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A Structure Theorem for Right[1] Adequate Semigroups of Type F

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Abstract

A right adequate semigroup of type F means a right adequate semigroup which is an F-rpp semigroup. We obtain the structure theorem for right adequate semigroups: a semigroup is a right adequate semigroup of type F if and only if it is isomorphic to some $\mathcal{F}(M, Y)$, where (M, Y) is an F-pair. As its applications, we establish a structure for adequate semigroups of type F. Our result extends the results on F-inverse semigroups.

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1 Introduction

An inverse semigroup S is called *F-inverse* if there exists a group congruence σ on S such that each σ-class has greatest element with respect to the natural

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partial order $\leq$ on S. McFadden and O'Carroll [8] have pointed out that the concept of F-inverse semigroups is indeed a generalization of residuated inverse semigroups. Later on, Edwards [1] defined analogously F-regular semigroups and F-orthodox semigroups and showed that an F-regular semigroup is indeed an F-orthodox semigroup.

A semigroup is called *rpp* if for any $a \in S$, aS^1, regarded as an S-system, is projective. Dually, lpp semigroups may be defined. In [5], Fountain has pointed out a semigroup S is rpp if and only if every $\mathcal{L}^*$-class of S contains at least one idempotent. A rpp semigroup S is said to be a *right adequate semigroup* if the set of idempotents of S forms a commutative subsemigroup, that is a semilattice. A semigroup is called *abundant* if and only if it is both rpp and lpp. Regular semigroups are abundant semigroups and inverse semigroups are right adequate semigroups.

In order to generalize the F-regular semigroups, Guo [11] defined F-abundant semigroup. So-called an *F-abundant semigroup* is an abundant semigroup in which there exists a cancellative congruence σ such that each σ-class contains a greatest element with respect to the Lawson order $\leq$. In the same reference, Guo established the structure of a class of F-abundant semigroups namely strongly F-abundant semigroups by utilizing an SF-system. In [9], Ni, Chen and the second author obtained a structure of general F-abundant semigroups.

Parallelizing F-inverse semigroups, Li, Shum and the second author [12] defined F-rpp semigroups. We call an rpp semigroup S an *F-rpp semigroup* if there exists a left cancellative monoid congruence ρ on S such that each $\rho - class$ contains a greatest element with respect to the Lawson partial order $\leq_\ell$ on S (for the Lawson orders, see [7]). By introducing SFR-systems, they established a structure for strongly F-rpp semigroups. Recently, Huang, Chen and the second author [3] obtained the structure of general F-rpp semigroups.

In this paper, we shall research right adequate semigroups which are F-rpp, called *right adequate semigroups of type* F throughout this paper. By using a similar method in [10] and [2], we establish a structure of right adequate semigroups of type F.

2 Preliminaries

Throughout this paper we use the notions and terminologies of Fountain [5] and Howie [6]. Now, we provide some known results repeatedly used without mentions in the sequel.

Lemma 2.1 [5] *Let S be a semigroup and $a, b \in S$. Then the following statements are equivalent:*

(1) $a\mathcal{L}^*b$.

(2) *For all $x, y \in S^1$, $ax = ay$ if and only if $bx = by$.*

Evidently, $\mathcal{L}^*$ is a right congruence while $\mathcal{R}^*$ is a left congruence. In general, we have $\mathcal{L} \subseteq \mathcal{L}^*$ and $\mathcal{R} \subseteq \mathcal{R}^*$. But if a, b are regular elements, $a\mathcal{L}^*b$ $[a\mathcal{R}^*b]$ if and only if $a\mathcal{L}b$ $[a\mathcal{R}b]$. For the sake of convenience, we use a^* to denote a typical idempotent $\mathcal{L}^*$-related to a, and $a^\dagger$ to denote that $\mathcal{R}^*$-related to a.

Proposition 2.2 [5] *If S is a right adequate semigroup with semilattice of idempotents E, then*

(1) *For all $a, b \in S, (ab)^* = (a^*b)^*$.*
(2) *For all $a, b \in S, (ab)^*\omega b^*$.*

Let S be a rpp semigroup. As in [7], we define a relation S by

$x \leq_\ell y$ if and only if $L^*(x) \subseteq L^*(y)$ and there exists $f \in E(S) \bigcap L^*_x$ such that $x = yf$,

where $L^*(x)$ is the left *-ideal generated by x (see [4]) and L^*_x denotes the $\mathcal{L}^*$-class of S containing x. Then $\leq_\ell$ is a partial order on S. Dually, we may define the partial order $\leq_r$ on an lpp semigroup. If S is an abundant semigroup, we define the partial order $\leq$ on S as $\leq_\ell \cap \leq_r$. In equivalently, for $a, b \in S$, $a \leq b$ if and only if there exist $e, f \in E(S)$ such that $a = eb = bf$ (see [7]).

Lemma 2.3 [7] *Let S be a rpp semigroup. For $x, y \in S$ and $e \in E(S)$. Then the following statements holds:*

(1) *$\leq_\ell$ is a partial order on S, in particular, $\leq_\ell$ coincide with the usual idempotent order ω on $E(S)$, that is, $e\omega f$ if and only if $e = ef = fe$.*
(2) *If $x \leq_\ell e$, then $x^2 = x$ in S.*
(3) *If $x \leq_\ell y$ and y is a regular element in S, then x is also a regular element in S.*
(4) *Let $y^* \in L^*_y \bigcap E(S)$ and $\omega(y^*) = \{f : f\omega y^*\}$. Then $x \leq_\ell y$ if and only if for all [some] y^*, there exists $f \in \omega(y^*)$ such that $x = yf$.*
(5) *If $x \leq_\ell y$ and $x\mathcal{L}^*y$, then $x = y$.*

A congruence ρ on a semigroup S is called a *left cancellative monoid congruence* on S if S/ρ is a left cancellative monoid.

By an *F-rpp semigroup*, we mean a rpp semigroup in which there exists a left cancellative monoid congruence σ on S such that each σ-class of S contains a greatest element with respect to the Lawson order $\leq_\ell$. In this case, the σ is indeed the smallest left cancellative monoid congruence on S (see [12]). In what follows, we use σ to denote the smallest left cancellative monoid congruence on S if have.

Assume that S is an F-rpp semigroup. We denote by M the set of greatest elements in all σ-classes of S. In general, M does not form a subsemigroup of S. Now define a multiplication $\circ$ as follows:

$m \circ n =$ the greatest element of the σ-class of S containing mn.

It is not difficult to check that $(M, \circ)$ is a semigroup isomorphic to S/σ. This fact will be used in the proof of Theorem 3.4.

Lemma 2.4 *If S is a right adequate semigroup of type F, then*

(1) *S is a right type A semigroup (that is, a right adequate semigroup in which for any $a, e^2 = e \in S$, $ea = a(ea)^*$).*

(2) *The smallest left cancellative monoid congruence σ on S is equal to $\{(a,b) \in S \times S : (\exists e^2 = e \in S)\ ae = be\}$.*

(3) *$\sigma \cap \mathcal{L}^* = id$ (the identity relation on S).*

Proof. (1) Let $a \in S, e \in E(S)$. Since $a = aa^*$, we have $ea = (ea)a^*$ and $(ea)^* = (ea)^*a^*$. On the other hand, since $ea \in \sigma_a$ and by Lemma 2.3 (4), we have

$$ea = m_a(ea)^* = m_a(ea)^*a^* = m_a a^*(ea)^* = a(ea)^*,$$

where m_a denotes the greatest element in σ_a. Thus S is a right type-A semigroup.

(2) The proof follows from [14, Lemma 2.1].

(3) If $(a,b) \in \sigma \cap \mathcal{L}^*$, then since $(a,b) \in \sigma$ and by Lemma 2.3, $a = m_a a^*$ and $b = m_a b^*$. By $a\mathcal{L}^*b$, $a^* = b^*$. Thus $a = m_a b^* = b$ and whence $\sigma \cap \mathcal{L}^* = id$.
□

3 The structure theorem

The aim of this section is to establish the structure theorem for right adequate semigroups of type F.

Definition 3.1 *Let M be a left cancellative monoid with identity 1 and Y be a semilattice with identity i. Let M have an action on Y. We call such a pair (M, Y) an **F-pair** if the act of M on Y satisfying the following conditions: for all $m, n \in M, \alpha, \beta \in Y$,*

$(F1)$ $\alpha 1 = \alpha$.

$(F2)$ $(\alpha \wedge \beta)m = \alpha m \wedge \beta m$.

Given a (M, Y), form the set

$$\mathcal{F}(M, Y) = \{(m, \alpha) \in M \times Y \mid \alpha \leq im\}.$$

On $\mathcal{F}(M, Y)$, define a multiplication by

$$(m, \alpha) \star (n, \beta) = (mn, \alpha n \wedge \beta).$$

Then $\star$ is well defined since $\alpha n \wedge \beta \leq imn \wedge in \leq imn$.

Lemma 3.2 *$(\mathcal{F}(M,Y),\star)$ is a semigroup.*

Proof. Let $(m,\alpha),(n,\beta),(t,\gamma)\in(\mathcal{F}(M,Y),\star)$. Compute

$$\begin{aligned}[(m,\alpha)\star(n,\beta)]\star(t,\gamma) &= (mn,\alpha n\wedge\beta)\star(t,\gamma)\\ &= (mnt,\alpha nt\wedge\beta t\wedge\gamma)\\ &= (m,\alpha)\star(nt,\beta t\wedge\gamma)=(m,\alpha)\star[(n,\beta)\star(t,\gamma)].\end{aligned}$$

Then $\star$ is associative. Thus $(\mathcal{F}(M,Y),\star)$ is a semigroup. □

Proposition 3.3 *In the above monoid $(\mathcal{F}(M,Y),\star)$,*

(1) $E(\mathcal{F}(M,Y))=\{(1,\alpha):\alpha\in Y\}$ and is isomorphic to Y. Moreover, $E(\mathcal{F}(M,Y))$ is a semilattice.

(2) For elements $(m,\alpha),(n,\beta)\in\mathcal{F}(M,Y)$, $(m,\alpha)\mathcal{L}^*(n,\beta)$ if and only if $\alpha=\beta$.

(3) For elements $(m,\alpha),(n,\beta)\in\mathcal{F}(M,Y)$, $(m,\alpha)\sigma(n,\beta)$ if and only if $m=n$. Thus $\mathcal{F}(M,Y)/\sigma\cong M$.

(4) For elements $(m,\alpha),(n,\beta)\in\mathcal{F}(M,Y)$, $(m,\alpha)\leq_\ell(n,\beta)$ if and only if $m=n$ and $\alpha\leq\beta$.

(5) $\mathcal{F}(M,Y)$ is a right type A semigroup of type F.

Proof. (1) Let $(m,\alpha)\in E(\mathcal{F}(M,Y))$. Then $(m,\alpha)^2=(m^2,\alpha m\wedge\alpha)=(m,\alpha)$ and so $m^2=m$. It follows that $m=1$ since M is a left cancellative monoid. Conversely, since $(1,\alpha)^2=(1^2,\alpha 1\wedge\alpha)=(1,\alpha)$, we have $(1,\alpha)$ is an idempotent. Thus $E(\mathcal{F}(M,Y))=\{(1,\alpha):\alpha\in Y\}$. On the other hand, it is easy to know that the mapping

$$\theta:E(\mathcal{F}(M,Y))\to Y;\ (1,\alpha)\mapsto\alpha$$

is an isomorphism. Thus $E(\mathcal{F}(M,Y)$ is a semilattice.

(2) If $(t,\gamma),(z,\delta)\in\mathcal{F}(M,Y)$ and $(m,\alpha)(t,\gamma)=(m,\alpha)(z,\delta)$, that is, $(mt,\alpha t\wedge\gamma)=(mz,\alpha z\wedge\delta)$, so that $mt=mz$, and hence $t=z$ since M is a left cancellative monoid, thus $(1,\alpha)(t,\gamma)=(1,\alpha)(z,\delta)$ and therefore $(m,\alpha)\mathcal{L}^*(1,\alpha)$ by noticing $(m,\alpha)(1,\alpha)=(m,\alpha)$.

By the above proof, we have

$$\begin{aligned}(m,\alpha)\mathcal{L}^*(n,\beta)\quad&\text{if and only if } (1,\alpha)\mathcal{L}^*(1,\beta);\\ &\text{if and only if } \alpha\mathcal{L}^*\beta;\\ &\text{if and only if } \alpha=\beta.\end{aligned}$$

(3) By (2), $(\mathcal{F}(M,Y),\star)$ is a right adequate monoid. If $(m,\alpha)\in\mathcal{F}(M,Y)$ and $(1,\beta)\in E(\mathcal{F}(M,Y))$, then

$$\begin{aligned}(1,\beta)(m,\alpha) &= (m,\beta m\wedge\alpha)=(m,\alpha)(1,\beta m\wedge\alpha)\\ &= (m,\alpha)[(1,\beta)(m,\alpha)]^*\end{aligned}$$

and whence $(\mathcal{F}(M,Y),\star)$ is a right type-A monoid.

If $(m,\alpha)\sigma(n,\beta)$, then by Lemma 2.4 (3), $(m,\alpha)(1,\gamma)=(n,\beta)(1,\gamma)$ for some $\gamma\in Y$, so that $m=n$. Conversely, if $(m,\alpha),(m,\beta)\in\mathcal{F}(M,Y)$, then $\alpha\wedge\beta\in Y$ and $(m,\alpha)(1,\alpha\wedge\beta)=(m,\beta)(1,\alpha\wedge\beta)$. This means that $(m,\alpha)\sigma(n,\beta)$. We have now proved that $(m,\alpha)\sigma(n,\beta)$ if and only if $m=n$.

The rest follows from the fact that the mapping $\phi:\mathcal{F}(M,Y)/\sigma\to M$ defined by $\phi(m,\alpha)=m$ is an isomorphism.

(4) If $(m,\alpha)\leq_\ell(n,\beta)$, then there exists $(1,\gamma)$ such that $(m,\alpha)=(n,\beta)(1,\gamma)$, and so by comparing components, $m=n$ and $\alpha=\beta\gamma$, i.e., $\alpha\leq\beta$. Conversely, if $m=n$ and $\alpha\leq\beta$, then $(m,\alpha)=(m,\alpha\beta)=(n,\beta)(1,\alpha)$ and $(m,\alpha)\leq_\ell(n,\beta)$ by noting $(1,\alpha)\leq(1,\beta)=(n,\beta)^*$. This proves (4).

(5) By (3), $\sigma_{(m,\alpha)}=\{(m,\alpha)\in\mathcal{F}(M,Y):\alpha\in Y\}$ and further by (4), (m,im) is the greatest element in $\sigma_{(m,\alpha)}$ with respect to $\leq_\ell$. Thus $\mathcal{F}(M,Y)$ is a right type A semigroup of type F. □

We arrive now at the structure theorem for right adequate semigroups of type F.

Theorem 3.4 *If (M,Y) is an F-pair, then the semigroup $\mathcal{F}(M,Y)$ is a right adequate semigroup of type F. Conversely, any right adequate semigroup of type F can be constructed in this way.*

Proof. By Proposition 3.3, we only need to prove the converse part. For this purpose, we suppose that S is a right adequate semigroup of type F and E is the semilattice of idempotents of S. Then E is a semilattice with identity i. We denote by M the set of greatest elements in all σ-classes of S. Now define a multiplication $\circ$ as follows:

$$m\circ n=\text{the greatest element of the }\sigma\text{-class of }S\text{ containing }mn.$$

It is not difficult to check that $(M,\circ)$ is a semigroup isomorphic to S/σ. Hence M is a left cancellative monoid. Let 1 be the identity of M. It is easy to see that $1=i$. Also, we define an action of M on Y by $\alpha m=(\alpha m)^*$ for $\alpha\in E$ and $m\in M$.

Next, we shall prove that the pair (M,E) is indeed an F-pair. Now let $m,n\in M,\alpha,\beta\in E$.

- $\alpha 1=(\alpha i)^*=\alpha$. This means that $(F1)$ holds.
- Compute
$$\begin{aligned}(\alpha\wedge\beta)m&=((\alpha\wedge\beta)m)^*=(\alpha\beta m)^*\\&=(\alpha m(\beta m)^*)^*=(\alpha m)^*(\beta m)^*\\&=\alpha m\wedge\beta m.\end{aligned}$$

  Thus $(F2)$ holds.

We have now proved that the pair (M, E) is an F-pair.

It remains to verify that S is isomorphic to $\mathcal{F}(M, E)$. For this, we consider the mapping

$$\theta: S \to \mathcal{F}(M, E);\ s \mapsto (m_s, s^*),$$

where m_s is the greatest element of the σ-class containing s with respect to $\leq_\ell$. Obviously, θ is well defined.

- If $\theta(s) = \theta(t)$, then $m_s = m_t$ and $s^* = t^*$, hence $(s, t) \in \sigma \cap \mathcal{L}^*$, thus by Lemma 2.4, $s = t$. Consequently, θ is injective.

- Since $\theta(m\alpha) = (m, \alpha)$ for any $(m, \alpha) \in \mathcal{F}(M, E)$, θ is surjective.

- For $s, t \in S$, we have

$$\begin{aligned}\theta(s)\theta(t) &= (m_s, s^*)(m_t, t^*) = (m_s \circ m_t, (s^* m_t)^* \wedge t^*) \\ &= (m_{st}, (s^* m_t t^*)^*) = (m_{st}, (s^* t)^*) = (m_{st}, (st)^*) \\ &= \theta(st)\end{aligned}$$

  and whence θ is a homomorphism.

Up to now, we complete the proof. □

4 A special case: F-adequate semigroups

Recall from [11] that an *F-abundant semigroup* is defined as an abundant semigroup in which each σ-class contains a greatest element with respect to the Lawson order $\leq$.

Lemma 4.1 (1) *Let S be an adequate semigroup. If S is an F-adequate semigroup, then S is a type-A semigroup.*

(2) *If S is a type-A semigroup, then the smallest cancellative monoid congruence is equal to $\{(a, b) \in S \times S : (\exists e^2 = e \in S)\ ae = be\}$.*

(3) *An adequate semigroup is type-A if and only if $\leq_\ell = \leq_r$.*

Proof. (1): follows from [11, Proposition 2.2].

(2): follows from [14, Lemma 2.1].

(3): follows from [7, Theorem 2.6]. □

By Lemma 4.1, the following proposition is immediate.

Proposition 4.2 *Let S be a type-A semigroup. Then S is an F-adequate semigroup if and only if S is an F-rpp [F-lpp] semigroup.*

As in [13], an abundant semigroup S is called *left IC* if each $a \in S$ and for some $a^\dagger, a^*$, there exists a maping $\theta : \langle a^* \rangle \to \langle a^\dagger \rangle$ such that $ax = \theta(x)a$ for any $x \in \langle a^* \rangle$, where $\langle a^* \rangle$ ($\langle a^\dagger \rangle$) is the subsemigroup of S generated by the idempotents in the subsemigroup a^*Sa^* ($a^\dagger S a^\dagger$). Dually, right IC abundant semigroups can be defined.

Lemma 4.3 (1) *An abundant semigroup is left IC if and only if* $\leq_\ell = \leq$.

(2) *An adequate semigroup is a right type-A semigroup if and only if it is left IC.*

Proof. (1): is [13, Proposition 5.3].

(2): follows from [13, Proposition 5.2]. □

By Lemma 4.3, we have

Proposition 4.4 *A semigroup is an F-adequate semigroup if and only if it is an abundant semigroup which is a right adequate semigroup of type F.*

Proof. We need only to prove the converse part. Now, we suppose that an abundant semigroup S is a right adequate semigroup of type F. Then by Lemma 2.4, S is a right type-A semigroup and further an adequate semigroup. By Lemma 4.3, this implies $\leq_\ell = \leq$.

Now, to prove that S is an F-adequate semigroup, it suffices to show that the smallest left cancellative monoid congruence σ on S is a cancellative congruence. For this, we need only to verify that σ is right cancellative. Now let $a, b, c \in S$ and $(ba)\sigma = (ca)\sigma$. Then $ba = m_{ba}(ba)^*$ and $ca = m_{ba}(ca)^*$. On the other hand, since S is abundant, $ba = gm_{ba}$ for some $g \in E(S)$. Thus

$$(gb)a(ca)^* = gm_{ba}(ca)^* = gca(ca)^*$$

so that $gb(a(ca)^*)^\dagger = gc(a(ca)^*)^\dagger$ since $a(ca)^* \mathcal{R}^* (a(ca)^*)^\dagger$. Notice that $e\sigma$ is the identity of S/σ for all $e \in E(S)$, thus

$$b\sigma = (gb(a(ca)^*)^\dagger)\sigma = (gc(a(ca)^*)^\dagger)\sigma = c\sigma$$

and whence σ is a right cancellative congruence on S, as required. □

Lemma 4.5 *Let (M, Y) be an F-pair. If M is a cancellative monoid, then $\mathcal{F}(M, Y)$ is lpp if and only if for all $(m, \alpha) \in \mathcal{F}(M, Y)$, there exists $[m, \alpha] \in Y$ such that*

(L1) $\alpha \leq [m, \alpha]m$.

(L2) For all $\beta, \gamma \in Y$, $\beta m \wedge \alpha = \gamma m \wedge \alpha$ implies that $\beta \wedge [m, \alpha] = \gamma \wedge [m, \alpha]$.

Proof. Suppose that $\mathcal{F}(M,Y)$ is lpp. Let $(m,\alpha) \in \mathcal{F}(M,Y)$, there exists $[m,\alpha] \in Y$ such that $(1,[m,\alpha])\mathcal{R}^*(m,\alpha)$. Then $(1,[m,\alpha])(m,\alpha) = (m,\alpha)$ and so by comparing the components, $[m,\alpha]m \wedge \alpha = \alpha$. This means that $(L1)$ holds.

To verify $(L2)$, we let $\beta m \wedge \alpha = \gamma m \wedge \alpha$. Then $(1,\beta)(m,\alpha) = (1,\gamma)(m,\alpha)$ and by $(1,[m,\alpha])\mathcal{R}^*(m,\alpha)$, we have $(1,\beta)(1,[m,\alpha]) = (1,\gamma)(1,[m,\alpha])$, so that $\beta \wedge [m,\alpha] = \gamma \wedge [m,\alpha]$ by comparing components. Thus $(L2)$ holds.

The converse is clear by working backward through the calculations. □

By using Theorem 3.4, Proposition 4.4 and Lemma 4.5, we immediately obtain the structure theorem for F-adequate semigroups.

Theorem 4.6 *Let (M,Y) be an F-pair. If for all $(m,\alpha) \in \mathcal{F}(M,Y)$, there exists $[m,\alpha] \in Y$ such that*

$(L1)$ $\alpha \leq [m,\alpha]m$;

$(L2)$ For all $\beta,\gamma \in Y$, $\beta m \wedge \alpha = \gamma m \wedge \alpha$ implies that $\beta \wedge [m,\alpha] = \gamma \wedge [m,\alpha]$,

then $\mathcal{F}(M,Y)$ is an F-adequate semigroup. Conversely, any F-adequate semigroup can be constructed in this way.

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