International Mathematical Forum, 5, 2010, no. 44, 2149 - 2155

Fuzzy Local Languages

S. Gnanasekaran[1]

Department of Mathematics
Periyar Arts College
Cuddalore-607 001, India
sg_vianna@yahoo.co.in

Abstract

Berstel and Pin have characterized local languages by introducing local automaton. In this paper fuzzy local language is introduced and the fuzzy local languages recognized by partial fuzzy local automata in max-min and min-max compositions are studied. Some closure properties of fuzzy local languages are provided.

Mathematics Subject Classification: 68Q45

Keywords: local automaton, local language, fuzzy sets, fuzzy automaton, fuzzy language

1 Introduction

Fuzzy sets have been introduced by Zadeh [8] and since then have appeared in many fields of sciences. They have been studied within automata theory for the first time by Wee [7]. Fuzzy automata provide a reliable formal base for the theory of computing with words. Formal languages are precise while natural languages are quite imprecise. To reduce a gap between these two constructs, it becomes advantageous to introduce fuzziness into the structures of formal languages. This leads to the concept of fuzzy languages. Zadeh started the studies of fuzzy languages accepted by fuzzy automata. More on the recent development of algebraic theory of fuzzy automata and fuzzy languages can be found in the book by Mordeson and Malik [6]

Berstel and Pin [3] have defined local automata and shown that a language is local if and only if it is recognized by a local automaton. Beal [1] has given a more general definition of local automata. Caron [4] has made use of an

[1]The work reported here has been supported by UGC-File No. MRP-2529/08(UGC-SERO) dated 18.03.2008

equivalent definition in order to generalize the result stated by Berstel and Pin.

In this paper partial fuzzy local automaton is introduced. The fuzzy languages recognized by partial fuzzy local automata in max-min and min-max compositions,called fuzzy local languages are considered and some closure properties of these classes of languages are given.

2 Preliminaries

2.1 Local languages

Let Σ be a finite alphabet. We denote by Σ^*, the free monoid generated by Σ. The elements of Σ^* are called finite words. For each finite word $u \in \Sigma^*$, we denote by $P(u)$, the prefix of u of length 1 and by $F_2(u)$, the set of all factors of u of length 2. We denote by $S(u)$, the suffix of u of length 1. A subset of Σ^* is called a *language* over Σ. A language L $\subseteq \Sigma^*$ is called *local* if there exists a triple (I,C,J) where $I, J \subseteq \Sigma$ and $C \subseteq \Sigma^2$ such that

$$L = \{u \in \Sigma^* : P(u) \in I, F_2(u) \subseteq C, S(u) \in J\}.$$

The class of all local languages is closed under intersection but not closed under union. If L_1 and L_2 are local languages over disjoint alphabets, then $L_1 \cup L_2$ is also a local languages.

2.2 Fuzzy automata

A *deterministic finite automaton* (DFA) is a system M = (Q,Σ, δ, q_0,F) where Q is a finite set of states, Σ is a finite set of symbols, called an alphabet, $q_0 \in Q$ is the initial state, $F \subseteq Q$ is a set of final states and $\delta : Q \times \Sigma \to Q$ is a transition funtion. $L \subseteq \Sigma^*$ is recognized by M if $L = \{u \in \Sigma^* : \delta(q_0, u) \in F\}$ and we write $L = L(M)$. A deterministic finite automaton M = (Q,Σ, δ, q_0,F) is said to be *local*, if, for every $a \in \Sigma$, the set $\{\delta(q, a) : q \in Q\}$ contains at most one element.

A *fuzzy subset* of a set Σ is a mapping $\alpha : \Sigma \to [0, 1]$. A *fuzzy language* over Σ is a fuzzy subset of Σ^*. A *partial fuzzy automation* (pfa) is a tuple $(Q, \Sigma, \delta, i, \tau)$ where Q, Σ and $\delta : Q \times \Sigma \to Q$ are defined as in deterministic finite automaton, i is a fuzzy subset of Q, called initial states and τ is a fuzzy subset of Q, called final states.

3 Partial fuzzy local automaton

In this section partial fuzzy automaton is introduced and fuzzy local languages are defined.

Definition 3.1 *A partial fuzzy automaton (pfa)* $(Q, \Sigma, \delta, i, \tau)$ *is called a partial fuzzy local automaton (pfla) if,* $\forall a \in \Sigma$*, the set* $\{\delta(q, a) : q \in Q\}$ *contains at most one element.*

Definition 3.2 *Let* $(Q, \Sigma, \delta, i, \tau)$ *be a partial fuzzy local automaton. The max-min fuzzy local language by* M *is a fuzzy subset* $L_\vee(M)$ *of* Σ^* *defined by*

$$L_\vee(M)(u) = \bigvee_{q \in Q} (i(q) \wedge \tau(\delta(q, u)))$$

and min-max fuzzy local language by M *is a fuzzy subset* $L_\wedge(M)$ *of* Σ^* *defined by*

$$L_\wedge(M)(u) = \bigwedge_{q \in Q} (i(q) \vee \tau(\delta(q, u)))$$

Definition 3.3 *A fuzzy subset* λ *of* Σ^* *is called* $F_\vee$*-local (*$F_\wedge$*-local) if there exists a pfla* M *such that* $\lambda = L_\vee(M)(\lambda = L_\wedge(M))$*.*

Theorem 3.4 *Let* L *be a local language over* Σ*. Then the characteristic function* χ_L *of* L *is a* $F_\vee$*-local language.*

Proof Since L is a local language over Σ, there exists a deterministic finite local automaton, $M' = (Q, \Sigma, \delta, q_0, F)$, such that L is recognized by M'. Consider the pfla $M = (Q, \Sigma, \delta, i, \tau)$, where $i : Q \to [0, 1]$ is defined by $i(q_0) = 1$ and $i(q) = 0$ if $q \neq q_0$ and $\tau : Q \to [0, 1]$ is defined by $\tau(q) = 1$ if $q \in F$ and $\tau(q) = 0$ if $q \notin F$. Let $u \in \Sigma^*$. Then

$$\begin{aligned} L_\vee(M)(u) &= \bigvee_{q \in Q} (i(q) \wedge \tau(\delta(q, u))) \\ &= \tau\left(\delta(q_0, u)\right) \\ &= \begin{cases} 1 & \text{if } \delta(q_0, u) \in F \\ 0 & \text{if } \delta(q_0, u) \notin F \end{cases} \\ &= \chi_L(u). \end{aligned}$$

Thus $L_\vee(M) = \chi_L(u)$.

Theorem 3.5 *Let* $L \subset \Sigma^*$*. Suppose the characteristic function* χ_L *of* L *is a* $F_\vee$*-local language. Then* L *is a union of local languages over* Σ*.*

Proof Since χ_L of L is a $F_\vee$-local language, there exists a pfla, $M = (Q, \Sigma, \delta, i, \tau)$ such that $L_\vee(M) = \chi_L$. Thus

$$\bigvee_{q \in Q} (i(q) \wedge \tau(\delta(q, u))) = \begin{cases} 1 & \text{if } u \in L \\ 0 & \text{if } u \notin L \end{cases}$$

Now $u \in L$ if and only if there exists $q' \in Q$ such that $i(q') = 1$ and $\tau(\delta(q', u)) = 1$. Let

$$Q_0 = \{q \in Q : i(q) = 1\}$$

and for $q \in Q_0$,

$$F = \{\delta(q, u) \in Q : \tau(\delta(q, u)) = 1 \text{ for some } u \in L\}.$$

Then $Q_0 \neq \phi$ and $F \neq \phi$. For $q \in Q_0$, let $M_q = (Q, \Sigma, \delta, q, F)$ and let L_q be the language recognized by M_q. Then $L_q \subseteq L$ and each L_q is a local language, since each M_q is a deterministic local automaton. Hence $\bigcup_{q \in Q_0} L_q \subseteq L$. Let $u \in L$. Then $\bigvee_{q \in Q} (i(q) \wedge \tau(\delta(q, u))) = 1$. Hence there exists $q \in Q_0$ such that $i(q) \wedge \tau(\delta(q, u)) = 1$. Then $\tau(\delta(q, u)) = 1$ and so $\delta(q, u) \in F$. Thus $u \in L_q$. Hence $\bigcup_{q \in Q_0} L_q = L$ and so L is union of local languages.

Theorem 3.6 *Let L be a local language over Σ. Then the characteristic function χ_L of L is a $F_\wedge$-local language.*

Proof Since L is a local language over Σ, there exists a deterministic finite local automaton, $M' = (Q, \Sigma, \delta, q_0, F)$, such that L is recognized by M'. Consider the pfla $M = (Q, \Sigma, \delta, i, \tau)$, where $i : Q \to [0, 1]$ is defined by $i(q_0) = 0$ and $i(q) = 1$ if $q \neq q_0$ and $\tau : Q \to [0, 1]$ is defined by $\tau(q) = 1$ if $q \in F$ and $\tau(q) = 0$ if $q \notin F$. Let $u \in \Sigma^*$. Now

$$L_\wedge(M)(u) = \bigwedge_{q \in Q} (i(q) \vee \tau(\delta(q, u))).$$

Let $u \notin L$. Then $\chi_L(u) = 0$ and $\delta(q_0, u) \notin F$. Hence $i(q_0) = 0$ and $\tau(\delta(q_0, u)) = 0$. Thus $L_\wedge(M)(u) = 0$. Suppose $u \in L$. Then $\chi_L(u) = 1$. If $q = q_0$, then $i(q_0) = 0$ and $\tau(\delta(q_0, u)) = 1$ and therefore $i(q) \vee \tau(\delta(q, u)) = 1$. If $q \neq q_0$, then $i(q_0) = 1$ and therefore $i(q) \vee \tau(\delta(q, u)) = 1$. Thus $L_\wedge(M) = \chi_L$.

Theorem 3.7 *Let $L \subset \Sigma^*$. Suppose the characteristic function χ_L of L is a $F_\wedge$-local language. Then L is a local language over Σ.*

Proof Since χ_L of L is a $F_\wedge$-local language, there exists a pfla, $M = (Q, \Sigma, \delta, i, \tau)$, such that $L_\wedge(M) = \chi_L$. Thus

$$\bigwedge_{q \in Q} (i(q) \vee \tau(\delta(q, u))) = \begin{cases} 1 & \text{if } u \in L \\ 0 & \text{if } u \notin L \end{cases}$$

Now $u \in L$ if and only if for all $q \in Q$ either $i(q) = 1$ or $\tau(\delta(q, u)) = 1$. Let

$$Q_0 = \{q \in Q : i(q) = 1\}$$

and for all $q \in Q \setminus Q_0$,

$$F_q = \bigcup_{u \in \Sigma^*} \{\delta(q, u) \in Q : \tau(\delta(q, u)) = 1\}.$$

For all $q \in Q \setminus Q_0$, let $M_q = (Q, \Sigma, \delta, q, F_q)$ and let L_q be the language recognized by M_q. Then each L_q is a local language, since each M_q is a deterministic local automata. Then

$$\begin{aligned} y \in \bigcap_{q \in Q\setminus Q_0} L_q &\Leftrightarrow y \in L_q, \forall q \in Q \setminus Q_0 \\ &\Leftrightarrow \delta(q, y) \in F_q, \forall q \in Q \setminus Q_0 \\ &\Leftrightarrow \tau(\delta(q, y)) = 1, \forall q \in Q \setminus Q_0 \\ &\Leftrightarrow y \in L. \end{aligned}$$

Hence $L = \bigcap_{q \in Q\setminus Q_0} L_q$ and therefore L is a local language.

Theorem 3.8 *If λ is a $F_\vee$-local language over Σ, then $\overline{\lambda} = 1 - \lambda$ is a $F_\wedge$-local language over Σ.*

Proof Since λ is a $F_\vee$-local language, there exists a pfla, $M = (Q, \Sigma, \delta, i, \tau)$ such that $L_\vee(M) = \lambda$. Let $M^c = (Q, \Sigma, \delta, \overline{i}, \overline{\tau})$ where $\overline{i} = 1 - i$ and $\overline{\tau} = 1 - \tau$. Then M^c is a pfla and $\forall u \in \Sigma^*$,

$$\begin{aligned} L_\wedge(M^c)(u) &= \bigwedge_{q \in Q} \left(\overline{i}(q) \vee \overline{\tau}(\delta(q, u))\right) \\ &= \bigwedge_{q \in Q} ((1 - i(q)) \vee (1 - \tau(\delta(q, u)))) \\ &= 1 - \bigvee_{q \in Q} (i(q) \wedge \tau(\delta(q, u))) \\ &= 1 - \lambda(u) \\ &= \overline{\lambda}(u) \end{aligned}$$

Then $\mathrm{L}_\wedge(M^c) = \overline{\lambda}$.

Thus $\overline{\lambda}$ is a $F_\wedge$-local language.

Theorem 3.9 *If λ_1 and λ_2 are $F_\vee$-local languages over Σ, then $\lambda_1 \wedge \lambda_2$ is a $F_\vee$-local language over Σ.*

Proof Since λ_1 and λ_2 are $F_\vee$-local languages, there exists partial fuzzy local automata, $M_1 = (Q_1, \Sigma, \delta_1, i_1, \tau_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, i_2, \tau_2)$ such that $L_\vee(M_1) = \lambda_1$ and $L_\vee(M_2) = \lambda_2$. Let

$$M = (Q_1 \times Q_2, \Sigma, \delta_1 \times \delta_2, i_1 \times i_2, \tau = \tau_1 \times \tau_2).$$

Now

$$L_\vee(M)(u) = \bigvee_{(p,q) \in Q_1 \times Q_2} ((i_1 \times i_2)(p, q) \wedge \tau((\delta_1 \times \delta_2)((p, q), u)))$$

$$
\begin{aligned}
&= \bigvee_{(p,q)\in Q_1\times Q_2} (i_1(p) \wedge i_2(q) \wedge \tau_1 \times \tau_2(\delta_1(p,u), \delta_2(q,u))) \\
&= \bigvee_{(p,q)\in Q_1\times Q_2} (i_1(p) \wedge i_2(q) \wedge \tau_1(\delta_1(p,u)) \wedge \tau_2(\delta_2(q,u))) \\
&= \left(\bigvee_{p\in Q_1} (i_1(p) \wedge \tau_1(\delta_1(p,u)))\right) \bigwedge \left(\bigvee_{p\in Q_2} (i_2(p) \wedge \tau_2(\delta_2(p,u)))\right) \\
&= L_\vee(M_1)(u) \wedge L_\vee(M_2)(u) \\
&= \lambda_1(u) \wedge \lambda_2(u) \\
&= (\lambda_1 \wedge \lambda_2)(u).
\end{aligned}
$$

Hence $L_\vee(M) = \lambda_1 \wedge \lambda_2$.

Theorem 3.10 *If λ_1 and λ_2 are $F_\wedge$-local languages over Σ, then $\lambda_1 \wedge \lambda_2$ is a $F_\wedge$-local language over Σ.*

Proof Since λ_1 and λ_2 are $F_\wedge$-local languages, there exists partial fuzzy local automata, $M_1 = (Q_1, \Sigma, \delta_1, i_1, \tau_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, i_2, \tau_2)$ such that $L_\wedge(M_1) = \lambda_1$ and $L_\wedge(M_2) = \lambda_2$. Let

$$M = (Q_1 \times Q_2, \Sigma, \delta_1 \times \delta_2, i_1 \times i_2, \tau = \tau_1 \times \tau_2).$$

Now

$$
\begin{aligned}
L_\vee(M)(u) &= \bigwedge_{(p,q)\in Q_1\times Q_2} ((i_1 \times i_2)(p,q) \wedge \tau((\delta_1 \times \delta_2)((p,q),u))) \\
&= \bigwedge_{(p,q)\in Q_1\times Q_2} (i_1(p) \wedge i_2(q) \wedge \tau_1 \times \tau_2(\delta_1(p,u), \delta_2(q,u))) \\
&= \bigwedge_{(p,q)\in Q_1\times Q_2} (i_1(p) \wedge i_2(q) \wedge \tau_1(\delta_1(p,u)) \wedge \tau_2(\delta_2(q,u))) \\
&= \left(\bigwedge_{p\in Q_1} (i_1(p) \wedge \tau_1(\delta_1(p,u)))\right) \bigwedge \left(\bigwedge_{p\in Q_2} (i_2(p) \wedge \tau_2(\delta_2(p,u)))\right) \\
&= L_\wedge(M_1)(u) \wedge L_\wedge(M_2)(u) \\
&= \lambda_1(u) \wedge \lambda_2(u) \\
&= (\lambda_1 \wedge \lambda_2)(u).
\end{aligned}
$$

Hence $L_\wedge(M) = \lambda_1 \wedge \lambda_2$.

References

[1] M.P. Beal, Codes circulaires, automates locaux et entropic, *Theoretical Computer Science,* **57** (1988), 283-302.

[2] D. Beauquier and J.E. Pin, Languages and scanners, *Theoretical Computer Science,* **84** (1991), 3-21.

[3] J. Berstel and J.E. Pin, Local languages and the Berry-Sethi algorithm, *Theoretical Computer Science,* **155** (1996), 439-446.

[4] P. Caron, Families of locally testable languages, *Theoretical Computer Science,* **242** (2000), 361-376.

[5] J.E. Hopcroft and J.D. Ullman *Introduction to Automata Theory, Languages and Computation*, Addison-Wesley, 1979.

[6] J.N. Mordeson and D.S. Malik *Fuzzy automata and languages*, Chapman & Hall/CRC, 2002.

[7] W.G. Wee and K.S. Fu, A formation of fuzzy automata and its application as a model of learning system *IEEE Trans. Syst. Sci. Cybern.,* **5(3)**(1969), 215-223.

[8] L.A. Zadeh, Fuzzy sets, *Information and Control*, **8** (1965), 338-353.

Received: February, 2010