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Mathematical Modelling of Mechanical Combination Lock Systems

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Abstract. In this study, mathematical modelling of the mechanical combination lock systems and special kinds of lock mechanisms has been carried out. Formation of the codes in these systems are based on the consecutive rotation motions. The basic structures allowing the motions in the systems are the lever and the round plaque mechanisms. The letter and the numbers on the plaque mechanisms are effective in the envisioning of the code. The basic information concerning with the rotation motion was received from [2] and the according to this information, the code matrix was completed. Movement conditions in the linear, quadratic, cubic and exponential functions of the code matrix whose general research was completed, show difference. Combination lock mechanisms in this study have been studied in kinematic point of view rather than the probability calculations. Many different designs of the combination lock systems formed by consecutive rotation motions can be created in mathematical logic frame and by creating different codes.

Mathematics Subject Classification: 70B15

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1. INTRODUCTION

Rotation. If $X = (x, y)$ are the coordinates of a point $P \in \mathbb{R}^2$ in the moving body measured in the coordinate frame M, then the coordinates of P measured in coordinate frame F can be given by

$$D : F \rightarrow M.$$

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This transformation is given by,

$$\vec{X} = [A]\vec{x} + d,$$

where $\vec{x}$ is the coordinate vector of a point in M and $\vec{X}$ is the coordinate vector of the same point but measured in F. If the moving body is of dimension n (usually $n = 2$ or 3), then $[A]$ is an $n \times n$ matrix and d is n-dimensional vector. Let $n = 2$ be, so

$$[A] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, d = \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix}.$$

The pair (A, d) defines this transformation, and is called a planar displacement [2]. The matrix $[A]$ has $[A^T]$ as its inverse, therefore it is an orthogonal matrix; and, because its determinant is 1, it defines a rotation.It is interesting to examine the 2×2 orthogonal matrices that are not rotations [2].

The fixed origin point $O = (0, \ldots ., 0) \in \mathbb{R}^n$. Let $R\theta(0)$ be the θ degree rotation, leaving the O point fixed. Then the rotation equations for $n = 2$ can be given by,

$$x' = x\cos\theta + y\sin\theta$$

$$y' = -x\sin\theta + y\cos\theta$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Briefly, it can be written as

$$\vec{X} = R\theta\vec{x}$$

$R\theta$ matrix here, is a matrix having the $R\theta^T = R\theta^{-1}$ property and $R\theta$ is an orthogonal matrix.

Definition 1.1. Let A be an nn matrix. A real or complex number λ is called an eigenvalue of A if, for some nonzero $nx1$ matrix X, $AX = \lambda X$. We usually think of X as a vector written as a column, and call it an eigenvector associated with λ [4].

Theorem 1.2. *Let A be 3×3 orthogonal matrix. A has three eigenvalues. $\lambda_1 = 1$ and $\lambda_{2/3} = \cos\theta \pm i\sin\theta$. That is when A is a 3×3 orthogonal matrix, the linear transformation evident with A,*

a) *the direction evident with $\vec{X}$, acquiescing in eigenvector to the vector $\vec{X}$, evident with $\lambda = 1$ remains unchanged.*

b) *$\lambda_{2/3} = \cos\theta \quad \pm i\sin\theta$ are two imaginary proportional roots and describe a plane. A linear transformation leaves this plane invariant.*

c) *Since the eigenvectors matching different eigenvalues are vertical, the direction determined by* $\vec{X}$ *are vertical to the plane, evident by* x_2, x_3 *matching* λ_2 *and* λ_3 [1].

Result 1.3. The rotations in $\mathbb{R}^3$ are planar movements and the rotations appear on the planes vertical to the eigenvector matching to $\lambda = 1$ [3].

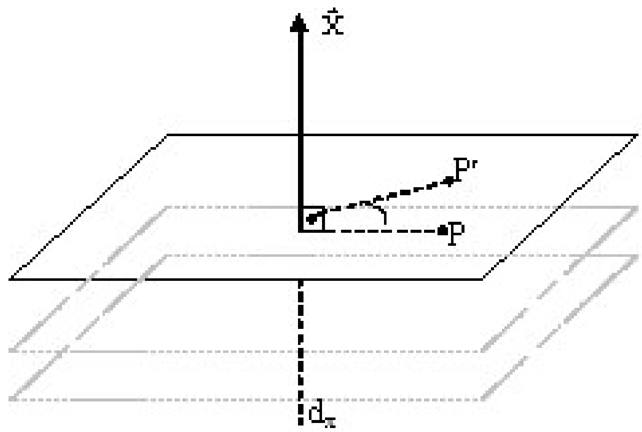

FIGURE 1. The plane evident with X_2, X_3.

The d_x line evident with $\vec{X}$ vector is called the rotation axis of the rotation R.

2. Combination Locks and the Mechanism Kinematics

Mechanic combination locks are the locks having a low probability front mechanism, allowing the system to recognize the key before opened by the key among the other mechanic locks. Many combination lock mechanism samples dating back in the past can be seen. The code in some of those have been expressed by numbers or by letters for the others.

And the mechanisms in the combination locks known as safe locks and most widely used are compassed of the united motion of many plaques The process of this system is given and the figures belonging to the mechanism and a plaque sample are shown below.

The lock systems with a mechanic codes are basically depend on consecutive rotation motions. Some of the structures allowing those rotational movements are the level structure and a round plaque mechanism. The ciphers are created by utilizing the numbers and the letters on the plaque

For example, let's show the mechanism by installing the system on a dial with numbers from 1 to 12:

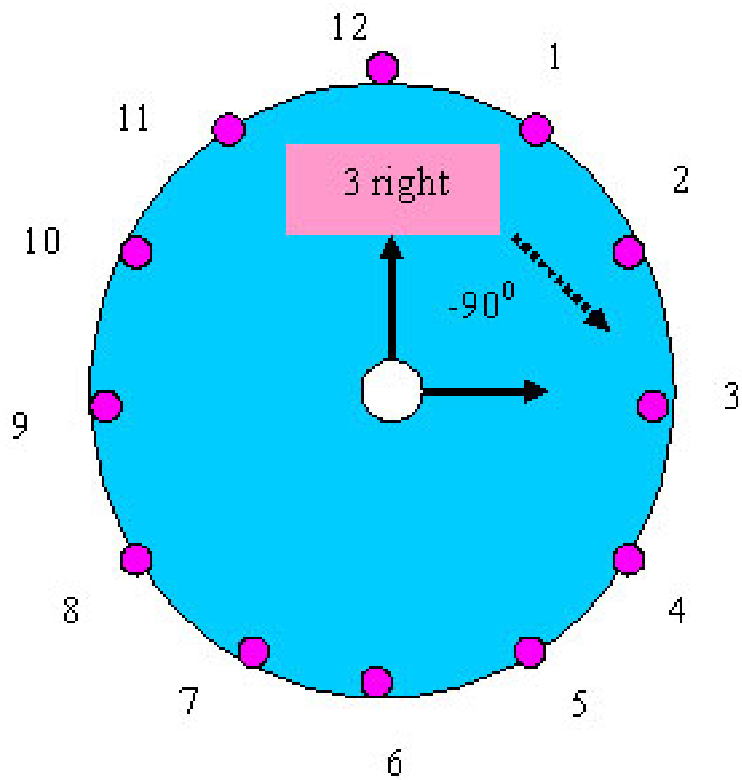

FIGURE 2. A dial mechanism of 12.

If the code here is set as,

2 left

5 right

7 left

2 right

The background condition in the system, the angle values in terms of the O oriented rotations are as the following. The angle values in this mechanism are given in terms of $360^0/12^0 = 30^0$.

3 right	...	$3 \times 30 = 90^0$	$\theta = -90^0$	$\left[R_1(-90^0)\right]$
2 left	...	$2 \times 30 = 60^0$	$\theta = +60^0$	$\left[R_2(+60^0)\right]$
5 right	...	$5 \times 30 = 150$	$\theta = -150$	$\left[R_3(-150^0)\right]$
7 left	...	$7 \times 30 = 210$	$\theta = +210$	$\left[R_4(+210^0)\right]$
2 left	...	$2 \times 30 = 60^0$	$\theta = -60^0$	$\left[R_5(-60^0)\right]$

Therefore, after the $\mathbf{K}_1$, $\mathbf{K}_2$, $\mathbf{K}_3$, $\mathbf{K}_4$ mechanism system is: $[R_1(-90^0)]$, $[R_2(+60^0)]$, $[R_3(-150^0)]$, $[R_4(+210^0)]$, $[R_5(-60^0)]$.

Rotations the cipher plaques get in a certain order and the opening of the lock will be provided.

Consequently, the kinematic expression of the system formed according to the given example is,

$$\prod_{i=1}^{5} [R_i(\theta_i)]$$

In this situation, the matrices for $\forall i$ values are the rotation matrices matching to these motions.

$$[R_i(\theta_i)] \quad = \quad \begin{bmatrix} \cos\theta_i & \sin\theta_i \\ -\sin\theta_i & \cos\theta_i \end{bmatrix}$$

If the $[R_i(\theta_i)]$ matrices in the example are written accordingly,

Then the matrices,

$$[R_1(\theta_1)] = \left[R_1(-90^0)\right] = \begin{bmatrix} \cos(-90^0) & \sin(-90^0) \\ -\sin(-90^0) & \cos(-90^0) \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ +1 & 0 \end{bmatrix}$$

$$[R_2(\theta_2)] = \left[R_2(60^0)\right] = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$[R_3(\theta_3)] = \left[R_3(-150^0)\right] = \begin{bmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ +\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

$$[R_4(\theta_4)] = \left[R_4(210^0)\right] = \begin{bmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

$$[R_5(\theta_5)] = R_5(-60^0) = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

are obtained.

The matrix description of a safe lock with mechanic ciphered that we will utilize and describe the programming and the ciphering in electronic medium and that shows the consecutive rotations can be shown as the following.

Description 2.1. Imagine $f : I \subset \mathbb{R} \to \mathbb{R}$ showing a given function.

$$[A] = \begin{bmatrix} \cos((-1)^{(|f(x)|]}.\theta(x)) & \sin((-1)^{[|f(x)|]}.\theta(x)) \\ \sin((-1)^{[|f(x)|]}.\theta(x)) & \cos((-1)^{(|f(x)|]}.\theta(x)) \end{bmatrix}$$

Then the $[A]$ matrix above is called as code matrix.

Example 2.2. The rotation motion series with five studied above can be shown with the identified code matrix as the following:

If $f : \mathbb{R} \to \mathbb{R}$, then imagine

$$f(x) = \begin{cases} 1, & x = 0 \\ 2, & x = \frac{1}{2} \\ 3, & x = 1 \\ 4, & x = 2 \\ 1, & x = \frac{1}{4} \end{cases}$$

and $[A]$,

$$[A] = \begin{bmatrix} \cos((-1)^{(|f(x)|]}.2\pi x) & \sin((-1)^{[|f(x)|]}.2\pi x) \\ \sin((-1)^{[|f(x)|]}.2\pi x) & \cos((-1)^{(|f(x)|]}.2\pi x) \end{bmatrix}$$

$$[A_1] = \begin{bmatrix} \cos 0^0 & \sin 0^0 \\ -\sin 0^0 & \cos 0^0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[A_2] = \begin{bmatrix} \cos \pi & \sin \pi \\ -\sin \pi & \cos \pi \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$[A_3] = \begin{bmatrix} \cos(-2\pi) & \sin(-2\pi) \\ -\sin(-2\pi) & \cos(-2\pi) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[A_4] = \begin{bmatrix} \cos(4\pi) & \sin(4\pi) \\ -\sin(4\pi) & \cos(4\pi) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[A_5] = \begin{bmatrix} \cos(-\pi/2) & \sin(-\pi/2) \\ -\sin(-\pi/2) & \cos(-\pi/2) \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

is found.

The example we handled since the aim is to explain the duty of $f(x)$ was especially chosen.

Now according to the selection of $f(x)$ function type the conditions of the code matrix will be carried out.

The form in terms of the components matching the code matrix bound to x and $f(x)$, research will be done for the special functions, accordingly, primarily by composing a general condition.

TABLE 1. A general study of code matrix.

x	x_1	x_2	x_3	x_4
$f(x)$	0	1	2	3
$[\|f(x)\|]$	0	1	2	3
$\theta \rightarrow (-1)^{[\|f(x)\|]}.2\pi x$	0	-2πx_2	-2πx_3	-2πx_4
$\cos\theta$	1	$\cos(-2\pi x_2)$	$\cos(-2\pi x_3)$	$\cos(-2\pi x_4)$
$\sin\theta$	0	$\sin(-2\pi x_2)$	$\sin(-2\pi x_3)$	sin(-2πx_4)

$[A] = \quad \ldots [A_3] . [A_2] . [A_1]$ rotation matrices can be studied.

2.1. $f(x) = ax$ **linear function condition.**

TABLE 2. Linear function condition of code matrix.

x	0	1/a	2/a	3/a
$f(x)$	0	1	2	3
$[\|f(x)\|]$	0	1	2	3
$\theta \rightarrow (-1)^{[\|f(x)\|]}.2\pi x$	0	$-2\pi/a$	$4\pi/a$	$-6\pi/a$
$\cos\theta$	1	$\cos(-2\pi/a)$	$\cos(4\pi/a)$	$\cos(-6\pi/a)$
$\sin\theta$	0	$\sin(-2\pi/a)$	$\sin(4\pi/a)$	$\sin(-6\pi/a)$

Imagine $a = 4$ special choose to be applied on $f(x) = ax$ general condition. That is, for $f(x) = 4x$ function, the condition above is studied as the following.

2.2. $f(x) = ax^2$, $(a \neq 1)$, **square function condition.**

(the positive values of x will be studied.)

TABLE 3. Linear function $a = 4$ special condition of the code matrix.

x	0	1/4	2/4	3/4
$f(x)$	0	1	2	3
$[\|f(x)\|]$	0	1	2	3
$\theta \to (-1)^{[\|f(x)\|]}.2\pi x$	0	$-\pi/2$	π	$-3\pi/2$
$\cos\theta$	1	$\cos(-\pi/2)$	$\cos(\pi)$	$\cos(-3\pi/2)$
$\sin\theta$	0	$\sin(-\pi/2)$	$\sin(\pi)$	$\sin(-3\pi/2)$

TABLE 4. Square function condition of the code matrix.

x	0	$\sqrt{\frac{1}{a}}$	$\sqrt{\frac{2}{a}}$	$\sqrt{\frac{3}{a}}$
$f(x)$	0	1	2	3
$[\|f(x)\|]$	0	1	2	3
$\theta = (-1)^{[\|f(x)\|]}.2\pi x$	0	$-2\sqrt{1/a}\pi$	$+2\sqrt{\frac{2}{a}}\pi$	$-2\sqrt{\frac{3}{a}}\pi$
$\cos\theta$	1	$\cos(-2\sqrt{1/a}\pi)$	$\cos(2\sqrt{\frac{2}{a}}\pi)$	$\cos(2\sqrt{\frac{3}{a}})\pi$
$\sin\theta$	0	$\sin(-2\sqrt{\frac{1}{a}}\pi)$	$\sin(2\sqrt{\frac{2}{a}}\pi)$	$\sin(2\sqrt{\frac{3}{a}})\pi$

Accordingly, if it is studied for $f(x) = 4x^2$ special condition,

TABLE 5. Square function special condition of the code matrix.

x	0	1/2	$\sqrt{1/2}$	$\sqrt{3}/2$
$f(x)$	0	1	2	3
$[\|f(x)\|]$	0	1	2	3
$\theta = (-1)^{[\|f(x)\|]}.2\pi x$	0	$-\pi$	$+\sqrt{2}\pi$	$-\sqrt{3}\pi$
$\cos\theta$	1	$\cos(-\pi)$	$\cos(\sqrt{2}\pi)$	$\cos(-\sqrt{3}\pi)$
$\sin\theta$	0	$\sin(-\pi)$	$\sin(\sqrt{2}\pi)$	$\sin(-\sqrt{3}\pi)$

2.3. $f(x) = ax^3$ **cubic function condition**

In this case, if it is studied for $f(x) = 8x^3$ special condition, the following results will be obtained:

Table 6. Cubic function condition of the code matrix.

x	0	$\sqrt[3]{\frac{1}{a}}$	$\sqrt[3]{\frac{2}{a}}$	$\sqrt[3]{\frac{3}{a}}$
$f(x)$	0	1	2	3
$[\lvert f(x)\rvert]$	0	1	2	3
$\theta =(\text{-}1)^{[\lvert f(x)\rvert]}.2\pi x$	0	$-2\sqrt[3]{\frac{1}{a}}\pi$	$+2\sqrt[3]{\frac{2}{a}}\pi$	$-2\sqrt[3]{\frac{3}{a}}\pi$
$\cos\theta$	1	$\cos(-2\sqrt[3]{\frac{1}{a}}\pi)$	$\cos(+2\sqrt[3]{\frac{2}{a}}\pi)$	$\cos(-2\sqrt[3]{\frac{3}{a}}\pi)$
$\sin\theta$	0	$\sin(-2\sqrt[3]{\frac{1}{a}}\pi)$	$\sin(+2\sqrt[3]{\frac{2}{a}}\pi)$	$\sin(-2\sqrt[3]{\frac{3}{a}}\pi)$

Table 7. Cubic function special condition of the code matrix.

x	0	$\frac{1}{2}$	$\frac{1}{\sqrt[3]{4}}$	$\frac{\sqrt[3]{3}}{2}$
$f(x)$	0	1	2	3
$[\lvert f(x)\rvert]$	0	1	2	3
$\theta =(\text{-}1)^{[\lvert f(x)\rvert]}.2\pi x$	0	$(-\pi)$	$\left(2/\sqrt[3]{4}\right)\pi$	$\left(-2\frac{\sqrt[3]{3}}{2}\pi\right)$
$\cos\theta$	1	$\cos(-\pi)$	$\cos\left(\frac{2}{\sqrt[3]{4}}\pi\right)$	$\cos\left(-\frac{\sqrt[3]{3}}{2}\pi\right)$
$\sin\theta$	0	$\sin(-\pi)$	$\sin\left(\frac{2}{\sqrt[3]{4}}\pi\right)$	$\sin\left(-\frac{\sqrt[3]{3}}{2}\pi\right)$

2.4. $f(x) = a^x$, $(a \neq 1)$ **exponential function condition**

Table 8. Exponential function condition of the code matrix.

x		0	$\log_a 2$	$\log_a 3$
$f(x)$	0	1	2	3
$[\lvert f(x)\rvert]$	0	1	2	3
$\theta = (-1)^{[\lvert f(x)\rvert]}.\ 2\pi x$	0	0	$+2\log_a 2\pi$	$(\ -2\log_a 3\pi)$
$\cos\theta$	1	1	$\cos(2\log_a 2\pi)$	$\cos(\ -2\log_a 3\pi)$
$\sin\theta$	0	0	$\sin(2\log_a 2\pi)$	$\sin(\ -2\log_a 3\pi)$

$$f(x) = a^x \Rightarrow \log_a(f(x)) = x$$
$$a^x = 2 \Rightarrow \log_a 2 = x$$

Let's study $f(x) = 2^x$ exponential function as a special condition:

3. CONCLUSION

In this study, combination lock mechanisms, being one of the lock mechanism kinds, have been studied in kinematic point of view. The fundamental logic

TABLE 9. Exponential function special condition of the code matrix.

x		0	1	$\log_2 3$
$f(x)$	0	1	2	3
$[\|f(x)\|]$	0	1	2	3
$\theta = (-1)^{[\|f(x)\|]}.\ 2\pi x$	0	0	$(+2\pi)$	$(-2\log_2 3)\,\pi$
$\cos\theta$	1	1	$\cos(2\pi)$	$\cos(\ -2\log_2 3\pi)$
$\sin\theta$	0	0	$\sin(2\pi)$	$\sin(-2\log_2 3\pi)$

in ciphered lock mechanisms has been recognized to be a rotation motion, by completing a rotation motion which is itself a planar motion. The mechanism of these systems based on consecutive rotation has been studied on a dial and thus the rotational motions have been expressed in matrices. The code matrix belonging to this motion system being a base for the programming and coding in mechanical and electronical condition. Linear, quadratic, cubic, exponential function conditions of the code matrix have been studied and that different motion systems appear in different function types has observed. In these functions, the rotational motion results obtained with different angles were shown on the tables and explained.

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