International Mathematical Forum, 5, 2010, no. 45, 2205 - 2211

On Foldness of Intuitionistic Fuzzy H-Ideals in BCK-Algebras

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Abstract

In this paper, we introduce the concept of intuitionistic fuzzy n-foldness of H-ideals in BCK-algebras and obtain some results.

Mathematics Subject Classification: 03E72,03F55, 03G25

Keywords: Intuitionistic fuzzy sets, BCK-algebras, n-fold H-ideals

1 Introduction and Preliminaries

After the introduction of the concept of fuzzy sets by Zadeh [9], several researchers were conducted on the generalization of the notion of fuzzy sets. The idea of intuitionistic fuzzy set was introduced by K.T.Atanassov [1, 2], as a generalization of the notion of fuzzy sets. In [5], Zhan and Tan introduced the fuzzy H-ideals of a BCK-algebras. In this paper, we introduce the concept of intuitionistic fuzzy n-fold H-ideals of BCK-algebras and obtained some related results.

A BCK-algebra is a non-empty set X with a binary operation $*$ and a constant 0 satisfying the following axioms:

(1). $(x * y) * (x * z) \leq (z * y)$,

(2). $x * (x * y) \leq y$,

(3). $x \leq x$,

(4). $x \leq y$, $y \leq x \Rightarrow x = y$,

(5). $0 \leq x$, where $x \leq y$ is defined by $x * y = 0$.

A non-empty subset I of a BCK-algebra X is called an ideal of X if $0 \in I$ and if $x * y, y \in I \Rightarrow x \in I$. We note that if x is in an ideal I of X and $y \leq x$, then $y \in I$.

An ideal I of a BCK-algebra X is called closed if $0 * x \in I$ whenever $x \in I$.
A mapping $f : X \to Y$ of BCK-algebras is called a homomorphism if $f(x*y) = f(x) * f(y)$ for all $x, y \in X$.
For any element $x, y \in X$, let us write $x * y^k$ for $(...((x * y) * y) * ...) * y$ where y occurs k times.

Definition 1.1 *An intuitionistic fuzzy set A in a non-empty set X is an object having the form* $A = \{(x, \mu_A(x), \lambda_A(x))/x \in X\}$, *where the function* $\mu_A : X \to [0, 1]$ *and* $\lambda_A : X \to [0, 1]$ *denoted the degree of membership (namely* $\mu_A(x)$*) and the degree of non membership (namely* $\lambda_A(x)$*) of each element* $x \in X$ *to the set A respectively, and* $0 \le \mu_A(x) + \lambda_A(x) \le 1$ *for all* $x \in X$.

Definition 1.2 *An intuitionistic fuzzy set* $A = (X, \mu_A, \lambda_A)$ *of X is called an intuitionistic fuzzy ideal of X, if it satisfies the following axioms:*
(IF1) $\mu_A(0) \ge \mu_A(x)$ *and* $\lambda_A(0) \le \lambda_A(x)$,
(IF2) $\mu_A(x) \ge min\{\mu_A(x * y), \mu_A(y)\}$,
(IF3) $\lambda_A(x) \le max\{\lambda_A(x * y), \lambda_A(y)\}$, *for all* $x, y \in X$.

Definition 1.3 *An intuitionistic fuzzy set* $A = (X, \mu_A, \lambda_A)$ *of X is called an intuitionistic fuzzy closed ideal of X, if it satisfies (IF2),(IF3) and the following:*
(IF4) $\mu_A(0 * x) \ge \mu_A(x)$ *and* $\lambda_A(0 * x) \le \lambda_A(x)$, *for all* $x \in X$.

Definition 1.4 *A non-empty subset I of BCK-algebra X is called an H-ideal of X if* $0 \in I$ *and* $x * (y * z), y \in I \Rightarrow x * z \in I$.

Definition 1.5 *A fuzzy subset* μ *of a BCK-algebra X is called a fuzzy H-ideal of X, if* $\mu(0) \ge \mu(x)$ *and* $\mu(x * z) \ge min\{\mu(x * (y * z)), \mu(y)\}$ *for all* $x, y, z \in X$.

Definition 1.6 *Let* μ *be a fuzzy set of X. The complement of* μ *is denoted by* $\bar{\mu}$ *and is defined as* $\bar{\mu}(x) = 1 - \mu(x)$, *for all* $x \in X$.

Definition 1.7 *Let* $A = (X, \mu_A, \lambda_A)$ *be an intuitionistic fuzzy set of X. Then (i).* $\neg A = (X, \mu_A, \bar{\mu_A})$ *and (ii).* $\diamond A = (X, \bar{\lambda_A}, \lambda_A)$.

Definition 1.8 *An intuitionistic fuzzy set* $A = (X, \mu_A, \lambda_A)$ *of a BCK-algebra X is called an intuitionistic fuzzy H-ideal of X, if*
(IFH 1) $\mu_A(0) \ge \mu_A(x)$ *and* $\lambda_A(0) \le \lambda_A(x)$,
(IFH 2) $\mu_A(x * z) \ge min\{\mu_A(x * (y * z)), \mu_A(y)\}$,
(IFH 3) $\lambda_A(x * z) \le max\{\lambda_A(x * (y * z)), \lambda_A(y)\}$, *for all* $x, y, z \in X$.

Definition 1.9 *An intuitionistic fuzzy set* $A = (X, \mu_A, \lambda_A)$ *of a BCK-algebra X is called an intuitionistic fuzzy closed H-ideal of X, if it satisfies (IFH 2),(IFH 3) and the following:*
(IFH 4) $\mu_A(0 * x) \ge \mu_A(x)$ *and* $\lambda_A(0 * x) \le \lambda_A(x)$, *for all* $x \in X$.

Definition 1.10 *Let $A = (X, \mu_A, \lambda_A)$ be an intuitionistic fuzzy set of a BCK-algebra X. The set $U(\mu_A; s) = \{x \in X/\mu_A(x) \geq s\}$ is called upper s-level of μ_A and the set $L(\lambda_A; t) = \{x \in X/\lambda_A(x) \leq t\}$ is called lower t-level of λ_A.*

2 Intuitionistic Fuzzy n-fold H-ideals

Definition 2.1 *A non-empty sub set I of a BCK-algebra X is called an n-fold H-ideal of X if (i) $0 \in I$ and (ii) there exists a fixed $n \in N$ such that $x * (y * z^n), y \in I \Rightarrow x * z^n \in I$ for all $x, y, z \in X$.*

Note: Every n-fold H-ideal is an ideal.

Definition 2.2 *A fuzzy sub set μ of a BCK-algebra X is called a fuzzy n-fold H-ideal of X if (i) $\mu(0) \geq \mu(x)$ and (ii) there exists a fixed $n \in N$ such that $\mu(x * z^n) \geq min\{\mu(x * (y * z^n)), \mu(y)\}$ for all $x, y, z \in X$.*

Definition 2.3 *An intuitionistic fuzzy set $A = (X, \mu_A, \lambda_A)$ of BCK-algebra X is called an intuitionistic fuzzy n-fold H-ideal of X if*
(IFNH 1) $\mu_A(0) \geq \mu_A(x)$ and $\lambda_A(0) \leq \lambda_A(x)$, there exists a fixed $n \in N$ such that (IFNH 2) $\mu_A(x * z^n) \geq min\{\mu_A(x * (y * z^n)), \mu_A(y)\}$
and (IFNH 3) $\lambda_A(x * z^n) \leq max\{\lambda_A(x * (y * z^n)), \lambda_A(y)\}$ for all $x, y, z \in X$.

Definition 2.4 *An intuitionistic fuzzy set $A = (X, \mu_A, \lambda_A)$ of a BCK-algebra X is said to be an intuitionistic fuzzy n-fold closed H-ideal of X, if it satisfies (IFNH 2),(IFNH 3) and the following:*
(IFNH 4) $\mu_A(0 * x) \geq \mu_A(x)$ and $\lambda_A(0 * x) \leq \lambda_A(x)$, for all $x \in X$.

Proposition 2.5 *Every intuitionistic fuzzy n-fold H-ideal is an intuitionistic fuzzy ideal.*

Lemma 2.6 *If $A = (X, \mu_A, \lambda_A)$ is an intuitionistic fuzzy H-ideal of a BCK-algebra X, then we have the following $x \leq a \Rightarrow \mu_A(x) \geq \mu_A(a)$ and $\lambda_A(x) \leq \lambda_A(a)$, for all $x, a \in X$.*

Proof. Let $x, a \in X$ such that $x \leq a \Rightarrow x * a = 0$. Consider $\mu_A(x) = \mu_A(x * 0) \geq min\{\mu_A(x * (a * 0)), \mu_A(a)\} = min\{\mu_A(x * a), \mu_A(a)\} = \mu_A(a)$ and $\lambda_A(x) = \lambda_A(x * 0) \leq max\{\lambda_A(x * (a * 0)), \lambda_A(a)\} = max\{\lambda_A(x * a), \lambda_A(a)\} = \lambda_A(a)$.

Theorem 2.7 *Let $A = (X, \mu_A, \lambda_A)$ be an intuitionistic fuzzy n-fold H-ideal of a BCK-algebra X. Then so is $\neg A = (X, \mu_A, \bar{\mu_A})$.*

Proof. We have

$$\mu_A(0) \geq \mu_A(x) \Rightarrow 1 - \bar{\mu}_A(0) \geq 1 - \bar{\mu}_A(x) \Rightarrow \bar{\mu}_A(0) \leq \bar{\mu}_A(x),$$

for any $x \in X$. Consider, for any $x, y, z \in X$,

$$\begin{aligned}
&\mu_A(x * z^n) \geq \min\{\mu_A(x * (y * z^n)), \mu_A(y)\} \\
&\Rightarrow 1 - \bar{\mu}_A(x * z^n) \geq \min\{1 - \bar{\mu}_A(x * (y * z^n)), 1 - \bar{\mu}_A(y)\} \\
&\Rightarrow \bar{\mu}_A(x * z^n) \leq 1 - \min\{1 - \bar{\mu}_A(x * (y * z^n)), 1 - \bar{\mu}_A(y)\} \\
&\Rightarrow \bar{\mu}_A(x * z^n) \leq \max\{\bar{\mu}_A(x * (y * z^n)), \bar{\mu}_A(y)\}.
\end{aligned}$$

Hence $\neg A = (X, \mu_A, \bar{\mu_A})$ is an intuitionistic fuzzy n-fold H-ideal of X.

Theorem 2.8 *Let $A = (X, \mu_A, \lambda_A)$ be an intuitionistic fuzzy n-fold H-ideal of a BCK-algebra X. Then so is $\diamond A = (X, \bar{\lambda_A}, \lambda_A)$.*

Proof. We have

$$\lambda_A(0) \leq \lambda_A(x) \Rightarrow 1 - \bar{\lambda}_A(0) \leq 1 - \bar{\lambda}_A(x) \Rightarrow \bar{\lambda}_A(0) \geq \bar{\lambda}_A(x),$$

for any $x \in X$. Consider, for any $x, y, z \in X$,

$$\begin{aligned}
&\lambda_A(x * z^n) \leq \max\{\lambda_A(x * (y * z^n)), \lambda_A(y)\} \\
&\Rightarrow 1 - \bar{\lambda}_A(x * z^n) \leq \max\{1 - \bar{\lambda}_A(x * (y * z^n)), 1 - \bar{\lambda}_A(y)\} \\
&\Rightarrow \bar{\lambda}_A(x * z^n) \geq 1 - \max\{1 - \bar{\lambda}_A(x * (y * z^n)), 1 - \bar{\lambda}_A(y)\} \\
&\Rightarrow \bar{\lambda}_A(x * z^n) \geq \min\{\bar{\lambda}_A(x * (y * z^n)), \bar{\lambda}_A(y)\}.
\end{aligned}$$

Hence $\diamond A = (X, \bar{\lambda_A}, \lambda_A)$ is an intuitionistic fuzzy n-fold H-ideal of X.

Theorem 2.9 *$A = (X, \mu_A, \lambda_A)$ be an intuitionistic fuzzy n-fold H-ideal of a BCK-algebra X if and only if $\neg A = (X, \mu_A, \bar{\mu_A})$ and $\diamond A = (X, \bar{\lambda_A}, \lambda_A)$ are intuitionistic fuzzy n-fold H-ideals of X.*

Theorem 2.10 *If $A = (X, \mu_A, \lambda_A)$ is an intuitionistic fuzzy n-fold closed H-ideal of a BCK-algebra X, then so is $\neg A = (X, \mu_A, \bar{\mu_A})$.*

Proof. For any $x \in X$, we have

$$\mu_A(0 * x) \geq \mu_A(x) \Rightarrow 1 - \bar{\mu}_A(0 * x) \geq 1 - \bar{\mu}_A(x) \Rightarrow \bar{\mu}_A(0 * x) \leq \bar{\mu}_A(x).$$

Hence $\neg A = (X, \mu_A, \bar{\mu_A})$ is an intuitionistic fuzzy n-fold closed H-ideal of X.

Theorem 2.11 *If $A = (X, \mu_A, \lambda_A)$ is an intuitionistic fuzzy n-fold closed H-ideal of a BCK-algebra X, then so is $\diamond A = (X, \bar{\lambda_A}, \lambda_A)$.*

Proof. For any $x \in X$, We have

$$\lambda_A(0 * x) \leq \lambda_A(x) \Rightarrow 1 - \bar{\lambda}_A(0 * x) \leq 1 - \bar{\lambda}_A(x) \Rightarrow \bar{\lambda}_A(0 * x) \geq \bar{\lambda}_A(x).$$

Hence, $\diamond A = (X, \bar{\lambda_A}, \lambda_A)$ is an intuitionistic fuzzy n-fold closed H-ideal of X.

Theorem 2.12 *$A = (X, \mu_A, \lambda_A)$ is an intuitionistic fuzzy n-fold closed H-ideal of a BCK-algebra X if and only if $\neg A = (X, \mu_A, \bar{\mu_A})$ and $\diamond A = (X, \bar{\lambda_A}, \lambda_A)$ are intuitionistic fuzzy n-fold closed H-ideals of BCK-algebra X.*

Theorem 2.13 *$A = (X, \mu_A, \lambda_A)$ is an intuitionistic fuzzy n-fold H-ideal of a BCK-algebra X if and only if the non-empty upper s-level cut $U(\mu_A; s)$ and the non-empty lower t-level cut $L(\lambda_A; t)$ are n-fold H-ideals of X, for any $s, t \in [0, 1]$.*

Proof. Suppose $A = (X, \mu_A, \lambda_A)$ is an intuitionistic fuzzy n-fold H-ideal of X. For any $s, t \in [0, 1]$, define the sets $U(\mu_A; s) = \{x \in X/\mu_A(x) \geq s\}$ and $L(\lambda_A; t) = \{x \in X/\lambda_A(x) \leq t\}$. Since $L(\lambda_A; t) \neq \phi$, for $x \in L(\lambda_A; t) \Rightarrow \lambda_A(x) \leq t \Rightarrow \lambda_A(0) \leq t \Rightarrow 0 \in L(\lambda_A; t)$. Let $x * (y * z^n) \in L(\lambda_A; t)$ and $y \in L(\lambda_A; t)$ implies $\lambda_A(x*(y*z^n)) \leq t$ and $\lambda_A(y) \leq t$. Since, for all $x, y, z \in X$, $\lambda_A(x * z^n) \leq max\{\lambda_A(x * (y * z^n)), \lambda_A(y)\} \leq max\{t, t\} = t \Rightarrow \lambda_A(x * z^n) \leq t$. Therefore $x * z^n \in L(\lambda_A; t)$, for all $x, y, z \in X$. Hence $L(\lambda_A; t)$ is n-fold H-ideal of X. Similarly, we can prove $U(\mu_A; s)$ is an n-fold H-ideal of X.
Conversly, suppose that $U(\mu_A; s)$ and $L(\lambda_A; t)$ are n-fold H-ideal of X, for any $s, t \in [0, 1]$. If possible, assume $a, b, c \in X$ such that $\mu_A(0) < \mu_A(a)$ and $\lambda_A(0) > \lambda_A(b)$. Put

$$s_o = \frac{1}{2}[\mu_A(0) + \mu_A(a)] \Rightarrow s_o < \mu_A(a), 0 \leq \mu_A(0) < s_o < 1 \Rightarrow a \in U(\mu_A; s_o).$$

Since $U(\mu_A; s_o)$ is n-fold H-ideal of X, we have $0 \in U(\mu_A; s_o) \Rightarrow \mu_A(0) \geq s_o$, which is contradiction. Therefore $\mu_A(0) \geq \mu_A(a)$, for all $a \in X$. Similarly by taking $t_o = \frac{1}{2}[\lambda_A(0) + \lambda_A(b)]$, we can show $\lambda_A(0) \leq \lambda_A(b)$, for any $b \in X$. If possible assume $a, b, c \in X$ such that $\mu_A(a*c^n) < min\{\mu_A(a*(b*c^n)), \mu_A(b)\}$.
Put $s_o = \frac{1}{2}[\mu_A(a * c^n) + min\{\mu_A(a * (b * c^n)), \mu_A(b)\}]$
$\Rightarrow s_o > \mu_A(a * c^n)$ and $s_o < min\{\mu_A(a * (b * c^n)), \mu_A(b)\}$
$\Rightarrow s_o > \mu_A(a * c^n), s_o < \mu_A(a * (b * c^n))$ and $s_o < \mu_A(b)$
$\Rightarrow a * c^n \notin U(\mu_A; s_O), a * (b * c^n) \in U(\mu_A; s_o)$ and $b \in U(\mu_A; s_o)$,
which is contradiction to n-fold H-ideal $U(\mu_A; s_o)$.
Therefore $\mu_A(a*c^n) \geq min\{\mu_A(a*(b*c^n)), \mu_A(b)\}$, for any $a, b, c \in X$. Similarly we can prove $\lambda_A(a * c^n) \leq max\{\lambda_A(a * (b * c^n)), \lambda_A(b)\}$, for any $a, b, c \in X$.
Hence $A = (X, \mu_A, \lambda_A)$ is an intuitionistic fuzzy n-fold H-ideal of a BCK-algebra X.

Theorem 2.14 *$A = (X, \mu_A, \lambda_A)$ is an intuitionistic fuzzy n-fold closed H-ideal of a BCK-algebra X if and only if the non-empty upper s-level cut $U(\mu_A; s)$ and the non-empty lower t-level cut $L(\lambda_A; t)$ are n-fold closed H-ideal of X, for any $s, t \in [0, 1]$.*

Corollary 2.15 *If $A = (X, \mu_A, \lambda_A)$ is an intuitionistic fuzzy n-fold H-ideal of X, then the sets $J = \{x \in X/\mu_A(x) = \mu_A(0)\}$ and $K = \{x \in X/\lambda_A(x) = \lambda_A(0)\}$ are n-fold H-ideal of X.*

Proof. Since $0 \in X, \mu_A(0) = \mu_A(0)$ and $\lambda_A(0) = \lambda_A(0)$ implies $0 \in J$ and $0 \in K$, So $J \neq \Phi$ and $K \neq \Phi$. Let $x*(y*z^n) \in J$ and $y \in J \Rightarrow \mu_A(x*(y*z^n)) = \mu_A(0)$ and $\mu_A(y) = \mu_A(0)$. Since $\mu_A(x * z^n) \geq min\{\mu_A(x * (y * z^n)), \mu_A(y)\} = \mu_A(0) \Rightarrow \mu_A(x*z^n) \geq \mu_A(0)$, but $\mu_A(0) \geq \mu_A(x*z^n)$. It follows that $x*z^n \in J$, for all $x, y, z \in X$. Hence J is n-fold H-ideal of X. Similarly we can prove K is n-fold H-ideal of X.

Definition 2.16 *Let $f : X \to X'$ be a homomorphism of BCK-algebras. For any intuitionistic fuzzy set $A = (X', \mu_A, \lambda_A)$, we define a new intuitionistic fuzzy set $A^f = (X, \mu_A^f, \lambda_A^f)$ in X by $\mu_A^f(x) = \mu_A(f(x))$, $\lambda_A^f(x) = \lambda_A(f(x))$, $x \in X$.*

Theorem 2.17 *Let X and X' be BCK-algebras and f a homomorphism from X onto X'.*
(1). If $A = (X, \mu_A, \lambda_A)$ is an intuitionistic fuzzy n-fold H-ideal of X' then $A^f = (X, \mu_A^f, \lambda_A^f)$ is an intuitionistic fuzzy n-fold H-ideal of X.
(2). If $A^f = (X, \mu_A^f, \lambda_A^f)$ is an intuitionistic fuzzy n-fold H-ideal of X then $A = (X, \mu_A, \lambda_A)$ is an intuitionistic fuzzy n-fold H-ideal of X'.

Proof. (1) For $x' \in X'$ there exists $x \in X$ such that $f(x) = x'$, we have $\mu_A^f(0) = \mu_A(f(0)) = \mu_A(0') \geq \mu_A(x') = \mu_A(f(x)) = \mu_A^f(x)$ and $\lambda_A^f(0) = \lambda_A(f(0)) = \lambda_A(0') \leq \lambda_A(x') = \lambda_A(f(x)) = \lambda_A^f(x)$.

Let $x, z, \in X, y' \in X'$ then there exists $y \in X$ such that $f(y) = y'$. We have
$\mu_A^f(x * z^n) = \mu_A(f(x * z^n)) = \mu_A(f(x) * (f(z))^n)$
$\geq min\{\mu_A(f(x) * (y' * (f(z))^n)), \mu_A(y')\}$
$=min\{\mu_A(f(x) * (f(y) * (f(z))^n), \mu_A(f(y))\}$
$=min\{\mu_A^f((x * (y * z^n)), \mu_A^f(y)\}$
and
$\lambda_A^f(x * z^n) = \lambda_A(f(x * z^n)) = \lambda_A(f(x) * (f(z))^n)$
$\leq max\{\lambda_A(f(x) * (y' * (f(z))^n)), \lambda_A(y')\}$
$=max\{\lambda_A(f(x) * (f(y) * (f(z))^n), \lambda_A(f(y))\}$
$=max\{\lambda_A^f((x * (y * z^n)), \lambda_A^f(y)\}$.
Hence $A^f = (X, \mu_A^f, \lambda_A^f)$ is an intuitionistic fuzzy n-fold H-ideal of X.
(2) Since $f : X \to X'$ onto, for $x, y, z \in X'$ there exists $a, b, c \in X$ such that $f(a) = x, f(b) = y$ and $f(c) = z$.
$\mu_A(x * z^n) = \mu_A(f(a) * (f(c))^n) = \mu_A(f(a * c^n)) = \mu_A^f(a * c^n)$
$\geq min\{\mu_A^f(a * (b * c^n)), \mu_A^f(b)\} = min\{\mu_A(f(a) * (f(b) * (f(c))^n), \mu_A(f(b))\} = min\{\mu_A(x * (y * z)), \mu_A(y)\}$ and $\lambda_A(x * z^n) = \lambda_A(f(a) * (f(c))^n) = \lambda_A(f(a * c^n)) = \lambda_A^f(a * c^n) \leq max\{\lambda_A^f(a * (b * c^n)), \lambda_A^f(b)\} = max\{\lambda_A(f(a) * (f(b) * (f(c))^n), \lambda_A(f(b))\} = min\{\lambda_A(x * (y * z)), \lambda_A(y)\}$.
Hence $A = (X, \mu_A, \lambda_A)$ is an intuitionistic fuzzy n-fold H-ideal of X'.

Definition 2.18 *Let $A = (X, \mu_A, \lambda_A)$ be an intuitionisitc fuzzy set of BCK-algebra X then we say that μ_A has 'Sup' property, if for any sub-set $T \subseteq X$*

there exist $x_o \in T$ *such that*

$$\mu_A(x_o) = \underset{t \in T}{Sup}\, \mu_A(t)$$

and we say that λ_A *has 'inf' property, if for any sub-set* $S \subseteq X$ *there exists* $y_o \in S$*such that*

$$\lambda_A(y_o) = \inf_{s \in S} \lambda_A(s)$$

.

Theorem 2.19 *Let* $f : X \to X'$ *be an onto homomorphism of BCK-algebras. If* $A = (X, \mu_A, \lambda_A)$ *is an intuitionistic n-fold H-ideal of* X *with* μ_A *has 'Sup' property and* λ_A *has 'inf' property then the immage of* A *under* f *is also intuitionistic n-fold H-ideal of* X'.

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Received: January, 2010