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The Homomorphism and Anti-Homomorphism of Level Subgroups of Fuzzy Subgroups

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Abstract

In this paper, we introduce some properties of level subgroups of fuzzy subgroups of a group with respect to the homomorphism and anti-homomorphism.

Keywords: Fuzzy subset , fuzzy subgroup , level subset of fuzzy subset , level subgroup of fuzzy subgroup, homomorphism ,anti-homomorphism

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INTRODUCTION

After the introdution of fuzzy sets by L.A.Zadeh[7] , several researchers explored on the generalization of the notion of fuzzy set. Choudhury.F.P. and Chakraborty.A.B. and Khare.S.S.[1] defined a fuzzy subgroup and fuzzy homomorphism. We introduce some properties of a fuzzy subgroup of a group with homomorphism and anti-homomorphism.

1. PRELIMINARIES

1.1 Definition :
Let X be a non–empty Universal set. A fuzzy subset A of X is a function $A : X \to [0,1]$.

1.2 Definition:
Let G be a group. A fuzzy subset A of G is said to be a fuzzy subgroup of G if it is satisfying the following axioms:
(i) $A(xy) \geq \min \{ A(x), A(y) \}$,
(ii) $A(x^{-1}) \geq A(x)$ for all $x, y \in G$.

1. 3 Definition :
Let A be a fuzzy subset of a set X. For $t \in [0, 1]$, the level subset of A is the set, $A_t = \{ x \in X : \mu_A(x) \geq t \}$. This is called a fuzzy level subset of A.

1.4 Definition :
Let A be a fuzzy subgroup of a group G. The subgroup A_t of G, for $t \in [0,1]$ such that $t \leq \mu_A(e)$ is called a level subgroup of A.

1.5 Definition :
If (G, **.**) and (G^l, **.**) are any two groups, then the function $f : G \to G^l$ is called a **group homomorphism** if $f(xy) = f(x)f(y)$, for all x and $y \in G$.

1.6 Definition :
If (G, **.**) and (G^l, **.**) are any two groups, then the function $f : G \to G^l$ is called a **group anti-homomorphism** if $f(xy) = f(y)f(x)$, for all x and $y \in G$.

1.7 Definition :
Let X and X^1 be any two sets. Let $f : X \to X^1$ be any function and let A be a fuzzy subset in X, V be a fuzzy subset in $f(X)= X^1$, defined by

$$\mu_V(y) = \sup_{x \in f^{-1}(y)} \mu_A(x) ,$$

for all $x \in X$ and $y \in X^1$. A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

1.1 Theorem :
Let G, G^l be any two groups with identity. Let $f : G \to G^l$ be a homomorphism. Then , (i) $f(1) = 1^l$ where 1 and 1^l are the identities of G and G^l respectively. (ii) $f(a^{-1}) = [f (a)]^{-1}$, for all $a \in G$.

Proof: It is trivial.

1.2 Theorem :
Let G, G^l be any two groups with identity. Let $f : G \to G^l$ be an anti-homomorphism. Then , (i) $f(1) = 1^l$ where 1 and 1^l are the identities of G and G^l respectively, and(ii) $f(a^{-1}) = [f(a)]^{-1}$, for all $a \in G$.

Proof : It is trivial.

SOME PROPOSITIONS:

1.1 Proposition:

Let A be a fuzzy subgroup of a group G. Then for $t \in [0,1]$ such that $t \leq \mu_A(e)$, A_t is a subgroup of G.

1.2 Proposition:

The homomorphic image of a fuzzy subgroup of a group G is a fuzzy subgroup of a group G^l.

1.3 Proposition :

The homomorphic pre-image of a fuzzy subgroup of a group G^l is a fuzzy subgroup of a group G.

1.4 Proposition :

The anti-homomorphic image of a fuzzy subgroup of a group G is a fuzzy subgroup of a group G^l.

1.5 Proposition :

The anti-homomorphic pre-image of a fuzzy subgroup of a group G^l is a fuzzy subgroup of a group G.

1.6 Proposition:

The homomorphic image of a level subgroup of a fuzzy subgroup of a group G is a level subgroup of a fuzzy subgroup of a group G^l.

Proof:

Let G and G^l be any two groups.
Let $f : G \rightarrow G^l$ be a homomorphism.
That is $f(xy) = f(x)f(y)$ for all x and $y \in G$.
Let $V = f(A)$, where A is a fuzzy subgroup of a group G.
Clearly V is a fuzzy subgroup of a group G^l.
Let x and $y \in G$, implies f(x) and f(y) in G^l.
Clearly A_t is a level subgroup of A.
That is $A(x) \geq t$ and $A(y) \geq t$:
$A(xy^{-1}) \geq t$.
We have to prove that $f(A_t)$ is a level subgroup of V.
Now, $V(f(x)) \geq A(x) \geq t$, implies that $V(f(x)) \geq t$:
$V(f(y)) \geq A(y) \geq t$, implies that $V(f(y)) \geq t$; and

$$V(f(x)(f(y))^{-1}) = V(f(x)f(y^{-1})), \text{ as f is a homomorphism}$$
$$= V(f(xy^{-1})), \text{ as f is a homomorphism}$$
$$\geq A(xy^{-1}) \geq t ,$$

which implies that $V(f(x)(f(y))^{-1}) \geq t$.
Therefore $V(f(x)(f(y))^{-1}) \geq t$.
Hence $f(A_t)$ is a level subgroup of a fuzzy subgroup V of a group G^l.

1.7 Proposition :

The homomorphic pre-image of a level subgroup of a fuzzy subgroup of a group G^l is a level subgroup of a fuzzy subgroup of a group G.

Proof:

Let G and G^l be any two groups.
Let f : $G \rightarrow G^l$ be a homomorphism.
That is $f(xy) = f(x)f(y)$ for all x and $y \in G$.
Let V=f(A), where V is a fuzzy subgroup of a group G^l.
Clearly A is a fuzzy subgroup of a group G.
Let f(x) and f(y) $\in G^l$, implies x and y in G.
Clearly $f(A_t)$ is a level subgroup of V.
That is $V(f(x)) \geq t$ and $V(f(y)) \geq t$;
$V(f(x)(f(y))^{-1}) \geq t$.
We have to prove that A_t is a level subgroup of A.
Now, $A(x) = V(f(x)) \geq t$, implies that $A(x) \geq t$:
$A(y) = V(f(y)) \geq t$, implies that $A(y) \geq t$; and
$A(xy^{-1}) = V(f(xy^{-1}))$,
$= V(f(x)f(y^{-1}))$, as f is a homomorphism
$= V(f(x)(f(y))^{-1})$, as f is a homomorphism
$\geq t$,
which implies that $A(xy^{-1}) \geq t$.
Therefore $A(xy^{-1}) \geq t$.
Hence A_t is a level subgroup of a fuzzy subgroup A of a group G.

1.8 Proposition:

The anti-homomorphic image of a level subgroup of a fuzzy subgroup of a group G is a level subgroup of a fuzzy subgroup of a group G^l.

Proof:

Let G and G^l be any two groups.
Let f : $G \rightarrow G^l$ be an anti-homomorphism.
That is $f(xy) = f(y)f(x)$ for all x and $y \in G$.
Let V=f(A), where A is a fuzzy subgroup of a group G.
Clearly V is a fuzzy subgroup of a group G^l.
Let x and y $\in G$, implies f(x) and f(y) in G^l.
Clearly A_t is a level subgroup of A.
That is $A(x) \geq t$ and $A(y) \geq t$:
$A(y^{-1}x) \geq t$.
We have to prove that $f(A_t)$ is a level subgroup of V.
Now, $V(f(x)) \geq A(x) \geq t$, implies that $V(f(x)) \geq t$:
$V(f(y)) \geq A(y) \geq t$, implies that $V(f(y)) \geq t$; and
$V(f(x)(f(y))^{-1}) = V(f(x)f(y^{-1}))$, as f is an anti-homomorphism
$= V(f(y^{-1}x))$, as f is an anti-homomorphism
$\geq A(y^{-1}x) \geq t$,
which implies that $V(f(x)(f(y))^{-1}) \geq t$.
Therefore $V(f(x)(f(y))^{-1}) \geq t$.
Hence $f(A_t)$ is a level subgroup of a fuzzy subgroup V of a group G^l.

1.9 Proposition :

The anti-homomorphic pre-image of a level subgroup of a fuzzy subgroup of a group G^l is a level subgroup of a fuzzy subgroup of a group G.

Proof:

Let G and G^{l} be any two groups.
Let $f : G \to G^{l}$ be an anti-homomorphism.
That is $f(xy) = f(y)f(x)$, for all x and $y \in G$.
Let $V = f(A)$, where V is a fuzzy subgroup of a group G^{l}.
Clearly A is a fuzzy subgroup of a group G.
Let f(x) and $f(y) \in G^{l}$, implies x and y in G.
Clearly $f(A_t)$ is a level subgroup of V.
That is $V(f(x)) \geq t$ and $V(f(y)) \geq t$;
$V((f(y))^{-1}f(x)) \geq t$.
We have to prove that A_t is a level subgroup of A.
Now, $A(x) = V(f(x)) \geq t$, implies that $A(x) \geq t$:
$A(y) = V(f(y)) \geq t$, implies that $A(y) \geq t$; and
$A(xy^{-1}) = V(f(xy^{-1}))$,
$= V(f(y^{-1})f(x))$, as f is an anti-homomorphism
$= V((f(y))^{-1}f(x))$, as f is an anti-homomorphism
$\geq t$,
which implies that $A(xy^{-1}) \geq t$.
Therefore $A(xy^{-1}) \geq t$.
Hence A_t is a level subgroup of a fuzzy subgroup A of a group G.

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