

# Rough Intuitionistic Fuzzy Sets in a Lattice

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## Abstract

In this paper basic notions of the rough intuitionistic fuzzy set theory will be given. In fact the paper concerns a relationship between rough intuitionistic fuzzy sets and lattice theory. We shall introduce the notion of rough intuitionistic fuzzy lattice (resp rough intuitionistic fuzzy ideals). We also defined intuitionistic fuzzy rough set corresponding to a rough set and given conditions for intuitionistic fuzzy rough lattice. We prove that an intuitionistic fuzzy rough set in  $\text{Apr}(X)$  is an intuitionistic fuzzy rough lattice iff it's level rough set  $(\underline{A}_{(\alpha,\beta)}, \overline{A}_{(\alpha,\beta)})$  is a rough sub lattice of  $\text{Apr}(X)$ .

**Mathematics Subject Classification:** 03B52, 03F55, 06D72

**Keywords:** Rough set, Intuitionistic fuzzy set (IFS), Rough intuitionistic fuzzy set (RIFS), Intuitionistic fuzzy lattice (IFL), Rough intuitionistic fuzzy lattice (RIFL), Intuitionistic fuzzy rough set (IFRS), Intuitionistic fuzzy rough lattice (IFRL), Level rough sets

## 1 Introduction

Rough set theory has been proposed by Pawlak [7] as a tool to conceptualize, organize and analyze various types of data in data mining. It has recently received wide attention in real life applications and theoretical research. This

method is especially useful for dealing with uncertain and vague knowledge in information systems. The concept of a fuzzy set was introduced by Zadeh and it is now a rigorous area of research with applications in various fields. With the work of fuzzy sets, 1986 Atanassov presented intuitionistic fuzzy sets which are very effective to deal with vagueness. The concept of intuitionistic fuzzy sets is a generalization of one of the fuzzy sets.

Recently many researchers applied the notion of intuitionistic fuzzy sets to relations, group theory, ring theory, lattice theory etc. In paper [9] we studied the rough sets corresponding to an ideal of a lattice and introduced rough sub lattice, rough ideals and their characterizations and in [8] we studied intuitionistic fuzzy sub lattices and ideals. This paper is a blend of the above papers where we studied relationship between rough sets and intuitionistic fuzzy sets. Here we give the rough approximation of an intuitionistic fuzzy set and introduced rough intuitionistic fuzzy sub lattices, ideals etc. Also we defined intuitionistic fuzzy rough sets, intuitionistic fuzzy rough sub lattices, and ideals and studied their properties.

## 2 Preliminaries

In this section we recall the following definitions and results which are used in the sequel.

**Definition 2.1.** [1] Let  $X$  be a non-empty set. An intuitionistic fuzzy set [IFS]  $A$  of  $X$  is an object of the following form  $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$  where  $\mu_A : X \rightarrow [0, 1]$  and  $\nu_A : X \rightarrow [0, 1]$  define the *degree of membership* and the *degree of non membership* of the element  $x \in X$ ,  $0 \leq \mu_A(x) + \nu_A(x) \leq 1$

The set of all IFS's on  $X$  is denoted by  $\text{IFS}(X)$ .

**Definition 2.2.** [1] If  $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$  and  $B = \{\langle x, \mu_B(x), \nu_B(x) \rangle / x \in X\}$  are any two IFS of  $X$  then,

$$(i) A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x) \text{ and } \nu_A(x) \geq \nu_B(x) \forall x \in X.$$

$$(ii) A = B \Leftrightarrow \mu_A(x) = \mu_B(x) \text{ and } \nu_A(x) = \nu_B(x) \forall x \in X.$$

$$(iii) \bar{A} = \{\langle x, \nu_A(x), \mu_A(x) \rangle / x \in X\}$$

$$(iv) [A] = \{\langle x, \mu_A(x), \mu_A^c(x) \rangle / x \in X\} \text{ where } \mu_A^c(x) = 1 - \mu_A(x)$$

$$(v) \langle A \rangle = \{\langle x, \nu_A^c(x), \nu_A(x) \rangle / x \in X\} \text{ where } \nu_A^c(x) = 1 - \nu_A(x)$$

$$(vi) A \cap B = \{\langle x, \mu_{A \cap B}(x), \nu_{A \cap B}(x) \rangle / x \in X\} \text{ where}$$

$$\mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(x)\} \text{ and } \nu_{A \cap B}(x) = \max\{\nu_A(x), \nu_B(x)\}$$

$$(vii) A \cup B = \{\langle x, \mu_{A \cup B}(x), \nu_{A \cup B}(x) \rangle / x \in X\} \text{ where}$$

$$\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\} \text{ and } \nu_{A \cup B}(x) = \min\{\nu_A(x), \nu_B(x)\}$$

**Definition 2.3.** [8] Let  $L$  be a lattice and  $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in L\}$  be an IFS of  $L$ . Then  $A$  is called an *Intuitionistic fuzzy sublattice* (IFL) of  $L$  if the following conditions are satisfied:

- (i)  $\mu_A(x \vee y) \geq \min\{\mu_A(x), \mu_A(y)\}$ , (ii)  $\mu_A(x \wedge y) \geq \min\{\mu_A(x), \mu_A(y)\}$
- (iii)  $\nu_A(x \vee y) \leq \max\{\nu_A(x), \nu_A(y)\}$ , (iv)  $\nu_A(x \wedge y) \leq \max\{\nu_A(x), \nu_A(y)\} \forall x, y \in L$ .

The set of all IFL's of  $L$  is denoted as IFL ( $L$ ).

**Definition 2.4.** [8] An IFS  $A$  of  $L$  is called an *intuitionistic fuzzy ideal* (IFI) of  $L$ , if the following conditions are satisfied:

- (i)  $\mu_A(x \vee y) \geq \min\{\mu_A(x), \mu_A(y)\}$ , (ii)  $\mu_A(x \wedge y) \geq \max\{\mu_A(x), \mu_A(y)\}$
- (iii)  $\nu_A(x \vee y) \leq \max\{\nu_A(x), \nu_A(y)\}$ , (iv)  $\nu_A(x \wedge y) \leq \min\{\nu_A(x), \nu_A(y)\} \forall x, y \in L$

The set of all IFI's of  $L$  is denoted as IFI ( $L$ ).

**Theorem 2.5.** [8] If  $A$  and  $B$  are two IFL's (IFI's) of a lattice  $L$  then  $A \cap B$  is an IFL (IFI) of  $L$ .

**Definition 2.6.** [9] Let  $U$  is a nonempty set and  $\theta$  an equivalence relation on  $U$ . Then the pair  $(U, \theta)$  is called an *approximation space*.

**Definition 2.7.** [9] Let  $(U, \theta)$  be an approximation space and  $X$  any nonempty subset of  $U$ . Then the sets,  $\underline{Apr}(X) = \{x \in U / [x]_\theta \subseteq X\}$  and  $\overline{Apr}(X) = \{x \in U / [x]_\theta \cap X \neq \emptyset\}$  are called the lower and upper rough approximations of the set  $X$ .

Then  $Apr(X) = (\underline{Apr}(X), \overline{Apr}(X))$  is called a *rough set* in  $(U, \theta)$ .

**Definition 2.8.** [9] A rough set  $Apr(X) = (\underline{Apr}(X), \overline{Apr}(X))$  in a lattice  $L$  is called a *rough lattice* if both  $\underline{Apr}(X)$ , and  $\overline{Apr}(X)$  are sublattices of  $L$ .

### 3 Rough Intuitionistic Fuzzy Set (RIFS)

In this section we define Rough intuitionistic fuzzy set and defined operations on them. Rough intuitionistic fuzzy lattices (RIFL) and ideals (RIFI) are introduced and certain properties of them are studied.

**Definition 3.1.** Let  $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$  be an IFS of  $X$ . Then the *rough*

*intuitionistic fuzzy set* of  $A$  (RIFS) denoted as  $Apr(A) = (\underline{Apr}(A), \overline{Apr}(A))$  and defined as  $\underline{Apr}A = \{\langle x, \underline{\mu}_A(x), \underline{\nu}_A(x) \rangle\}$  where  $\underline{\mu}_A(x) = \bigwedge_{x' \in [x]_\theta} \mu_A(x')$  and

$$\underline{\nu}_A(x) = \bigvee_{x' \in [x]_\theta} \nu_A(x')$$

$$\overline{Apr}A = \{\langle x, \overline{\mu}_A(x), \overline{\nu}_A(x) \rangle\} \text{ where } \overline{\mu}_A(x) = \bigvee_{x' \in [x]_\theta} \mu_A(x') \text{ and } \overline{\nu}_A(x) = \bigwedge_{x' \in [x]_\theta} \nu_A(x')$$

Where  $\theta$  is an equivalence relation on  $X$  and  $[x]_\theta$  is the equivalence class containing  $x$ .

**Definition 3.2.** Let  $\underline{Apr}(A) = (\underline{Apr}(A), \overline{Apr}(A))$ , and  $\underline{Apr}(B) = (\underline{Apr}(B), \overline{Apr}(B))$  are two RIFS, Then

- (1)  $\underline{Apr}(A) \cup \underline{Apr}(B) = (\underline{Apr}(A) \cup \underline{Apr}(B), \overline{Apr}(A) \cup \overline{Apr}(B))$
- (2)  $\underline{Apr}(A) \cap \underline{Apr}(B) = (\underline{Apr}(A) \cap \underline{Apr}(B), \overline{Apr}(A) \cap \overline{Apr}(B))$
- (3)  $\underline{Apr}(A) \subseteq \underline{Apr}(B)$  if and only if  $\underline{Apr}(A) \subseteq \underline{Apr}(B), \overline{Apr}(A) \subseteq \overline{Apr}(B)$
- (4) The complement of  $\underline{Apr}(A)$  denoted as  $\underline{Apr}^c(A) = (\underline{Apr}^c(A), \overline{Apr}^c(A))$

**Proposition 3.3.** For every approximation space  $(X, \theta)$  and for every IFS's  $A, B \in X$ , we have,

- (1)  $\underline{Apr}(A) \subseteq A \subseteq \overline{Apr}(A)$
- (2) If  $A \subseteq B$  then  $\underline{Apr}(A) \subseteq \underline{Apr}(B)$  and  $\overline{Apr}(A) \subseteq \overline{Apr}(B)$
- (3)  $\underline{Apr}^c(A) = (\underline{Apr}^c(A), \overline{Apr}^c(A))$
- (4)  $\underline{Apr}[\underline{Apr}(A)] = \underline{Apr}(A)$
- (5)  $\overline{Apr}[\overline{Apr}(A)] = \overline{Apr}(A)$
- (6)  $\underline{Apr}[\overline{Apr}(A)] = \overline{Apr}(A)$
- (7)  $\overline{Apr}[\underline{Apr}(A)] = \underline{Apr}(A)$
- (8)  $\overline{Apr}(A) = [\underline{Apr}(A^c)]^c$
- (9)  $\underline{Apr}(A) = [\overline{Apr}(A^c)]^c$
- (10)  $\underline{Apr}(A \cap B) = \underline{Apr}(A) \cap \underline{Apr}(B)$
- (11)  $\overline{Apr}(A \cap B) \subseteq \overline{Apr}(A) \cap \overline{Apr}(B)$
- (12)  $\underline{Apr}(A \cup B) \supseteq \underline{Apr}(A) \cup \underline{Apr}(B)$
- (13)  $\overline{Apr}(A \cup B) = \overline{Apr}(A) \cup \overline{Apr}(B)$

*Proof.* Follow from Definitions 2.2, and 3.2

**Proposition 3.4.** Let  $L$  be a lattice and  $A$  is an IFL (IFI) of  $L$ . Then  $\underline{Apr}(A)$  and  $\overline{Apr}(A)$  are also IFL's (IFI's) of  $L$ .

*Proof.* We will prove the case of IFL. Proof for IFI is similar. We have  $\underline{\mu}_A(x \vee y) = \bigwedge_{x' \vee y' \in [x \vee y]_\theta} \mu(x' \vee y') \geq \bigwedge_{x' \vee y' \in [x \vee y]_\theta} [\min\{\mu(x'), \mu(y')\}]$ , since  $A$  IFL of  $L$ .  
 $\geq \min\{\bigwedge_{x' \in [x]_\theta} \mu(x'), \bigwedge_{y' \in [y]_\theta} \mu(y')\} = \min\{\underline{\mu}_A(x), \underline{\mu}_A(y)\}$ . i.e  $\underline{\mu}_A(x \vee y) \geq \min\{\underline{\mu}_A(x), \underline{\mu}_A(y)\}$ .

Similarly  $\underline{\mu}_A(x \wedge y) = \bigwedge_{x' \wedge y' \in [x \wedge y]_\theta} \mu(x' \wedge y') \geq \bigwedge_{x' \wedge y' \in [x \wedge y]_\theta} [\min\{\mu(x'), \mu(y')\}]$ , since

$A$  IFL of  $L$ .

$\geq \min\{\bigwedge_{x' \in [x]_\theta} \mu(x'), \bigwedge_{y' \in [y]_\theta} \mu(y')\} = \min\{\underline{\mu}_A(x), \underline{\mu}_A(y)\}$ . i.e  $\underline{\mu}_A(x \wedge y) \geq \min\{\underline{\mu}_A(x), \underline{\mu}_A(y)\}$ .

Now  $\underline{\nu}_A(x \vee y) = \bigvee_{x' \vee y' \in [x \vee y]_\theta} \nu(x' \vee y') \leq \bigvee_{x' \vee y' \in [x \vee y]_\theta} [\max\{\nu(x'), \nu(y')\}]$ , since  $A$

IFL of  $L$ .

$\leq \max\{\bigvee_{x' \in [x]_\theta} \nu(x'), \bigvee_{y' \in [y]_\theta} \nu(y')\} = \max\{\underline{\nu}_A(x), \underline{\nu}_A(y)\}$ . i.e  $\underline{\nu}_A(x \vee y) \leq \max\{\underline{\nu}_A(x), \underline{\nu}_A(y)\}$ .

Similarly  $\underline{\nu}_A(x \wedge y) \leq \max\{\underline{\nu}_A(x), \underline{\nu}_A(y)\}$ . Hence  $\underline{Apr}(A)$  is an IFL of  $L$ .

Now  $\overline{\mu}_A(x \vee y) = \bigvee_{x' \vee y' \in [x \vee y]_\theta} \mu(x' \vee y') \geq \bigvee_{x' \vee y' \in [x \vee y]_\theta} [\min\{\mu(x'), \mu(y')\}]$ , since  $A$

IFL of  $L$ .

$\geq \min\{\bigvee_{x' \in [x]_\theta} \mu_A(x'), \bigvee_{y' \in [y]_\theta} \mu_A(y')\}$  since  $x'$  and  $y'$  vary independently  
 $\geq \min\{\bar{\mu}_A(x), \bar{\mu}_A(y)\}$ . i.e  $\bar{\mu}_A(x \vee y) \geq \min\{\bar{\mu}_A(x), \bar{\mu}_A(y)\}$ .  
 Similarly  $\bar{\mu}_A(x \wedge y) \geq \min\{\bar{\mu}_A(x), \bar{\mu}_A(y)\}$   
 Also  $\bar{\nu}_A(x \vee y) = \bigwedge_{x' \vee y' \in [x \vee y]_\theta} \nu_A(x' \vee y') \leq \bigwedge_{x' \vee y' \in [x \vee y]_\theta} [\max\{\nu_A(x'), \nu_A(y')\}]$ , since  $A$  is an IFL of  $L$ .  
 $\leq \max\{\bigwedge_{x' \in [x]_\theta} \nu_A(x'), \bigwedge_{y' \in [y]_\theta} \nu_A(y')\}$  since  $x'$  and  $y'$  vary independently  
 $\leq \max\{\bar{\nu}_A(x), \bar{\nu}_A(y)\}$ . i.e  $\bar{\nu}_A(x \vee y) \leq \max\{\bar{\nu}_A(x), \bar{\nu}_A(y)\}$ .  
 Similarly  $\bar{\nu}_A(x \wedge y) \leq \max\{\bar{\nu}_A(x), \bar{\nu}_A(y)\}$ . Hence  $\overline{Apr}(A)$  is an IFL of  $L$ .

**Definition 3.5.** A rough intuitionistic fuzzy set  $Apr(A)$  of  $L$  is called a *rough intuitionistic fuzzy lattice* (RIFL) [*rough intuitionistic fuzzy ideal* (RIFI)] if both  $\underline{Apr}(A)$  and  $\overline{Apr}(A)$  are IFL's (IFI's) of  $L$ .

**Theorem 3.6.** If  $A$  is an IFL (IFI) of  $L$  then  $Apr(A)$  is a RIFL (RIFI) of  $L$ .

*Proof.* Follow from Proposition 3.4.

**Theorem 3.7.** If  $Apr(A)$  and  $Apr(B)$  are RIFL's (RIFI's), then  $Apr(A) \cap Apr(B)$  is also a RIFL (RIFI).

*Proof.* We have  $Apr(A) \cap Apr(B) = (\underline{Apr}(A) \cap \underline{Apr}(B), \overline{Apr}(A) \cap \overline{Apr}(B))$ . Since  $Apr(A)$  and  $Apr(B)$  are RIFL's (RIFI's) we have  $\underline{Apr}(A)$ ,  $\overline{Apr}(A)$ ,  $\underline{Apr}(B)$ , and  $\overline{Apr}(B)$  are IFL's (IFI's). Then  $\underline{Apr}(A) \cap \underline{Apr}(B)$  and  $\overline{Apr}(A) \cap \overline{Apr}(B)$  are IFL's (IFI's) by Theorem 2.5. So  $Apr(A) \cap Apr(B)$  is a RIFL (RIFI) by Def 3.5.

**Remark 3.8.** The union of two RIFI's need not be a RIFI.

Consider the lattice  $L = \{1, 2, 3, 4, 6, 12\}$  of divisors of 12. Let  $\theta = \{1, 2, 3, 6, 4, 12\}$  be the equivalence class. We define  $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in L\}$  by  $\langle 1, .7, .2 \rangle$ ,  $\langle 2, .5, .5 \rangle$ ,  $\langle 3, .6, .3 \rangle$ ,  $\langle 4, .4, .5 \rangle$ ,  $\langle 6, .5, .5 \rangle$ ,  $\langle 12, .4, .5 \rangle$  and  $B = \{\langle x, \mu_B(x), \nu_B(x) \rangle / x \in L\}$  by  $\langle 1, .6, .2 \rangle$ ,  $\langle 2, .6, .4 \rangle$ ,  $\langle 3, .5, .5 \rangle$ ,  $\langle 4, .5, .4 \rangle$ ,  $\langle 6, .4, .5 \rangle$ ,  $\langle 12, .4, .5 \rangle$ . Here  $A$  and  $B$  are IFI's of  $L$ . Now  $Apr(A) = (\underline{Apr}(A), \overline{Apr}(A))$ , where  $\underline{Apr}A = \{\langle x, \underline{\mu}_A(x), \underline{\nu}_A(x) \rangle\}$  is  $\langle 1, .5, .5 \rangle$ ,  $\langle 2, .5, .5 \rangle$ ,  $\langle 3, .5, .5 \rangle$ ,  $\langle 4, .4, .5 \rangle$ ,  $\langle 6, .5, .5 \rangle$ ,  $\langle 12, .4, .5 \rangle$  and  $\overline{Apr}A = \{\langle x, \bar{\mu}_A(x), \bar{\nu}_A(x) \rangle\}$  is  $\langle 1, .7, .2 \rangle$ ,  $\langle 2, .7, .2 \rangle$ ,  $\langle 3, .6, .3 \rangle$ ,  $\langle 4, .4, .5 \rangle$ ,  $\langle 6, .6, .3 \rangle$ ,  $\langle 12, .4, .5 \rangle$ . Also  $Apr(B) = (\underline{Apr}(B), \overline{Apr}(B))$ , where  $\underline{Apr}B = \{\langle x, \underline{\mu}_B(x), \underline{\nu}_B(x) \rangle\}$  is  $\langle 1, .6, .4 \rangle$ ,  $\langle 2, .6, .4 \rangle$ ,  $\langle 3, .4, .5 \rangle$ ,  $\langle 4, .5, .4 \rangle$ ,  $\langle 6, .4, .5 \rangle$ ,  $\langle 12, .4, .5 \rangle$  and  $\overline{Apr}B = \{\langle x, \bar{\mu}_B(x), \bar{\nu}_B(x) \rangle\}$  is  $\langle 1, .6, .2 \rangle$ ,  $\langle 2, .6, .2 \rangle$ ,  $\langle 3, .5, .5 \rangle$ ,  $\langle 4, .5, .4 \rangle$ ,  $\langle 6, .5, .5 \rangle$ ,  $\langle 12, .4, .5 \rangle$ . Clearly  $Apr(A)$  and  $Apr(B)$  are RIFI's. Now  $Apr(A) \cap Apr(B) = (\underline{Apr}(A) \cap \underline{Apr}(B), \overline{Apr}(A) \cap \overline{Apr}(B))$  is given by,  $\underline{Apr}(A) \cap \underline{Apr}(B) = \langle 1, .6, .4 \rangle$ ,  $\langle 2, .6, .4 \rangle$ ,  $\langle 3, .5, .5 \rangle$ ,  $\langle 4, .5, .4 \rangle$ ,  $\langle 6, .5, .5 \rangle$ ,  $\langle 12, .4, .5 \rangle$ .  $\overline{Apr}(A) \cap \overline{Apr}(B) = \langle 1, .7, .2 \rangle$ ,  $\langle 2, .7, .2 \rangle$ ,  $\langle 3, .6, .3 \rangle$ ,  $\langle 4, .5, .4 \rangle$ ,  $\langle 6, .6, .3 \rangle$ ,  $\langle 12, .4, .5 \rangle$ . Here  $\mu_{\underline{Apr}(A) \cap \underline{Apr}(B)}(3 \vee 4) = \mu_{\underline{Apr}(A) \cap \underline{Apr}(B)}(12) = .4 \not\geq \min\{\mu_{\underline{Apr}(A) \cap \underline{Apr}(B)}(3), \mu_{\underline{Apr}(A) \cap \underline{Apr}(B)}(4)\} = .5$ . Hence  $Apr(A) \cap Apr(B)$  is not an RIFI.

**Remark 3.9.** Every RIFI is a RIFL. But the converse is not true.

Consider the lattice and the equivalence relation given in the Result 3.8. Let  $B = \{\langle x, \mu_B(x), \nu_B(x) \rangle / x \in L\}$  be given by,  $\langle 1, .2, .7 \rangle, \langle 2, .4, .4 \rangle, \langle 3, .2, .5 \rangle, \langle 4, .3, .6 \rangle, \langle 6, .5, .5 \rangle, \langle 12, .6, .3 \rangle$ . Now  $\text{Apr}(B) = (\underline{\text{Apr}}(B), \overline{\text{Apr}}(B))$ , where  $\underline{\text{Apr}}B = \{\langle x, \underline{\mu}_B(x), \underline{\nu}_B(x) \rangle\}$  is  $\langle 1, .2, .7 \rangle, \langle 2, .2, .7 \rangle, \langle 3, .2, .5 \rangle, \langle 4, .3, .6 \rangle, \langle 6, .2, .5 \rangle, \langle 12, .6, .3 \rangle$ . and  $\overline{\text{Apr}}B = \{\langle x, \overline{\mu}_B(x), \overline{\nu}_B(x) \rangle\}$  is  $\langle 1, .4, .4 \rangle, \langle 2, .4, .4 \rangle, \langle 3, .5, .5 \rangle, \langle 4, .3, .6 \rangle, \langle 6, .5, .5 \rangle, \langle 12, .6, .3 \rangle$ . It is easily verified that  $\text{Apr}(B)$  is a RIFL, But not RIFI, because  $\underline{\mu}_B(4 \wedge 6) = \underline{\mu}_B(2) = .2 \not\geq \max\{\underline{\mu}_B(4), \underline{\mu}_B(6)\} = .3$

**Remark 3.10.** If  $\text{Apr}(A)$  is a RIFI and  $\text{Apr}(B)$  is a RIFL. Then  $\text{Apr}A \cap \text{Apr}B$  is a RIFL, but not a RIFI.

Consider the the RIFI  $\text{Apr}(A)$  given in Result 3.8 and the RIFL  $\text{Apr}(B)$  given in Result 3.9. Then  $\text{Apr}(A) \cap \text{Apr}(B) = (\underline{\text{Apr}}(A) \cap \underline{\text{Apr}}(B), \overline{\text{Apr}}(A) \cap \overline{\text{Apr}}(B))$  is given by,  $\underline{\text{Apr}}(A) \cap \underline{\text{Apr}}(B) = \langle 1, .2, .7 \rangle, \langle 2, .2, .7 \rangle, \langle 3, .2, .5 \rangle, \langle 4, .3, .6 \rangle, \langle 6, .2, .5 \rangle, \langle 12, .4, .5 \rangle$  and  $\overline{\text{Apr}}(A) \cap \overline{\text{Apr}}(B) = \langle 1, .4, .4 \rangle, \langle 2, .4, .4 \rangle, \langle 3, .5, .5 \rangle, \langle 4, .3, .6 \rangle, \langle 6, .5, .5 \rangle, \langle 12, .4, .5 \rangle$ . Here it is easily verified that  $\text{Apr}(A) \cap \text{Apr}(B)$  is a RIFL, but not a RIFI because  $\underline{\mu}_B(4 \wedge 6) = \underline{\mu}_B(2) = .2 \not\geq \max\{\underline{\mu}_B(4), \underline{\mu}_B(6)\} = .3$

## 4 Intuitionistic Fuzzy Rough Set (IFRS)

In this section we introduce intuitionistic fuzzy rough sublattices and ideals, and certain characterization of intuitionistic fuzzy rough sublattice (ideal) in terms of level rough set is given.

**Definition 4.1.** Let  $\text{Apr}(A)$  is a rough set in  $(U, \theta)$  then an intuitionistic fuzzy rough set (IFRS)  $\text{Apr}(A) = (\underline{\text{Apr}}(A), \overline{\text{Apr}}(A))$ , in  $\text{Apr}(X)$  is obtained by the maps  $\mu_{\text{Apr}(A)} : \underline{\text{Apr}}(X) \rightarrow [0, 1]$  and  $\nu_{\text{Apr}(A)} : \underline{\text{Apr}}(X) \rightarrow [0, 1]$  where  $0 \leq \mu_{\text{Apr}(A)} + \nu_{\text{Apr}(A)} \leq 1$  and  $\mu_{\overline{\text{Apr}}(A)} : \overline{\text{Apr}}(X) \rightarrow [0, 1]$  and  $\nu_{\overline{\text{Apr}}(A)} : \overline{\text{Apr}}(X) \rightarrow [0, 1]$  where  $0 \leq \mu_{\overline{\text{Apr}}(A)} + \nu_{\overline{\text{Apr}}(A)} \leq 1$  and also with property  $\mu_{\text{Apr}(A)}(x) \leq \mu_{\overline{\text{Apr}}(A)}(x) \forall x \in \underline{\text{Apr}}(X)$  and  $\nu_{\overline{\text{Apr}}(A)}(x) \leq \nu_{\text{Apr}(A)}(x) \forall x \in \overline{\text{Apr}}(X)$ .

**Definition 4.2.** Let  $\text{Apr}(X)$  be a rough set and  $\text{Apr}(A) = (\underline{\text{Apr}}(A), \overline{\text{Apr}}(A))$ , is an IFRS in  $\text{Apr}(X)$ . Then we can define an *interval valued intuitionistic fuzzy set*

$$A = \{\langle x, [\underline{\mu}_{\text{Apr}(A)}(x), \mu_{\text{Apr}(A)}(x)], [\nu_{\overline{\text{Apr}}(A)}(x), \overline{\nu}_{\text{Apr}(A)}(x)] \rangle\} \text{ where,}$$

$$\underline{\mu}_{\text{Apr}(A)}(x) = \mu_{\text{Apr}(A)}(x) \text{ if } x \in \underline{\text{Apr}}(X) \quad \text{and} \quad \overline{\nu}_{\text{Apr}(A)}(x) = \nu_{\text{Apr}(A)}(x) \text{ if } x \in \underline{\text{Apr}}(X)$$

$$= 0 \text{ if } x \in \widehat{\text{Apr}}(X), \quad \quad \quad = 1 \text{ if } x \in \widehat{\text{Apr}}(X),$$

where  $\widehat{\text{Apr}}(X) = \overline{\text{Apr}}(X) - \underline{\text{Apr}}(X)$  and we denote  $\widetilde{\mu}_A(x) = [\underline{\mu}_{\text{Apr}(A)}(x), \mu_{\overline{\text{Apr}}(A)}(x)]$  and  $\widetilde{\nu}_A(x) = [\nu_{\overline{\text{Apr}}(A)}(x), \overline{\nu}_{\text{Apr}(A)}(x)]$ .

**Definition 4.3.** The family of all closed subintervals  $[0,1]$  is denoted by  $D$   $[0,1]$ . If  $D_1 = [a, b]$ ,  $D_2 = [c, d]$  then  $\max\{D_1, D_2\} = [a \vee c, b \vee d]$  and  $\min\{D_1, D_2\} = [a \wedge c, b \wedge d]$ .

**Definition 4.4.** Let  $\text{Apr}(X)$  be a rough lattice and  $\text{Apr}(A) = (\underline{\text{Apr}}(A), \overline{\text{Apr}}(A))$  is an IFRS in  $\text{Apr}(X)$ . Then  $\text{Apr}(A)$  is called a *intuitionistic fuzzy rough sublattice* (IFRL) if  $\forall x \in \overline{\text{Apr}}(X)$  the following hold.

- (i)  $\tilde{\mu}_A(x \vee y) \geq \min\{\tilde{\mu}_A(x), \tilde{\mu}_A(y)\}$ , (ii)  $\tilde{\mu}_A(x \wedge y) \geq \min\{\tilde{\mu}_A(x), \tilde{\mu}_A(y)\}$   
 (iii)  $\tilde{\nu}_A(x \vee y) \leq \max\{\tilde{\nu}_A(x), \tilde{\nu}_A(y)\}$ , (iv)  $\tilde{\nu}_A(x \wedge y) \leq \max\{\tilde{\nu}_A(x), \tilde{\nu}_A(y)\} \forall x, y \in L$ .

If conditions (ii) and (iv) are replaced by  $\tilde{\mu}_A(x \wedge y) \geq \max\{\tilde{\mu}_A(x), \tilde{\mu}_A(y)\}$  and  $\tilde{\nu}_A(x \wedge y) \leq \min\{\tilde{\nu}_A(x), \tilde{\nu}_A(y)\}$ . Then  $\text{Apr}(A)$  is called an *intuitionistic fuzzy rough ideal* (IFRI).

**Definition 4.5.** Let  $\text{Apr}(X)$  be a rough lattice and  $\text{Apr}(A) = (\underline{\text{Apr}}(A), \overline{\text{Apr}}(A))$  a IFRS in  $\text{Apr}(X)$ . Then we define  $\underline{A}_{(\alpha, \beta)} = \{x \in \underline{\text{Apr}}(X) / \mu_{\underline{\text{Apr}}(A)}(x) \geq \alpha \text{ and } \nu_{\underline{\text{Apr}}(A)}(x) \leq \beta\}$  and  $\overline{A}_{(\alpha, \beta)} = \{x \in \overline{\text{Apr}}(X) / \mu_{\overline{\text{Apr}}(A)}(x) \geq \alpha \text{ and } \nu_{\overline{\text{Apr}}(A)}(x) \leq \beta\}$ . Then  $(\underline{A}_{(\alpha, \beta)}, \overline{A}_{(\alpha, \beta)})$  is called a *level rough set*.

**Theorem 4.6.** Let  $\text{Apr}(X)$  be a rough lattice and  $\text{Apr}(A) = (\underline{\text{Apr}}(A), \overline{\text{Apr}}(A))$  is an IFRS in  $\text{Apr}(X)$ . Then  $\text{Apr}(A)$  is an IFRL iff  $(\underline{A}_{(\alpha, \beta)}, \overline{A}_{(\alpha, \beta)})$  is a rough sublattice of  $\text{Apr}(X)$ .

*Proof.* First assume that  $(\underline{A}_{(\alpha, \beta)}, \overline{A}_{(\alpha, \beta)})$  is a rough sublattice in  $\text{Apr}(X)$ . We have to prove that  $\text{Apr}(A)$  is a IFRL of  $\text{Apr}(X)$ . Set  $\min\{\tilde{\mu}_A(x), \tilde{\mu}_A(y)\} = [\alpha_0, \alpha_1]$  and  $\max\{\tilde{\nu}_A(x), \tilde{\nu}_A(y)\} = [\beta_0, \beta_1]$ . Then  $\min\{\mu_{\underline{\text{Apr}}(A)}(x), \mu_{\underline{\text{Apr}}(A)}(y)\} = \alpha_0$  and  $\min\{\mu_{\overline{\text{Apr}}(A)}(x), \mu_{\overline{\text{Apr}}(A)}(y)\} = \alpha_1$   $\max\{\nu_{\underline{\text{Apr}}(A)}(x), \nu_{\underline{\text{Apr}}(A)}(y)\} = \beta_0$  and  $\max\{\nu_{\overline{\text{Apr}}(A)}(x), \nu_{\overline{\text{Apr}}(A)}(y)\} = \beta_1$ . Then  $\mu_{\underline{\text{Apr}}(A)}(x) \geq \alpha_1, \mu_{\underline{\text{Apr}}(A)}(y) \geq \alpha_1, \nu_{\underline{\text{Apr}}(A)}(x) \leq \beta_0, \nu_{\underline{\text{Apr}}(A)}(y) \leq \beta_0$ . Hence  $x, y \in \overline{A}_{(\alpha_1, \beta_0)} \Rightarrow x \vee y, x \wedge y \in \overline{A}_{(\alpha_1, \beta_0)}$ . So  $\mu_{\overline{\text{Apr}}(A)}(x \vee y) \geq \alpha_1, \mu_{\overline{\text{Apr}}(A)}(x \wedge y) \geq \alpha_1, \nu_{\overline{\text{Apr}}(A)}(x \vee y) \leq \beta_0, \nu_{\overline{\text{Apr}}(A)}(x \wedge y) \leq \beta_0$ . Let  $x, y \in \widehat{\overline{\text{Apr}}(X)} \Rightarrow$  either  $x$  or  $y \in \widehat{\overline{\text{Apr}}(X)}$  or  $x$  and  $y \notin \widehat{\overline{\text{Apr}}(X)}$ . If  $x$  or  $y \in \widehat{\overline{\text{Apr}}(X)}$  then  $\alpha_0 = 0$  and  $\beta_1 = 1$ . So that  $\mu_{\underline{\text{Apr}}(A)}(x \vee y) \geq 0 = \alpha_0, \mu_{\underline{\text{Apr}}(A)}(x \wedge y) \geq 0 = \alpha_0, \nu_{\underline{\text{Apr}}(A)}(x \vee y) \leq 1 = \beta_1, \nu_{\underline{\text{Apr}}(A)}(x \wedge y) \leq 1 = \beta_1$ . If  $x$  and  $y \notin \widehat{\overline{\text{Apr}}(X)}$ , then  $\mu_{\underline{\text{Apr}}(A)}(x) = \mu_{\overline{\text{Apr}}(A)}(x), \mu_{\underline{\text{Apr}}(A)}(y) = \mu_{\overline{\text{Apr}}(A)}(y), \nu_{\underline{\text{Apr}}(A)}(x) = \nu_{\overline{\text{Apr}}(A)}(x), \nu_{\underline{\text{Apr}}(A)}(y) = \nu_{\overline{\text{Apr}}(A)}(y)$ . So  $\min\{\mu_{\underline{\text{Apr}}(A)}(x), \mu_{\underline{\text{Apr}}(A)}(y)\} = \alpha_0$  and  $\max\{\nu_{\underline{\text{Apr}}(A)}(x), \nu_{\underline{\text{Apr}}(A)}(y)\} = \beta_1 \Rightarrow \mu_{\underline{\text{Apr}}(A)}(x) \geq \alpha_0, \mu_{\underline{\text{Apr}}(A)}(y) \geq \alpha_0, \nu_{\underline{\text{Apr}}(A)}(x) \leq \beta_1, \nu_{\underline{\text{Apr}}(A)}(y) \leq \beta_1 \Rightarrow x, y \in \overline{A}_{(\alpha_0, \beta_1)} \Rightarrow x \vee y, x \wedge y \in \overline{A}_{(\alpha_0, \beta_1)}$ , since  $\overline{A}_{(\alpha_0, \beta_1)}$  is a sublattice. So  $\mu_{\overline{\text{Apr}}(A)}(x \vee y) \geq \alpha_0, \mu_{\overline{\text{Apr}}(A)}(x \wedge y) \geq \alpha_0, \nu_{\overline{\text{Apr}}(A)}(x \vee y) \leq \beta_1, \nu_{\overline{\text{Apr}}(A)}(x \wedge y) \leq \beta_1$ . Hence

$$\begin{aligned} \tilde{\mu}_A(x \vee y) &= [\mu_{\underline{\text{Apr}}(A)}(x \vee y), \mu_{\overline{\text{Apr}}(A)}(x \vee y)] \geq [\alpha_0, \alpha_1] = \min\{\tilde{\mu}_A(x), \tilde{\mu}_A(y)\} \\ \tilde{\mu}_A(x \wedge y) &= [\mu_{\underline{\text{Apr}}(A)}(x \wedge y), \mu_{\overline{\text{Apr}}(A)}(x \wedge y)] \geq [\alpha_0, \alpha_1] = \min\{\tilde{\mu}_A(x), \tilde{\mu}_A(y)\} \end{aligned}$$

$\tilde{\nu}_A(x \vee y) = [\nu_{\overline{Apr(A)}}(x \vee y), \overline{\nu}_{Apr(A)}(x \vee y)] \leq [\beta_0, \beta_1] = \max\{\tilde{\nu}_A(x), \tilde{\nu}_A(y)\}$   
 $\tilde{\nu}_A(x \wedge y) = [\nu_{\overline{Apr(A)}}(x \wedge y), \overline{\nu}_{Apr(A)}(x \wedge y)] \leq [\beta_0, \beta_1] = \max\{\tilde{\nu}_A(x), \tilde{\nu}_A(y)\}$ . So  $Apr(A)$  is a IFRL.

Conversely, assume that  $Apr(A)$  is an IFRL of  $Apr(X)$ . We have to prove that  $\underline{A}_{(\alpha, \beta)}$  and  $\overline{A}_{(\alpha, \beta)}$  are sublattices of  $L$ . Let  $x, y \in \underline{A}_{(\alpha, \beta)}$ , Then  $\mu_{\overline{Apr(A)}}(x) \geq \alpha, \mu_{\overline{Apr(A)}}(y) \geq \alpha, \nu_{\overline{Apr(A)}}(x) \leq \beta, \nu_{\overline{Apr(A)}}(y) \leq \beta$ . So  $\min\{\tilde{\mu}_A(x), \tilde{\mu}_A(y)\} \geq [\alpha, \min\{\mu_{\overline{Apr(A)}}(x), \mu_{\overline{Apr(A)}}(y)\}]$  and  $\max\{\tilde{\nu}_A(x), \tilde{\nu}_A(y)\} \leq [\max\{\nu_{\overline{Apr(A)}}(x), \nu_{\overline{Apr(A)}}(y)\}, \beta]$ .

Hence

$$\tilde{\mu}_A(x \vee y) \geq [\alpha, \min\{\mu_{\overline{Apr(A)}}(x), \mu_{\overline{Apr(A)}}(y)\}], \tilde{\mu}_A(x \wedge y) \geq [\alpha, \min\{\mu_{\overline{Apr(A)}}(x), \mu_{\overline{Apr(A)}}(y)\}],$$

$$\tilde{\nu}_A(x \vee y) \leq [\max\{\nu_{\overline{Apr(A)}}(x), \nu_{\overline{Apr(A)}}(y)\}, \beta] \text{ and } \tilde{\nu}_A(x \wedge y) \leq [\max\{\nu_{\overline{Apr(A)}}(x), \nu_{\overline{Apr(A)}}(y)\}, \beta].$$

From these inequalities we get  $\mu_{\overline{Apr(A)}}(x \vee y) \geq \alpha, \mu_{\overline{Apr(A)}}(x \wedge y) \geq \alpha, \nu_{\overline{Apr(A)}}(x \vee y) \leq \beta, \nu_{\overline{Apr(A)}}(x \wedge y) \leq \beta$ . Since  $x \vee y, x \wedge y \in \underline{Apr(X)}$  we have  $\mu_{\overline{Apr(A)}}(x \vee y) = \mu_{\overline{Apr(A)}}(x \vee y), \mu_{\overline{Apr(A)}}(x \wedge y) = \mu_{\overline{Apr(A)}}(x \wedge y), \nu_{\overline{Apr(A)}}(x \vee y) = \nu_{\overline{Apr(A)}}(x \vee y), \nu_{\overline{Apr(A)}}(x \wedge y) = \nu_{\overline{Apr(A)}}(x \wedge y)$ . Hence  $x \vee y, x \wedge y \in \underline{A}_{(\alpha, \beta)}$ . Thus  $\underline{A}_{(\alpha, \beta)}$  is a sublattice. Similarly if  $x, y \in \overline{A}_{(\alpha, \beta)}$  then  $\mu_{\overline{Apr(A)}}(x) \geq \alpha, \mu_{\overline{Apr(A)}}(y) \geq \alpha, \nu_{\overline{Apr(A)}}(x) \leq \beta, \nu_{\overline{Apr(A)}}(y) \leq \beta$ . So  $\min\{\tilde{\mu}_A(x), \tilde{\mu}_A(y)\} \geq [0, \alpha]$  and  $\max\{\tilde{\nu}_A(x), \tilde{\nu}_A(y)\} \leq [\beta, 1]$ . Hence  $\tilde{\mu}_A(x \vee y) \geq [0, \alpha], \tilde{\mu}_A(x \wedge y) \geq [0, \alpha], \tilde{\nu}_A(x \vee y) \leq [\beta, 1], \tilde{\nu}_A(x \wedge y) \leq [\beta, 1]$ . Hence  $\mu_{\overline{Apr(A)}}(x \vee y) \geq \alpha, \mu_{\overline{Apr(A)}}(x \wedge y) \geq \alpha, \nu_{\overline{Apr(A)}}(x \vee y) \leq \beta, \nu_{\overline{Apr(A)}}(x \wedge y) \leq \beta$ . So  $x \vee y, x \wedge y \in \overline{A}_{(\alpha, \beta)}$ . Thus  $\overline{A}_{(\alpha, \beta)}$  is a sublattice.

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**Received: November, 2010**