In consequence B is m_X-g-closed.
The other proofs are similar. □

The proof of the following Theorem 2.14 is easy and hence omitted.

Theorem 2.14. *Let (X, m_X) be an m-space satisfying the property of Maki and $A \subseteq X$. Then the following properties hold:*
(i) *A is m_X-g-open if and only if $F \subset m_X$-Int(A) whenever F is m_X-closed and $F \subset A$.*
(ii) *A is m_X-π-g-open if and only if $F \subset m_X$-Int(A) whenever F is m_X-π-closed and $F \subset A$.*
(iii) *A is m_X-gp-open if and only if $F \subset m_X$-pInt(A) whenever F is m_X-closed and $F \subset A$.*
(iv) *A is m_X-π-gp-open if and only if $F \subset m_X$-pInt(A) whenever F is m_X-π-closed and $F \subset A$.*
(v) *A is m_X-gs-open if and only if $F \subset m_X$-sInt(A) whenever F is m_X-closed and $F \subset A$.*
(vi) *A is m_X-π-gs-open if and only if $F \subset m_X$-sInt(A) whenever F is m_X-π-closed and $F \subset A$.*

3. (m_X, m_Y)-Continuous maps and (m_X, m_Y)-Irresolute maps

In this section, we define different forms of continuity and irresoluteness on m-structures where the notions of g-closed set, gs-closed set, sg-closed set, π-closed set, πg-closed set, πgs-closed set, gp-closed set, πgp-closed set are involucrate.

Definition 3.1. A map $f : (X, m_X) \to (Y, m_Y)$ is called:
(i) *π-gs-(m_X, m_Y)-continuous if $f^{-1}(O)$ is m_X-π-gs-closed in (X, m_X) for every m_Y-closed set O of (Y, m_Y).*
(ii) *π-(m_X, m_Y)-continuous if $f^{-1}(O)$ is m_X-π-closed in (X, m_X) for every m_Y-closed set O of (Y, m_Y).*
1. *$\pi g - (m_X, m_Y)$ continuous if, $f^{-1}(O)$ is m_X-πg-closed in (X, m_X) for every m_Y-closed set O of (Y, m_Y).*

2. *$\pi gp(m_X, m_Y)$ continuous if, $f^{-1}(O)$ is m_X-πgp-closed in (X, m_X) for every m_Y-closed set O of (Y, m_Y).*
3. *$s-(m_X, m_Y)$ continuous if, $f^{-1}(O)$ is m_X-semiclosed in (X, m_X) for every m_Y-closed set O of (Y, m_Y).*
4. *$g-(m_X, m_Y)$ continuous if, $f^{-1}(O)$ is m_X-g-closed in (X, m_X) for every m_Y-closed set O of (Y, m_Y).*
5. *$gs-(m_X, m_Y)$ continuous if, $f^{-1}(O)$ is m_X-gs-closed in X for every m_Y-closed set O of (Y, m_Y).*
6. *$gp-(m_X, m_Y)$ continuous if, $f^{-1}(O)$ is m_X-gp-closed in (X, m_X) for every m_Y-closed set O of (Y, m_Y).*

Example 3.1. *In the Example 2.3, take $X = Y = \{a, b, c\}, m_X = m_Y = \{\emptyset, X, \{a\}, \{b\}\}$ and $f : (X, m_X) \mapsto (Y, m_Y)$ defined as: $f(a) = f(c) = c$ and $f(b) = a$. Then the function f satisfies all different notions of continuity described in Definition 3.1.*

From the above definition, easily we have the following theorem

Theorem 3.1. *Let $f : (X, m_X) \to (Y, m_Y)$, then:*

(i) *If f is (m_X, m_Y) continuous, then it is g-(m_X, m_Y)-continuous.*
(ii) *If f is (m_X, m_Y)-continuous, then it is s-(m_X, m_Y)-continuous.*
(iii) *If f is (m_X, m_Y)-continuous, then it is gp-(m_X, m_Y)-continuous.*
(iv) *If f is g-(m_X, m_Y)-continuous, then it is gp-(m_X, m_Y)-continuous.*
(v) *If f is s-(m_X, m_Y)-continuous, then it is gs-(m_X, m_Y)-continuous.*
(vi) *If f is g-(m_X, m_Y)-continuous, then it is gs-(m_X, m_Y)-continuous.*
(vii) *If f is g-(m_X, m_Y)-continuous, then it is gp-(m_X, m_Y)-continuous.*

and none of them are reversible.

Proof. The proof follows from Theorem 2.6. □

Theorem 3.2. *Let $f : (X, m_X) \to (Y, m_Y)$, where m_X satisfy the property of Maki, then If f is π-(m_X, m_Y)-continuous then f is (m_X, m_Y)-continuous.*

Proof. The proof follows from the fact that any π-closed set is closed in any m-structure. □

Example 3.2. *In the Example 2.3, take $X = Y = \{a, b, c\}, m_X = m_Y = \{\emptyset, X, \{a\}, \{b\}\}$ and $f : (X, m_X) \mapsto (Y, m_Y)$ defined as: $f(a) = f(b) = a$ and $f(c) = c$. Then f is π-(m_X, m_Y)-continuous but does not is (m_X, m_Y)-continuous.*

In the case that the $f : (X, m_X) \to (Y, m_Y)$ is a map, where m_X satisfy the condition of Maki, we have the following Theorem.

Theorem 3.3. *Let $f : (X, m_X) \to (Y, m_Y)$, where m_X satisfies the condition of Maki then:*

1. *If f is g-(m_X, m_Y)-continuous, then f if πg-(m_X, m_Y)-continuous.*
2. *If f is gs-(m_X, m_Y)-continuous, then f if π-gs-(m_X, m_Y)-continuous.*

3. *If f is gp-(m_X, m_Y)-continuous, then f if π-gp-(m_X, m_Y)-continuous.*

and none of them are reversible.

Proof. The proof follows from Theorem 2.7. □

In the following example, we shows that if the condition of Maki on m_X is omitted, then the Theorem 3.3 can be false

Example 3.3. *In the Example 2.3, take $X = Y = \{a, b, c\}$, $m_X = m_Y = \{\emptyset, X, \{a\}, \{b\}\}$ and $f : (X, m_X) \mapsto (Y, m_Y)$ defined as: $f(a) = c, f(b) = a$ and $f(c) = b$. Then:*

(i) *f is g-(m_X, m_Y)-continuous but not π-g-(m_X, m_Y)-continuous.*
(ii) *f is gs-(m_X, m_Y)-continuous but not πgs-(m_X, m_Y)-continuous.*

Example 3.4. *In the Example 2.3, take $X = \{a, b, c\}, m_X = \{\emptyset, X, \{a\}, \{b\}\}$ and $Y = \{x, y\}$, and$m_Y = \{\emptyset, Y, \{x\}\}$, $f : (X, m_X) \mapsto (Y, m_Y)$ defined as: $f(a) = f(c) = x$ and $f(b) = y$. Then f is π-gs-(m_X, m_Y)-continuous but not π-g-(m_X, m_Y)-continuous and π-gp-(m_X, m_Y)-continuous.*

Example 3.5. *In the Example 2.7, take $X = \{a, b, c, d\}, m_X = \{\emptyset, X, \{a, b\}, \{a, c\}, \{a\}, \{b\}, \{c\}\}$ and $Y = \{x, y\}$, and $m_Y = \{\emptyset, Y, \{x\}\}$, $f : (X, m_X) \mapsto (Y, m_Y)$ defined as: $f(b) = f(c) = f(d) = x$ and $f(a) = y$. Then f is π-gp-(m_X, m_Y)-continuous but not π-gs-(m_X, m_Y)-continuous and π-g-(m_X, m_Y)-continuous.*

Example 3.6. *In the Example 2.2, take $X = \{a, b, c, d\}, m_X = \{\emptyset, X, \{a\}, \{b\}, \{a, c\}\}$ and $Y = \{x, y\}$, and $m_Y = \{\emptyset, Y, \{x\}\}$.*
Define $f : (X, m_X) \mapsto (Y, m_Y)$ as: $f(a) = f(b) = f(c) = y$ and $f(d) = y$. Then f is gp-(m_X, m_Y)-continuous but none of π-gp-(m_X, m_Y)-continuous, π-gs-(m_X, m_Y)-continuous, π-g-(m_X, m_Y)-continuous and π-(m_X, m_Y)-continuous.

Definition 3.2. A map $f : (X, m_X) \to (Y, m_Y)$ is called:

(i) *(m_X, m_Y)-irresolute if $f^{-1}(O)$ is m_X-semiclosed in X for every m_Y-semiclosed set O of (Y, m_Y).*
(ii) *π-(m_X, m_Y)-irresolute if, $f^{-1}(O)$ is m_X-π-closed in (X, m_X) for every m_Y-π-closed set O of (Y, m_Y).*
(iii) *π-gp-(m_X, m_Y)-irresolute if $f^{-1}(O)$ is m_X-π-gp-closed in (X, m_X) for every m_Y-π-gp-closed set O of (Y, m_Y).*
(iv) *π-gs-(m_X, m_Y)-irresolute if, $f^{-1}(O)$ is m_X-π-gs-closed in (X, m_X) for every m_Y-π-gs-closed set O of (Y, m_Y).*
(v) *gs-(m_X, m_Y)-irresolute if, $f^{-1}(O)$ is m_X-gs-closed in (X, m_X) for every m_Y-gs-closed set O of (Y, m_Y).*
(vi) *gp-(m_X, m_Y)-irresolute if, $f^{-1}(O)$ is m_X-gp-closed in (X, m_X) for every m_Y-gp-closed set O of (Y, m_Y).*
(vii) *g-(m_X, m_Y)-irresolute if, $f^{-1}(O)$ is m_X-g-closed in (X, m_X) for every m_Y-g-closed set O of (Y, m_Y).*

Example 3.7. *In the Example 2.7, take* $X = \{a, b, c, d\}, m_X = \{\emptyset, X, \{a, b\}, \{a, c\}, \{a\}, \{b\}, \{c\}\}$. $Y = \{x, y\}$, *and* $m_Y = \{\emptyset, Y, \{x\}\}$, $f : (X, m_X) \mapsto (Y, m_Y)$ *defined as:* $f(b) = f(c) = f(d) = x$ *and* $f(a) = y$. *Then* f *is* π-*gp*-(m_X, m_Y)-*irresolute but none of* π-*gs*-(m_X, m_Y)-*irresolute and* π-*g*-(m_X, m_Y)-*irresolute.*

Example 3.8. *In the Example 2.4, take* $X = \{a, b, c, d\}, m_X = \{\emptyset, X, \{a, b, d\}, \{a, b, c\}, \{a\}, \{b\}\}$. $Y = \{x, y\}$, *and* $m_Y = \{\emptyset, Y, \{x\}\}$, $f : (X, m_X) \mapsto (Y, m_Y)$ *defined as:* $f(b) = f(c) = f(d) = x$ *and* $f(a) = y$. *Then* f *is* π-*gs*-(m_X, m_Y)-*irresolute but none of* π-*gp*-(m_X, m_Y)-*irresolute,* π-*g*-(m_X, m_Y)-*irresolute, g*-(m_X, m_Y)-*irresolute and gp*-(m_X, m_Y)-*irresolute.*

Example 3.9. *In the Example 2.2, take* $X = \{a, b, c, d\}, m_X = \{\emptyset, X, \{a\}, \{b\}, \{a, c\}\}$. $Y = \{x, y\}$, *and* $m_Y = \{\emptyset, Y, \{x\}\}$, $f : (X, m_X) \mapsto (Y, m_Y)$ *defined as:* $f(a) = f(b) = f(c) = y$ *and* $f(d) = x$. *Then* f *is gp*-(m_X, m_Y)-*irresolute but none of* π-*gp*-(m_X, m_Y)-*irresolute,* π-*gs*-(m_X, m_Y)-*irresolute,* π-*g*-(m_X, m_Y)-*irresolute and* π--(m_X, m_Y)-*irresolute.*

Definition 3.3. A map $f : (X, m_X) \to (Y, m_Y)$ is called:

(i) (m_X, m_Y)-*pre semiclosed if* $f(O)$ *is* m_Y-*semiclosed in* (Y, m_Y) *for all* m_X-*semiclosed set* O *of* (X, m_X).

(ii) (m_X, m_Y)-*pre semiopen if* $f(O)$ *is* m_Y-*semiopen in* (Y, m_Y) *for all* m_X-*semiopen set* O *of* (X, m_X).

(iii) (m_X, m_Y)-*regular open if,* $f(O)$ *is* m_Y-*regular open in* (Y, m_Y) *for every* m_X-*open set* O *of* (X, m_X).

Observation 3.1. *The following Lemma 3.1 generalize the Theorem 4.2, given in [1].*

Lemma 3.1. *Let* (X, m_X) *and* (Y, m_Y) *be two m-spaces where* m_X *satisfies the property of Maki. If* $f : (X, m_X) \to (Y, m_Y)$ *is* π-(m_X, m_Y)-*irresolute function and* $(m_X.m_Y)$-*pre semiclosed, then* $f(A)$ *is* m_Y-π-*gs-closed for every* m_X-π-*gs-closed set* A *in* X.

Proof. Let A be any m_X-π-gs-closed set in X, U an m_Y-π-closed set in y such that $f(A) \subseteq U$. By hypothesis $f^{-1}(U)$ is m_X-open set in X and $A \subseteq f^{-1}(U)$. Follows that m_X-sCl$(A) \subseteq f^{-1}(U)$, in consequence $f(m_X\text{-sCl}(A)) \subseteq U$. Since $A \subseteq m_X$-sCl(A), then $f(A) \subseteq f(m_X\text{-sCl}(A))$, in consequence, $m_Y\text{-sCl}(f(A)) \subseteq m_Y\text{-sCl}(f(m_X\text{-sCl}(A)))$. Since f is $(m_X, m_Y)-$pre semiclosed, $m_Y\text{-sCl}(f(m_X\text{-sCl}(A))) = f(m_X\text{-sCl}(A)$. Follows that $m_Y\text{-sCl}f((A)) \subseteq f(m_X\text{-sCl}(A) \subseteq U$. In consequence $f(A)$ is m_Y-πgs-closed set in Y. ☐

Observation 3.2. *The following Lemma 3.2 generalize the Theorem 4.3, given in [1].*

Lemma 3.2. *Let* (X, m_X) *and* (Y, m_Y) *be two m-spaces, where* m_Y *satisfies the property of Maki. If* $f : (X, m_X) \to (Y, m_Y)$ *is* (m_X, m_Y)-*irresolute,* (m_X, m_Y)-*regular open and bijective, then* f *is* π-*gs*-(m_X, m_Y)-*irresolute.*

Proof. Let F any m_Y-π-gs-closed set in Y and U any m_X-π-open set in X such that $f^{-1}(F) \subseteq U$. Follows that $F \subseteq f(U)$ since $f(U)$ is m_Y-π-open, then m_Y-sCl$(F) \subseteq f(U)$, therefore $f^{-1}(m_Y\text{-sCl}(F)) \subseteq U$. Since f is (m_X, m_Y)-irresolute, then $f^{-1}(m_Y\text{-sCl}(F))$ is m_X-semiclosed, in consequence $m_X\text{-sCl}(f^{-1}(F)) \subseteq m_X\text{-sCl}(f^{-1}(m_Y\text{-sCl}(F)) = (f^{-1}(m_Y\text{-sCl}(F)) \subseteq U$. Follows that $f^{-1}(F)$ is m_X-π-gs-closed in X. □

Lemma 3.3. *Let (X, m_X) and (Y, m_Y) be two m-spaces where m_Y satisfies the property of Maki. The following conditions are equivalent:*

(i) *$f : (X, m_X) \to (Y, m_Y)$ is (m_X, m_Y)-irresolute function.*
(ii) *For each subset $A \subseteq X$, $f(m_X\ \text{-}sCl(A)) \subseteq m_Y\text{-}sCl(f(A))$.*
(iii) *For each m_Y semiclosed subset $V \subseteq Y$, the inverse image $f^{-1}(V)$ is an m_X semiclosed in X.*
(iv) *For all $B \subseteq Y$, $m_X\text{-}sCl(f^{-1}(B)) \subseteq f^{-1}(m_Y\text{-}sCl(B))$.*

Proof. $(iii) \Rightarrow (ii)$: Let A be a subset of X and suppose that $y \notin m_Y$-sCl$(f(A))$, then there exists a m_Y-semi open set G in Y, such that $y \in G$ and $f(A) \cap G = \emptyset$, therefore, $f^{-1}(f(A) \cap G) = \emptyset$, it says that $A \cap f^{-1}(G) = \emptyset$. In consequence, $m_X\text{-sCl}(A) \subset (f^{-1}(G))^c$, follows that $f(m_X\text{-sCl}(A)) \cap G = \emptyset$; and therefore, $y \notin f(m_X\text{-sCl}(A))$. But it is said that $f(m_X\text{-sCl}(A)) \subset m_Y$-sCl$(f(A))$ for all subset A of X.
$(ii) \Rightarrow (iii)$: Let V any m_Y-semiclosed subset in Y, then $f^{-1}(V) \subseteq X$. By hypothesis $f(m_X\text{-sCl}(f^{-1}(V))) \subset m_Y\text{-sCl}(f(f^{-1}(V)))$, follows that $f(m_X\text{-sCl}(f^{-1}(V))) \subset m_Y\text{-sCl}(V)$. In consequence, $f(m_X\text{-sCl}(f^{-1}(V))) \subset V$, follows that $m_X\text{-}sCl(f^{-1}(V) \subset f^{-1}(V)$. Therefore $f^{-1}(V)$ is an m_X-semiclosed set.
$(ii) \Rightarrow (iv)$: Let B be a subset of Y, then $f^{-1}(B) \subseteq X$. Using the hypothesis, that $f(m_X\text{-}sCl(f^{-1}(B))) \subseteq m_Y\text{-}sCl(f(f^{-1}(B))) \subseteq m_Y\text{-}sCl(B)$, therefore, $m_X\text{-}sCl(f^{-1}(B)) \subseteq f^{-1}(m_Y\text{-}sCl(B))$.
$(iv \Rightarrow iii)$: Suppose that V is any m_Y-semiclosed set in Y. Then $f^{-1}(V) \subseteq X$, by hypothesis, we obtain that $m_X\text{-}sCl(f^{-1}(V)) \subseteq f^{-1}(m_Y\text{-}sCl(V))$. But V is a m_Y-semiclosed set, then $m_Y\text{-}sCl(V) = V$. In consequence, $m_X\text{-}sCl(f^{-1}(V)) \subseteq f^{-1}(V)$. But this says that $f^{-1}(V)$ is an m_X-semiclosed set in X.

The others implications $(i) \Rightarrow (iii)$ and $(iii) \Rightarrow (i)$, follow from the definition of (m_X, m_Y)-irresolute function and the complement of set. □

There are some relation between irresoluteness and continuity as we shows:

Theorem 3.4. *Let $f : (X, m_X) \to (Y, m_Y)$, then:*

(i) *If f is π-g-(m_X, m_Y)-irresolute, then f is π-g-(m_X, m_Y)-continuous.*
(ii) *If f is π-gs-(m_X, m_Y)-irresolute, then f is π-gs-(m_X, m_Y)-continuous.*
1. *If f is π-gp-(m_X, m_Y)-irresolute, then f is π-gp-(m_X, m_Y)-continuous.*

and none of them are reversible.

Proof. The proof follows from Theorem 2.6 and 2.7. □

The following example shows that there exists π-(m_X, m_Y)-irresolute maps but does not is π-(m_X, m_Y)-continuous.

Example 3.10. *Let* $X = Y = \{a, b, c, d\}$, $m_X = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{c, d\}\}$, *and* $m_Y = \{\emptyset, Y, \{a\}, \{b\}, \{a, c\}\}$. *Then the* m_X*-regular open sets of* (X, m_X) *are* $\emptyset$, X, $\{a\}$, $\{b\}$, $\{a, b\}$, $\{b, c\}$, $\{c, d\}$, $\{a, c, d\}$, $\{b, c, d\}$ *and the* m_X*-*π*-open sets are* $\emptyset$, X, $\{a\}$, $\{b\}$, $\{a, b\}$, $\{b, c\}$, $\{c, d\}$, $\{a, c, d\}$, $\{b, c, d\}$*,and* $\{a, b, c\}$. *Define a function* $f : (X, m_X) \mapsto (Y, m_Y)$ *as:* $f(a)=f(d)= d$, $f(b)=a$ *and* $f(c) = c$. *Then* f *is* π-(m_X, m_Y)*-irresolute but not* π-(m_X, m_Y)*-continuous.*

Bibliografía

[1] G. Aslim, A. Caksu Guler and T. Noiri, *On π-gs-Closed sets in topological spaces*, Acta Math. Hungar.112 (4)(2006), 275-283.

[2] C. Carpintero, E. Rosas and J. Vielma, *Espacios α-sg-T_i para $i = 0, 1/2, 1, 2, 3$*, Divulgaciones Matemáticas. 11(2)(2003), 137-147.

[3] C. Carpintero, E. Rosas and M. Salas, *Minimal Structure and separations properties*, Internat. J. of Pure. Appl. Math. (34)(4)(2007)473-488.

[4] J. Donchev, *Contra Continuous functions and strongly $S-$closed spaces*, Internat. J. Math. Math. Sci. 19(1996),303-310.

[5] D. S. Jankovic, I. L.Relly, *On semi separation properties*, Indian J. Pure Appl. Math., 16(1985).957-964.

[6] S. Kasahara, *Operator-Compact Spaces*, Mathematica Japonica, 21(1979), 97-105.

[7] N. Levine, *Semi Open Sets and Semi Continuity in Topological Spaces*, Amer. Math. Month., 70(1963), 36-41.

[8] H. Maki, R. Chandrasekhara and A. Nagoor Gani, *On generalizing semi-open sets and preopen sets*, Pure Appl. Math. Sci., 49 (1999), 17-29.

[9] T. Noiri and V. Popa, *A Unified Theory for Strong θ-Continuity for Functions*, Acta Math. Hungar., 106(3)(2005), 167-186.

[10] V. Popa and T. Noiri, *On Almost m-Continuous Functions*, Mathematicae Notae, 40(1999-2002), 75-94.

[11] V. Popa and T. Noiri, *A Unified Theory of Weak Continuity for Functions*, Rendiconti del circolo Matematico di Palermo, Serie II,Tomo LI(2002), 439-464.

Received: February, 2009