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Fuzzy Ideals of KU - Algebras

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Abstract

In this paper , we consider KU - ideals of KU- algebras and some fundamental properties to KU - algebra are discussed . The notion of fuzzy KU- ideals in KU - algebras are introduced , several appropriate examples are provided and their some properties are investigated . The image and the inverse image of fuzzy KU - ideals in KU - algebras are defined and how the image and the inverse image of fuzzy KU - ideals in KU - algebras become fuzzy KU - ideals are studied . Moreover , the cartesian product of fuzzy KU - ideals in cartesian product KU – algebras are given .

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1. Introduction

BCK - algebras form an important class of logical algebras introduced by K.Iseki and was extensively investigated by several researchers . The class of all BCK - algebras is quasi variety. k.Iseki posed an interesting problem (solved in[21]) whether the class of all BCK - algebras is a variety. In connection with this problem , Y.Komori introduced in [14] a notion of BCC algebras and W.A.Dudek (cf.[2],[3]) redefined the notion of BCC - algebras by using a dual form of the ordinary definition in the sense of Y. Komori .

In [18] , C.Prabpayak and U.Leerawat studied ideals and congruences of BCC – algebras ([8], [9]) and introduced a new algebraic structure which is called KU - algebra . They gave the concept of homomorphisms of KU - algebras and investigated some related properties .

L .A. Zadeh [23] introduced the notion of fuzzy sets . At present this concept has been applied to many mathematical branches , such as group , functional analysis , probability theory , topology, and so on . In 1991 , O. G. Xi [22] applied this concept to BCK - algebras , and he introduced the notion of fuzzy sub - algebras (ideals) of the BCK - algebras with respect to minimum , and since then Y.B. Jun et al studied fuzzy ideals (cf.[10] , [11] , [12] ,[13] ,[17]) , and moreover several fuzzy structures in BCC-algebras are considered (cf .[5] , [6] , [8] , [9]) .

In this paper , we introduce the notion of fuzzy KU - ideals of KU - algebras and then we investigate several basic properties which are related to fuzzy KU - ideals . we describe how to deal with the homomorphic image and inverse image of fuzzy KU - ideals . we have also prove that the cartesian product of fuzzy KU - ideals in cartesian product of fuzzy KU - algebras are fuzzy KU - ideals .

2. Preliminaries

By an KU - algebra we mean an algebra $(X , * , 0)$ of type $(2 , 0)$ with a single binary operation $*$ that satisfies the following identities : for any $x , y , z \in X$,

(ku_1) : $(x * y) * [(y * z) * (x * z)] = 0$,

(ku_2) : $x * 0 = 0$,

(ku_3) : $0 * x = x$,

(ku_4) : $x * y = 0 = y * x$ implies $x = y$.

In what follows, let $(X , * , 0)$ denote an KU - algebra unless otherwise specified . For brevity we also call X a KU - algebra . In X we can define a binary relation $\leq$ by :

$x \leq y$ if and only if $y * x = 0$.

Then $(X , * , 0)$ is a KU - algebra if and only if it satisfies that :

$(k`u_1)$: $(y * z) * (x * z) \leq (x * y)$,

$(k`u_2)$: $0 \leq x$,

(k\`u$_3$) : $x \leq y$, $y \leq x$ implies $x = y$,
(k\`u$_4$) : $x \leq y$ if and only if $y * x = 0$.

In an KU- algebra , the following identities are true : If we put in (ku$_1$) $y = x = 0$ we get $(0 * 0) * [(0 * z) * (0 * z)] = 0$, and it follows that : (KU$_5$) $z * z = 0$, and if we put $y = 0$ in (ku$_1$) , we get (p$_1$) $z * (x * z) = 0$.

Example 2.1. Let X = {0 , 1 , 2 , 3 , 4} in which * is defined by the following table :

*	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	1	0	3	3
3	0	0	2	0	2
4	0	0	0	0	0

It is easy to show that X is KU - algebra .

Definition 2.2 [19]. A subset S of KU - algebra X is called sub-algebra of X if $x * y \in S$, whenever $x , y \in S$.

Definition 2.3 [19]. A non - empty subset A of a KU-algebra X is called an KU ideal of X if it satisfies the following conditions :

(1) $0 \in A$,
(2) $x * (y * z) \in A$, $y \in A$ implies $x * z \in A$, for all $x , y , z \in X$.

Example 2.4 . Let X ={0 ,a,b,c,d,e}in which * is defined by the following table

*	0	a	b	c	d	e
0	0	a	b	c	d	e
a	0	0	b	b	d	e
b	0	0	0	a	d	e
c	0	0	0	0	d	e
d	0	0	0	a	0	e
e	0	0	0	0	0	0

Then (X , * , 0) is KU - algebra . It is easy to show that A_1 ={0 , a}, and A_2 ={0, a , b , c , d} are KU - ideals of X .

Lemma 2.5 . In a KU - algebra (X , * , 0) , the following hold :

$x \leq y$ imply $y * z \leq x * z$.

Proof . Since $x \leq y$ implies $y * x = 0$, by ku_1 , we obtain $(y * x)*[(x*z) * (y* z)] = 0$ but $y * x = 0$, then $0 * [(x * z) * (y * z)] = 0$,by ($ku_2$, ku_3),we get $(x * z) * (y * z) = 0$ i.e $y * z \leq x * z$.

Lemma 2.6. In KU - algebra X , we have

$$z * (y * x) = y * (z * x) \text{ , for all } x , y , z \in X$$

Proof . From (ku_1) we get $(0 * z)*[(z * x) * (0 * x)]= 0$, this implies $z*[(z * x)* x] = 0$

i.e $(z * x) * x \leq z$ ------------------- (a)

Making use of (a) and ($k`u_1$) , we get $z * (y * x) \leq [(z * x) * x] * (y * x) \leq y * (z * x)$ since x , y , z are arbitrary , interchanging y and z in the above inequality , we obtain $y * (z * x) \leq z * (y * x)$, By (ku_4) , we get $z * (y * x) = y * (z * x)$.

Lemma 2.7. If X is KU- algebra , then

$$y * [(y * x) * x] = 0 .$$

Proof . using lemma (2.6) , then $(y * x) * (y * x)= 0$.

Definition 2.8 [19]. Let $(X ,* ,0)$ and $(X`, *` , 0`)$ be KU – algebras a homomorphism is a map $f : X \to X`$ satisfying $f (x * y) = f (x) *` f (y)$ for all $x , y \in X$.

Theorem 2.9 [19] . Let f be a homomorphism of a KU - algebras X into a KU - algebra X`, then

(i) If 0 is the identity in X , then f (0) is the identity in X` .

(ii) If S is a KU - subalgebra of S , then f (S) is a KU- subalgrbra of X` .

(iii) If I is an KU- ideal of X , then f (I) is an KU- ideal in f (X) .

(vi) If S is a KU- subalgebra of f (X`) , then $f^{-1}(S)$ is a KU- algebra of X .

(v) If B is an KU - ideal in f (X) ,then $f^{-1}(B)$ is an KU- ideal in X .

(vi) If f is a homomorphism from KU- algebra X to a KU- algebra X` then f is one to one if and only if ker f = {0}

Proposition 2.10. Suppose $f : X \to X`$ is a homomorphism of KU - algebras , then

(1) $f (0) = 0`$,

(2) If $x \leq y$ implies $f (x) \leq f (y)$.

Proof . Since $x \leq y$ then $y * x = 0$, then $f (y * x) = f (y) *` f (x) = f(0)$ i.e $f (x) \leq f (y)$.

Proposition 2.11 . Let $(X , * , 0)$ and $(X`, * , 0)$ be KU - algebras and $f : X \to X`$ be a homomorphism , then ker f is KU- ideal of X .

Proof . $0 \in$ ker f , since $f (0) = 0`$. Let $x * (y * z) \in$ ker f , $y \in$ ker f , then

$f (x * (y * z)) = 0`$ , $f (y) = 0`$, since

$0` = f (x * (y * z) ) = f (x) *` f (y * z) = 0` = f (x) *` (f (y) *` f (z))$

$= f (y) *` (f (x) *` f (z))$,

(by lemma 2.6) together with $f (y) = 0`$, we get

$0` = (f (x) *` f (z)) = 0`$, this implies $f (x) *` f (z) = f (x * z) = 0`$.

i.e $x * z \in$ ker f , then ker f is an KU - ideal of X .

3. Fuzzy KU- ideals of kU-algebras

In this section , we will discuss and investigate a new notion called fuzzy KU - ideals of KU - algebras and study several basic properties which related to fuzzy KU - ideals .

Definition 3.1 [23] . let X be a set , a fuzzy set μ in X is a function $\mu : X \rightarrow [0,1]$.

Definition 3.2 . let X be a KU - algebra , a fuzzy set μ in X is called fuzzy sub-algebra if it satisfies:

(S_1) $\mu(0) \geq \mu(x)$,

(S_2) $\mu(x) \geq \{\mu(x * y) , \mu(y)\}$ for all x , y $\in$ X .

Definition 3.3 . let X be a KU-algebra , a fuzzy set μ in X is called a fuzzy KU-ideal of X if it satisfies the following conditions:

(F_1) $\mu(0) \geq \mu(x)$,

(F_2) $\mu(x * z) \geq \min\{\mu(x * (y * z)) , \mu(y)\}$.

Example 3.4 . Let X ={0 , 1 , 2 , 3 , 4 } in which $*$ is defined by the following table

*	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	3
2	0	1	0	1	4
3	0	0	0	0	3
4	0	0	0	0	0

Then (X , $*$, 0) is KU – algebra . Define a fuzzy set $\mu : X \rightarrow [0,1]$ by $\mu(0) = t_0$, $\mu(1) = \mu(2) = t_1$, $\mu(3) = \mu(4) = t_2$, where $t_0 , t_1 , t_2 \in [0,1]$ with $t_0 > t_1 > t_2$.

Routine calculation gives that μ is a fuzzy KU- ideal of KU- algebras X.

Lemma 3.5 . let μ be a fuzzy KU - ideal of KU - algebra X , if the inequality $x * y \leq z$ hold in X , Then $\mu(y) \geq \min\{\mu(x) , \mu(z)\}$.

Proof . Assume that the inequality $x * y \leq z$ holds in X , then $z * (x * y) = 0$ and by (F_2)$\mu(z * y) \geq \min\{\mu(z * (x * y)) , \mu(x)\}$, if we put z=0 then $\mu(0 * y) = \mu(y) \geq \min\{\mu(x * y) , \mu(x)\}$ (i).

but $\mu(x * y) \geq \min\{\mu(x * (z * y) , \mu(z)\}$

$= \min\{\mu(z * (x * y)) , \mu(z)\}$

$= \min\{\mu(0) , \mu(z)\} = \mu(z)$ (ii).

From (i) , (ii) , we get $\mu(y) \geq \min\{\mu(z) , \mu(x)\}$, this completes the proof .

Lemma 3.6 . If μ is a fuzzy KU - ideal of KU - algebra X and if $x \leq y$, then $\mu(x) \geq \mu(y)$.

Proof . if $x \leq y$,then $y * x = 0$, this together with $0 * x = x$ and $\mu(0) \geq \mu(y)$,we get

$$\mu(0 * x) = \mu(x) \geq \min\{\mu(0 * (y * x)), \mu(y)\} = \min\{\mu(0 * 0), \mu(y)\}$$
$$= \min\{\mu(0), \mu(y)\} = \mu(y).$$

Proposition 3.7. The intersection of any set of fuzzy KU - ideals of KU - algebra X is also fuzzy ideal .

Proof. let $\{\mu_i\}$ be a family of fuzzy KU - ideals of KU- algebra X , then for any $x, y, z \in X$,

$$(\cap\mu_i)(0) = \inf(\mu_i(0)) \geq \inf(\mu_i(x)) = (\cap\mu i)(x) \text{ and}$$
$$(\cap\mu_i)(x * z) = \inf(\mu_i(x * z)) \geq \inf(\min\{\mu_i(x * (y * z)), \mu_i(y)\})$$
$$= \min\{\inf(\mu_i(x * (y * z)), \inf(\mu_i(y)\}$$
$$= \min\{(\cap\mu_i)(x * (y * z)), (\cap\mu_i(y)\} .$$

This completes the proof .

Theorem3.8 . Let μ be a fuzzy set in X then μ is a fuzzy KU- ideal of X if and only if it satisfies :

For all $\alpha \in [0,1]$),$U(\mu, \alpha) \neq \phi$ implies $U(\mu, \alpha)$ is KU- ideal of X-----(A) where $U(\mu, \alpha) = \{x \in X / \mu(x) \geq \alpha\}$.

Proof . Assume that μ is a fuzzy ideal of X , let $\alpha \in [0, 1]$ be such that $U(\mu, \alpha) \neq \phi$, and let $x, y \in X$ be such that $x \in U(\mu, \alpha)$, then $\mu(x) \geq \alpha$ and so by (F_2) ,

$$\mu(y * 0) = \mu(0) \geq \min\{\mu(y * (x * 0)), \mu(x)\}$$
$$= \min\{\mu(y * 0), \mu(x)\} = \min\{\mu(0), \mu(x)\} = \alpha$$

thus $0 \in U(\mu, \alpha)$.

Let $x * (y * z) \in U(\mu, \alpha)$, $y \in U(\mu, \alpha)$, It follows from(F_2) that

$\mu(x * z) \geq \min\{\mu(x * (y * z)), \mu(y)\} = \alpha$, so that $x * z \in U(\mu, \alpha)$.

Hence $U(\mu, \alpha)$ is KU - ideal of X .

Conversely , suppose that μ satisfies (A) , let $x, y, z \in X$ be such that

$\mu(x * z) < \min\{\mu(x * (y * z)), \mu(y)\}$,taking

$\beta_0 = 1/2\{\mu(x * z) + \min\{\mu(x * (y * z)), \mu(y)\}$, we have

$\beta_0 \in [0,1]$ and $\mu(x * z) < \beta_0 < \min\{\mu(x * (y * z)), \mu(y)\}$

it follows that

$x * (y * z) \in U(\mu, \beta_0)$ and $x * z \notin U(\mu, \beta_0)$,

this is a contradiction and therefore μ is a fuzzy KU - ideal of X .

Proposition 3.9 . If μ is a fuzzy KU - ideal of X , then

$$\mu(x * (x * y)) \geq \mu(y)$$

proof . Taking $z = x * y$ in (F_2) and using (ku_2) and (F_1) , we get

$$\mu(x * (x * y)) \geq \min\{\mu(x * (y * (x * y)), \mu(y)\}$$
$$= \min\{\mu(x * (x * (y * y)), \mu(y)\}$$
$$= \min\{\mu(x * (x * 0)), \mu(y)\}$$
$$= \min\{\mu(0), \mu(y)\} = \mu(y) .$$

4. Characterization of fuzzy KU - ideal by their level KU - ideals

Theorem4.1 . A fuzzy subset μ of KU - algebra X is a fuzzy KU - ideal of X if and only if , for every t ∈ [0,1] , μ_t is either empty or an KU - ideal of X .

Proof . Assume that μ is a fuzzy KU - ideal of X , by (F1) , we have μ (0) ≥ μ (x) for all x ∈ X therefore μ (0) ≥ μ (x) ≥ t for x ∈ μt and so 0 ∈ μt . Let x ∗ (y ∗ z) ∈ μt and y ∈ μt , then μ (x ∗ (y ∗ z)) ≥ t and μ (y) ≥ t , since μ is a fuzzy KU - ideal it follows that μ (x ∗ z) ≥ min {μ (x ∗ (y ∗ z)) , μ (y)} ≥ t and that x ∗ z ∈ μt . Hence μt is an KU - ideal of X .

Conversely , we only need to show that (F_1) and (F_2) are true . If (F_1) is false , then there exist x`∈ X such that μ (0) < μ(x`). If we take t`= (μ (x`) + μ (0))/2 , then μ(0) < t` and 0 ≤ t` < μ (x`) ≤ 1 , thus x`∈ μ and μ ≠ ϕ As μ is an KU-ideal of X , we have 0 ∈ $\mu_{t`}$ and so μ (0) ≥ t`. This is a contradiction . Now , assume (F_2) is not true ,then there exist x` , y` and z` such that ,

μ (x`, z`) < min {μ (x`∗ (y`∗ z`) , μ (y`)}. Putting

t`=(μ (x`) + min {μ(x` ∗ (y` ∗ z`), μ (y`)}/2 , then μ (x` ∗ z`) < t` and 0 ≤ t` < min {μ (x` ∗ (y` ∗ z`) , μ (y`)} ≤ 1, hence

μ (x` ∗ (y` ∗ z`)) > t` and μ (y`) > t`,

which imply that x`∗ (y` ∗ z`) ∈ μ (t`) and y`∈ $\mu_{t`}$, since μ_t is an KU - ideal ,it follows that x` ∗ z` ∈ $\mu_{t`}$ and that μ (x` ∗ z`) ≥ t`, this is also a contradiction . Hence μ is a fuzzy KU – ideal of X .

Corollary4.2. If a fuzzy subset μ of KU - algebra X is a fuzzy KU - ideal , then for every t ∈ Im (μ) , μ_t is an KU - ideal of X .

Definition4.3 . let μ be a fuzzy KU - ideal of KU - algebra X ,.the KU - ideals μ_t t ∈ [0,1] are called level KU - ideal of μ .

Corollary4.4 . let I be an KU - ideal of KU - algebra X , then for any fixed number t in an open interval (0,1) , there exist a fuzzy KU - ideal μ of X such that μ_t = I .

proof . the proof is similar the corollary 4.4 [16] .

Definition 4. 5.

Let *f* be a mapping from the set *X* to a set *Y*. If μ is a fuzzy subset of *X*, then the fuzzy subset *B* of *Y* defined by

$$f(\mu)(y) = B(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu(x), & \text{if } f^{-1}(y) = \{x \in X, f(x) = y\} \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

is said to be the image of μ under *f*.

Similarly if β is a fuzzy subset of Y , then the fuzzy subset μ = β o f in X (i.e the fuzzy subset defined by μ (x) = β (f (x)) for all x ∈ X) is called the primage of β under f .

Theorem 4.6 . An onto homomorphic preimage of a fuzzy KU - ideal is also a fuzzy KU - ideal .

Proof . Let f : X → X\` be an into homomorphism of KU - algebras , β a fuzzy KU - ideal of X\` and μ the preimage of β under f , then β (f (x)) = μ (x) , for all x ∈ X . Let x ∈ X , then μ (0) = β (f (0)) ≥ β (f (x)) = μ (x). Now let x , y , z ∈ X then μ (x ∗ z) = β (f (x ∗ z))= β(f (x) ∗\` f (z)) ≥ min {β (f (x) ∗\` (f (y) ∗\` f(z)),β(f (y))} = min {β (f (x ∗ (y ∗ z)) , β (f (y))} = min {μ(x ∗ (y ∗ z)) , μ (y)} , and the proof is completed .

Definition 4.7 [16] .A fuzzy subset μ of X has sup property if for any subset T of X , there exist $t_0 \in$ T such that $\mu(t_0) = \underset{t \in T}{SUP}\ \mu(t)$.

Theorem 4.8. let X → Y be a homomorphism between KU - algebras X and Y . For every fuzzy KU - ideal μ in X , f (μ) is a fuzzy KU - ideal of Y .

Proof . By definition $B(y') = f(\mu)(y') = \sup_{x \in f^{-1}(y')} \mu(x)$ for all $y' \in Y$ and $\sup\phi = 0$

We have to prove that $B(x' * z') \geq \min\{B(x' * (y' * z')), B(y')\}$, ∀ x\`, y\`, z\` ∈ Y .

Let f : X → Y be an onto a homomorphism of KU - algebras , μ a fuzzy KU - ideal of X with sup property and β the image of μ under f , since μ is a fuzzy KU - ideal of X , we have μ(0) ≥ μ(x) for all x∈X . Note that 0 ∈ f^{-1} (0\`) , where 0 , 0\` are the zero of X and Y respectively

Thus, $B(0') = \sup_{t \in f^{-1}(0')} \mu(t) = \mu(0) \geq \mu(x),$ for all $x \in X$, which implies that

$B(0') \geq \sup_{t \in f^{-1}(x')} \mu(t) = B(x'),$ for any $x' \in Y$. For any $x', y', z' \in Y$, let

$x_0 \in f^{-1}(x')$, $y_0 \in f^{-1}(y')$, $z_0 \in f^{-1}(z')$ be Such that

$\mu(x_0 * z_0) = \sup_{t \in f^{-1}(x' * z')} \mu(t)$, $\mu(y_0) = \sup_{t \in f^{-1}(y')} \mu(t)$ and

$$\mu(x_0 * (y_0 * z_0)) = B\{f(x_0 * (y_0 * z_0))\} = B(x' * (y' * z'))$$

$$= \sup_{(x_0 * (y_0 * z_0)) \in f^{-1}(x' * (y' * z'))} \mu((x_0 * (y_0 * z_0)) = \sup_{t \in f^{-1}(x' * (y' * z'))} \mu(t).$$

Then $B(x' * z') = \sup_{t \in f^{-1}(x' * z')} \mu(t) = \mu(x_0 * z_0) \geq \min\{\mu(x_0 * (y_0 * z_0)), \mu(y_0)\} =$

$$\min\left\{\sup_{t \in f^{-1}(x' * (y' * z'))} \mu(t), \sup_{t \in f^{-1}(y')} \mu(t)\right\} = \min\{B(x' * (y' * z')) , B(y')\}.$$

Hence *B* is a fuzzy KU-ideal of *Y*.

5. Cartesian product of fuzzy KU-ideal

Definition 5.1[1] . A fuzzy μ is called a fuzzy relation on any set S , if μ is a fuzzy subset $\mu : S \times S \to [0,1]$.

Definition 5.2 [1] . If μ is a fuzzy relation a set S and β is a fuzzy subset of S , then μ is a fuzzy relation on β if $\mu (x , y) \le \min \{\beta (x) , \beta (y)\}$, $\forall$ x , y $\in$ S .

Definition 5.3 [1] . Let μ and β be fuzzy subset of a set S , the cartesian product of μ and β is define by $(\mu \times \beta) (x , y) = \min \{\mu (x) , \beta (y)\}$, $\forall$ x , y $\in$ S .

Lemma 5.4[1] . let μ and β be fuzzy subset of a set S then ,

(i) $\mu \times \beta$ is a fuzzy relation on S .

(ii) $(\mu \times \beta)_t = \mu_t \times \beta_t$ for all t $\in$ [0,1].

Definition 5.5 [1] . If β is a fuzzy subset of a set S , the strongest fuzzy relation on S , that is , a fuzzy relation on β is μ_β given by $\mu_\beta (x , y) = \min \{\beta (x) , \beta (y)\}$, $\forall$ x , y $\in$ S .

Lemma 5.6 . For a given fuzzy subset S , let μ_β be the strongest fuzzy relation on S then for t $\in$ [0,1] , we have $(\mu_\beta)_t = \beta_t \times \beta_t$.

Proposition5.7 . For a given fuzzy subset β of KU - algebra X , let μ_β be the strongest fuzzy relation on X . If μ_β is a fuzzy KU - ideal of X $\times$ X , then $\beta (x) \le \beta (0)$ for all x $\in$ X .

Proof . Since μ_β is a fuzzy KU- ideal of X $\times$ X , it follows from (F_1) that $\mu_\beta (x , x)$ $= \min \{\beta (x) , \beta (x)\} \le (0 , 0) = \min \{\beta (0) , \beta (0)\}$, where (0 , 0) $\in$ X $\times$ X then $\beta (x) \le \beta (0)$.

Remark 5.8 . Let X and Y be KU- algebras , we define $*$ on X $\times$ Y by :
For every (x , y), (u , v)$\in$X x Y , $(x , y) * (u , v) = (x * u , y * v)$, then clearly $(x * y , * , (0 , 0))$ is a KU- algebra .

Theorem 5.9 . let μ and β be a fuzzy KU- ideals of KU - algebra X ,then $\mu \times \beta$ is a fuzzy KU-ideal of X $\times$ X .

Proof . for any (x , y) $\in$X $\times$ X ,we have ,
$(\mu \times \beta) (0, 0) = \min \{\mu (0) , \beta (0)\} \ge \min \{\mu (x) , \beta (x)\} = (\mu \text{ x } \beta) (x , y)$.
Now let $(x_1 , x_2) , (y_1 , y_2) , (z_1 , z_2) \in X \times X$, then ,

$$
\begin{aligned}
(\mu \text{ x } \beta) (x_1 * z_1 , x_2 * z_2) &= \min \{\mu (x_1,z_1) , \beta (x_2 , z_2)\} \\
&\ge \min \{\min \{\mu (x_1 * (y_1 * z_1)) , \mu(y_1) \}\} , \min \{\beta (x_2 * (y_2 * z_2)) , \beta (y_2)\}\} \\
&= \min \{\min \{\mu (x_1 * (y_1 * z_1)) , \mu (x_2 * (y_2 * z_2))\} , \min \{ \mu(y_1), \beta(y_2)\}\} \\
&= \min \{(\mu \times \beta) (x_1 * (y_1 * z_1) , x_2 * (y_2 * z_2)) ,(\mu \times \beta)(y_1, y_2)\} .
\end{aligned}
$$

Hence $\mu \times \beta$ is a fuzzy KU- ideal of X $\times$ X .

Analogous to theorem 3.2 [15] , we have a similar results for KU- ideal , which can be proved in similar manner , we state the results without proof .

Theorem 5.10. let μ and β be a fuzzy subset of KU-algebra X ,such that $\mu \times \beta$ is a fuzzy KU-ideal of X $\times$ X , then

(i) either $\mu (x) \leq \mu (0)$ or $\beta (x) \leq \beta (0)$ for all $x \in X$,
(ii) if $\mu (x) \leq \mu (0)$ for all $x \in X$, then either $\mu (x) \leq \beta (0)$ or $\beta (x) \leq \beta(0)$,
(iii) if $\beta (x) \leq \beta (0)$ for all $x \in X$, then either $\mu (x) \leq \mu (0)$ or $\beta (x) \leq \mu(0)$,
(iv) either μ or β is a fuzzy KU- ideal of X .

Theorem 5.11. let β be a fuzzy subset of KU- algebra X and let μ_β be the strongest fuzzy relation on X , then β is a fuzzy KU - ideal of X if and only if μ_β is a fuzzy KU- ideal of $X \times X$.

proof . Assume that β is a fuzzy KU- ideal X , we note from (F_1) that :
$\mu_\beta (0, 0) = \min \{\beta (0) , \beta (0)\} \geq \min \{\beta (x) , \beta (y) \} = \mu_\beta (x , y)$.
Now, for any $(x_1,x_2) , (y_1,y_2) ,(z_1,z_2) \in X \text{ x } X$, we have from (F_2) :
$\mu_\beta (x_1 * z_1 , x_2 * z_2) = \min \{\beta (x_1 * z_1) , \beta (x_2 * z_2)\}$
$\geq \min \{\min\{\beta (x_1 * (y_1 * z_1)) , \beta (y_1)\} , \min \{\beta (x_2 * (y_2 * z_2)) , \beta(y_2)\}\}$
$= \min\{\min\{ \beta(x_1 * (y_1 * z_1)) , \beta (x_2 * (y_2 * z_2))\} , \min \{ \beta(y_1), \beta(y_2)\}\}$
$= \min \{\mu_\beta (x_1 * (y_1 * z_1) , x_2 * (y_2 * z_2)) , \mu_\beta (y_1 , y_2)\}$.
Hence μ_β is a fuzzy KU - ideal of $X \times X$.
Conversely : for all $(x , y) \in X \times X$, we have
$\text{Min} \{\beta (0) , \beta (0) \} = \mu_\beta (x , y) = \min \{\beta (x) , \beta (y)\}$ It follows that
$\beta (0) \geq \beta (x)$ for all $x \in X$, which prove (F_1).
Now, let $(x_1 , x_2) , (y_1 , y_2), (z_1 , z_2) \in X \times X$, then
$\min \{\beta (x_1 * z_1) , \beta (x_2 * z_2)\} = \mu_\beta (x_1 * z_1 , x_2 * z_2)$
$\geq \min \{\mu_\beta (x_1 , x_2) * ((y_1 , y_2) * (z_1, z_2)) , \mu_\beta (y_1,y_2))\}$
$= \min \{\mu_\beta (x_1 * (y_1 * z_1) , x_2 * (y_2 * z_2)) , \mu_\beta (y_1 , y_2)\}$
$= \min \{\min \{\beta (x_1 * (y_1 * z_1)) , \beta (x_2 * (y_2 * z_2))\} , \min \{\beta (y_1) , \beta (y_2)\}\}$
$= \min \{\min \{\beta (x_1 * (y_1 * z_1)) , \beta (y_1)\} , \min \{\beta(x_2 * (y_2 * z_2)) , \beta (y_2)\}\}$
In particular , if we take $x_2 = y_2 = z_2 =0$, then ,
$\beta (x_1 * z_1) \geq \min \{ \beta (x_1 * (y_1 * z_1)), \beta (y_1)\}$
This prove (F_1) and completes the proof .

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