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Subordinations for Some Classes of p-Valent Meromorphic Functions

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Abstract

We obtain certain subordinations for some classes of p-valent meromorphic functions. Some consequences are also mentioned. To see the sharpness of the results extremal functions are also provided.

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1 Introduction

Let M_p denotes the class of functions of the form

$$f(z) = \frac{1}{z^p} + \sum_{n=1}^{\infty} a_n z^n, \ p \in N = \{1, 2, ...\} \tag{1.1}$$

which are analytic in $U^* = \{z : 0 < |z| < 1\} = U \setminus \{0\}$.

For two functions $f(z)$ and $g(z)$ analytic in the unit disk U, we say that the function $f(z)$ is subordinate to $g(z)$ in U, written as $f(z) \prec g(z)$, $z \in U$ if there exists a Schwarz function $\omega(z)$, analytic in U with $\omega(0) = 0$ and $|\omega(z)| < 1$ such that $f(z) = g(\omega(z))$, $z \in U$. If g is univalent in U, $f \prec g$ means that $f(0) = g(0)$ and $f(U) \subset g(U)$.

Let A be the class of all functions of the form

$$p(z) = 1 + \sum_{n=1}^{\infty} p_n z^n \tag{1.2}$$

which are analytic in U. We denote by P, a class of functions $p(z) \in A$ satisfying $Re(p(z)) > 0$ or, in terms of subordination $p(z) \prec \left(\frac{1+z}{1+Bz}\right)^{\gamma}$for $0 < \gamma \leq 1, -1 \leq B < 1$, $z \in U$. In fact we denote by P_q, a class of functions $p(z) \in A$, if $p(z) \prec q(z)$ where q is univalent in U. Thus, if $q(z)=\left(\frac{1+Az}{1+Bz}\right)^{\gamma}$for $-1 < A \leq 1$, $-1 \leq B < 1$, $A \neq B$ and $0 < \gamma \leq 1$, then $P_q \equiv P(\gamma, A, B)$. Also $P(1, 1-2\alpha, -1) \equiv P(\alpha)$, $0 \leq \alpha < 1$. Note that $P(\gamma, 1, B) \equiv P$ for $0 < \gamma \leq 1$, $-1 \leq B < 1$. Clearly $P(0) \equiv P$.

A function $f(z) \in M_p$ is called p-valently meromorphic starlike of order $\alpha (0 \leq \alpha < 1)$ if $f(z) \neq 0$ and

$$-Re\left(\frac{zf'(z)}{pf(z)}\right) > \alpha \ (z \in U) \tag{1.3}$$

and the class of such functions is denoted by $M_p^s(\alpha)$.

Also, a function $f(z) \in M_p$ is called p-valently meromorphic close-to-starlike and close-to-convex respectively of order $\alpha (0 \leq \alpha < 1)$ if

$$Re\left(z^p f(z)\right) > \alpha \ (z \in U) \tag{1.4}$$

and

$$-Re\left(\frac{z^{p+1} f'(z)}{p}\right) > \alpha \ (z \in U) \tag{1.5}$$

and respective classes of such functions are denoted by $M_p^{cs}(\alpha)$ and $M_p^{cc}(\alpha)$.

Again, a function $f(z) \in M_p$ is called p-valently meromorphic strongly starlike of order $\gamma (0 < \gamma \leq 1)$ if $f(z) \neq 0$ and

$$|\arg\left(-\frac{zf'(z)}{pf(z)}\right)| < \gamma\frac{\pi}{2} \ (z \in U).$$

and its class is denoted by $\widetilde{M}_p^s(\gamma)$. A function $f(z) \in M_p$ is called p-valently meromorphic strongly close-to-starlike and strongly close-to-convex respectively of order $\gamma (0 < \gamma \leq 1)$ and its respective classes are denoted by $\widetilde{M}_p^{cs}(\gamma)$ and $\widetilde{M}_p^{cc}(\gamma)$ if

$$|\arg\left(z^p f(z)\right)| < \gamma\frac{\pi}{2} \ \text{ and } \ |\arg\left(-\frac{z^{p+1} f'(z)}{p}\right)| < \gamma\frac{\pi}{2}.$$

Note that $\widetilde{M}_p^s(1) \equiv \widetilde{M}_p^s, \widetilde{M}_p^{cs}(1) \equiv \widetilde{M}_p^{cs}$ and $\widetilde{M}_p^{cc}(1) \equiv \widetilde{M}_p^{cc}$.

Remark 1.1 *In view of equations* (1.3), (1.4) *and* (1.5) *if* $p(z) = -\frac{zf'(z)}{pf(z)}$, $z^p f(z)$ *and* $-\frac{z^{p+1}f'(z)}{p}$ *respectively, we denote the classes of* $p(z)$ *defined above by replacing* P *by* M_p^s, M_p^{cs} *and* M_p^{cc}.

In the theory of meromorphic functions Nunokawa and Ahuja [2], Ravichandran et al. [3] etc. have obtained some sufficient conditions for a meromorphic function to be in certain classes. Motivated with the work of Xu and Yang [4] in the theory of analytic functions, in this paper we find certain subordinations under which a p-valently meromorphic function belongs to some classes defined above. Sharpness of our results can be seen by considering some extremal functions.

In order to obtain our results, we need the following result of Miller and Mocanu [1].

Lemma 1.1 *Let* $q(z)$ *be univalent in the unit disk* U *and* θ *and* ϕ *be analytic in a domain* E *containing* $q(U)$ *with* $\phi(w) \neq 0$ *when* $w \in q(U)$. *set*

$$Q(z) = zq'(z)\phi(q(z)), \;\; h(z) = \theta(q(z)) + Q(z)$$

and suppose that either $Q(z)$ *is starlike or* $h(z)$ *is convex in* U.*In addition, assume that*

$$Re\frac{zh'(z)}{Q(z)} > 0, \;\; z \in U.$$

If $p(z)$ *is analytic in* U *with* $p(0) = q(0)$, $p(U) \subseteq E$ *and*

$$\theta(p(z)) + zp'(z)\phi(p(z)) \prec \theta(q(z)) + zq'(z)\phi(q(z))$$

then $p(z) \prec q(z)$ *and* $q(z)$ *is the best dominant.*

2 Main Results

We first derive following two Lemmas with the use of Lemma 1.1. Our results are direct consequence of these Lemmas.

Lemma 2.1 *Let* $q(z)$ *be univalent, convex in* U *and for* $\lambda > 0$

$$Re\left(2\lambda q(z) + 1 + \frac{zq''(z)}{q'(z)}\right) > 0 \;\; (z \in U).$$

If $p(z)$ *is analytic in* U *with* $p(0) = q(0)$ *and*

$$\lambda(p(z))^2 + zp'(z) \prec \lambda(q(z))^2 + zq'(z)$$

then $p(z) \in P_q$ *and* $q(z)$ *is the best dominant.*

Proof. Consider for $\lambda > 0$

$$\theta(w) = \lambda w^2, \ \ \phi(w) = 1, \ (w \in \mathbb{C})$$

which are analytic in $\mathbb{C}$ so that

$$\theta(p(z)) + zp'(z)\phi(p(z)) = \lambda(p(z))^2 + zp'(z).$$

Set,

$$Q(z) = zq'(z)\phi(q(z)), \ \ h(z) = \theta(q(z)) + Q(z)$$

we obtain that

$$Q(z) = zq'(z)$$

which is starlike in U and

$$\text{Re}\frac{zh'(z)}{Q(z)} = \text{Re}\left[2\lambda q(z) + 1 + \frac{zq''(z)}{q'(z)}\right] > 0.$$

Hence on applying Lemma 1.1 we get that $p(z) \prec q(z)$ or $p(z) \in P_q$ and $q(z)$ is the best dominant. ■

Lemma 2.2 *Let $q(z)$ be univalent in U with $q(z)q'(z) \neq 0$ and $\frac{zq'(z)}{q(z)}$ is starlike in U and for $\lambda > 0$*

$$\text{Re}\left[\lambda q(z) + 1 + \frac{zq''(z)}{q'(z)} - \frac{zq'(z)}{q(z)}\right] > 0 \ \ (z \in U).$$

If $p(z)$ is analytic in U with $p(z) \neq 0$, $p(0) = q(0)$ and satisfy

$$\lambda p(z) + \frac{zp'(z)}{p(z)}) \prec \lambda q(z) + \frac{zq'(z)}{q(z)}$$

then $p(z) \in P_q$ and $q(z)$ is the best dominant.
Proof. Consider for $\lambda > 0$

$$\theta(w) = \lambda w, \ \ \phi(w) = \frac{1}{w}, \ (w \in \mathbb{C}/\{0\})$$

which are analytic in $\mathbb{C}\backslash\{0\}$ so that

$$\theta(p(z)) + zp'(z)\phi(p(z)) = \lambda p(z) + \frac{zp'(z)}{p(z)}.$$

Set,

$$Q(z) = zq'(z)\phi(q(z)), \ \ h(z) = \theta(q(z)) + Q(z)$$

we get that

$$Q(z) = \frac{zq'(z)}{q(z)}$$

which is starlike in U and

$$\mathrm{Re}\frac{zh'(z)}{Q(z)} = \mathrm{Re}\left[\lambda q(z) + 1 + \frac{zq''(z)}{q'(z)} - \frac{zq'(z)}{q(z)}\right] > 0.$$

Hence by Lemma 1.1 we get that $p(z) \prec q(z)$ or $p(z) \in P_q$ and $q(z)$ is the best dominant. ■

Now applying Lemma 2.1 and Lemma 2.2 we prove following sufficient subordinate conditions for $f(z) \in M_p$ to be in aforementioned classes.

Theorem 2.1 *Let $f(z) \in M_p$ satisfies for $z \in U$, $f(z)f'(z) \neq 0$ and for $-1 < A \leq 1$, $-1 \leq B < 1$, $A \neq B$, $0 < \gamma \leq 1$, $\lambda > 0$*

$$\frac{zf'(z)}{pf(z)}\left[\left(\frac{\lambda}{p}+1\right)\frac{zf'(z)}{f(z)} - \frac{zf''(z)}{f'(z)} - 1\right] \prec h(z) \tag{2.1}$$

where

$$h(z) = \lambda\left(\frac{1+Az}{1+Bz}\right)^{2\gamma} + \frac{\gamma(A-B)z}{(1+Az)^{1-\gamma}(1+Bz)^{1+\gamma}}$$

then $f(z) \in M_p^s(\gamma, A, B)$, $\left(\frac{1+Az}{1+Bz}\right)^{\gamma}$ being best dominant and the result is sharp with extremal function

$$f(z) = z^{-p}\exp\left\{-p\int_0^z \frac{1}{t}\left[\left(\frac{1+At}{1+Bt}\right)^{\gamma} - 1\right]dt\right\} \tag{2.2}$$

Proof. Consider, for $z \in U$, $-1 < A \leq 1$, $-1 \leq B < 1$, $A \neq B$, $0 < \gamma \leq 1, q(z) = \left(\frac{1+Az}{1+Bz}\right)^{\gamma}$

$$q(z) = \left(\frac{1+Az}{1+Bz}\right)^{\gamma}$$

we get that

$$\begin{aligned}\mathrm{Re}\left\{1 + \frac{zq''(z)}{q'(z)}\right\} &= \mathrm{Re}\left\{1 + (\gamma-1)\frac{Az}{1+Az} - (\gamma+1)\frac{Bz}{1+Bz}\right\} \\ &= -1 + (1-\gamma)\mathrm{Re}\frac{1}{1+Az} + (1+\gamma)\mathrm{Re}\frac{1}{1+Bz} \\ &> -1 + \frac{1-\gamma}{1+|A|} + \frac{1+\gamma}{1+|B|} > 0.\end{aligned}$$

which shows that $q(z)$ is univalent, convex in U. And for $\lambda > 0$ we have

$$\begin{aligned}\mathrm{Re}\left(2\lambda q(z) + 1 + \frac{zq''(z)}{q'(z)}\right) &= \mathrm{Re}\left\{2\lambda\left(\frac{1+Az}{1+Bz}\right)^{\gamma}\right\} + \mathrm{Re}\left\{1 + \frac{zq''(z)}{q'(z)}\right\} \\ &> 2\lambda\left\{\mathrm{Re}\left(\frac{1+Az}{1+Bz}\right)\right\}^{\gamma} > 2\lambda\left(\frac{1-A}{1-B}\right)^{\gamma} \geq 0.\end{aligned}$$

Let $p(z) = -\frac{zf'(z)}{pf(z)}$ which is analytic in U with $p(0) = q(0) = 1$ and by (2.1)

$$\lambda(p(z))^2 + zp'(z) \prec \lambda(q(z))^2 + zq'(z)$$

therefore, by Lemma 2.1, we conclude that for $z \in U$,

$$-\frac{zf'(z)}{pf(z)} \prec \left(\frac{1+Az}{1+Bz}\right)^{\gamma}$$

or, $f(z) \in M_p^s(\gamma, A, B)$, $\left(\frac{1+Az}{1+Bz}\right)^{\gamma}$ is the best dominant and we see that equality attains for the function given by (2.2). ■

Remark 2.1 *(1)Taking $A = 1$ condition (2.1) is a sufficient condition for $f(z) \in M_p^s$. (2). Taking $A = 1$ and $B = -1$ condition (2.1) proves that*

$$-\frac{zf'(z)}{pf(z)} \prec \left(\frac{1+z}{1-z}\right)^{\gamma}$$

and hence

$$\left|\arg\left(-\frac{zf'(z)}{f(z)}\right)\right| < \gamma\left|\arg\left(\frac{1+z}{1-z}\right)\right| < \gamma\frac{\pi}{2}\,(z \in U)$$

which proves that $f(z) \in \widetilde{M}_p^s(\gamma)$.

Corollary 2.1 *Let $f(z) \in M_p$ satisfies for $z \in U$, $f(z)f'(z) \neq 0$ and for $\lambda > 0$, $0 \leq \alpha < 1$*

$$\frac{zf'(z)}{pf(z)}\left[\left(\frac{\lambda}{p} + 1\right)\frac{zf'(z)}{f(z)} - \frac{zf''(z)}{f'(z)} - 1\right] \prec h(z) \tag{2.3}$$

where

$$h(z) = \frac{\lambda(1-2\alpha)^2z^2 + 2\{(\lambda+1) - (2\lambda+1)\alpha\}z + \lambda}{(1-z)^2}$$

then $f(z) \in M_p^s(\alpha)$. The result is sharp for the function

$$f(z) = z^{-p}(1-z)^{2p(1-\alpha)} \tag{2.4}$$

Proof. Taking $A = 1 - 2\alpha$, $B = -1$ and $\gamma = 1$ in Theorem 2.1 and using (2.3), we get

$$-\frac{zf'(z)}{pf(z)} \prec \frac{1+(1-2\alpha)z}{1-z}$$

hence $f(z) \in M_p^s(\alpha)$. The equality attains for the function given by (2.4). ■

Corollary 2.2 *Let $f(z) \in M_p$ satisfies for $z \in U$, $f(z)f'(z) \neq 0$ and for $\lambda > 0$, $-1 < A \leq 1$*

$$\frac{zf'(z)}{pf(z)}\left[\left(\frac{\lambda}{p}+1\right)\frac{zf'(z)}{f(z)} - \frac{zf''(z)}{f'(z)} - 1\right] \prec \lambda A^2 z^2 + (2\lambda+1)Az + \lambda \tag{2.5}$$

then $f(z) \in M_p^s$. The extremal function is given by

$$f(z) = z^{-p}\exp\left(-pAz\right). \tag{2.6}$$

Proof. Taking $B = 0$ and $\gamma = 1$ in Theorem 2.1 and using (2.5), we get for $-1 < A \leq 1$,

$$-\frac{zf'(z)}{pf(z)} \prec 1 + Az$$

which proves that $|\frac{zf'(z)}{pf(z)} + 1| < |A|$ hence $f(z) \in M_p^s$. Equality attains for the function given by (2.6). ■

Theorem 2.2 *Let $f(z) \in M_p$ satisfies for $z \in U$, $f(z)f'(z) \neq 0$ and for $-1 < A \leq 1$, $-1 \leq B < 1$, $A \neq B$, $0 < \gamma \leq 1$, $\lambda > 0$*

$$1 + \frac{zf''(z)}{f'(z)} - \frac{zf'(z)}{f(z)}\left(\frac{\lambda}{p}+1\right) \prec \lambda\left(\frac{1+Az}{1+Bz}\right)^{\gamma} + \frac{\gamma(A-B)z}{(1+Az)(1+Bz)} \tag{2.7}$$

then $f(z) \in M_p^s(\gamma, A, B)$. The extremal function is given by (2.2).

Proof. Consider for $z \in U$, $-1 < A \leq 1$, $-1 \leq B < 1$, $A \neq B$, $0 < \gamma \leq 1$

$$q(z) = \left(\frac{1+Az}{1+Bz}\right)^{\gamma}$$

we have $q'(z) = q(z)\left[\frac{\gamma(A-B)}{(1+Az)(1+Bz)}\right]$, clearly, $q(z)q'(z) \neq 0$ and $Q(z) := \frac{zq'(z)}{q(z)}$ is starlike in U. Since for $-1 < A \leq 1$, $-1 \leq B < 1$, $A \neq B$,

$$\mathrm{Re}\left\{\frac{zQ'(z)}{Q(z)}\right\} = \mathrm{Re}\left\{\frac{1}{1+Az} - 1 + \frac{1}{1+Bz}\right\} > \frac{1}{1+|A|} - 1 + \frac{1}{1+|B|} > 0.$$

Also

$$\mathrm{Re}\left[\lambda q(z) + 1 + \frac{zq'(z)}{q'(z)} - \frac{zq'(z)}{q(z)}\right] = \lambda \mathrm{Re}\left(\frac{1+Az}{1+Bz}\right)^{\gamma} + \mathrm{Re}\left(\frac{zQ'(z)}{Q(z)}\right),\ z \in U.$$

$$> \lambda\left\{\mathrm{Re}\left(\frac{1+Az}{1+Bz}\right)\right\}^{\gamma} > \lambda\left(\frac{1-A}{1-B}\right)^{\gamma} \geq 0$$

Let $p(z) = -\frac{zf'(z)}{pf(z)}$ which is analytic in U with $p(0) = q(0) = 1$ and by (2.7)

$$\lambda p(z) + \frac{zp'(z)}{p(z)} \prec \lambda q(z) + \frac{zq'(z)}{q(z)}$$

therefore, by Lemma 2.2, we conclude that $p(z) \prec q(z)$ for $z \in U$ or,

$$-\frac{zf'(z)}{pf(z)} \prec \left(\frac{1+Az}{1+Bz}\right)^{\gamma}$$

hence $f(z) \in M_p^s(\gamma, A, B)$. The sharpness can be seen by the function given by (2.2). ■

We have similar Remark 2.1 for condition (2.7).

Taking $A = 1-2\alpha$, $B = -1$, $\gamma = 1$ in Theorem 2.2 we get following result.

Corollary 2.3 *If $f(z) \in M_p$ satisfies for $z \in U$, $f(z)f'(z) \neq 0$ and for $\lambda > 0$, $0 \leq \alpha < 1$*

$$1 + \frac{zf''(z)}{f'(z)} - \frac{zf'(z)}{f(z)}\left(\frac{\lambda}{p} + 1\right) \prec h(z)$$

where

$$h(z) = \frac{\lambda(1-2\alpha)^2 z^2 + 2\{(\lambda+1) - (2\lambda+1)\alpha\}z + \lambda}{1 - 2\alpha z - (1-2\alpha)z^2}$$

then $f(z) \in M_p^s(\alpha)$. The result is sharp for the function given by (2.4).

Letting $B = 0$ and $\gamma = 1$ in Theorem 2.2 we obtain following result.

Corollary 2.4 *If $f(z) \in M_p$ satisfies for $z \in U$, $f(z)f'(z) \neq 0$ and for $\lambda > 0$, $-1 < A \leq 1$*

$$1 + \frac{zf''(z)}{f'(z)} - \frac{zf'(z)}{f(z)}\left(\frac{\lambda}{p} + 1\right) \prec \frac{\lambda A^2 z^2 + (2\lambda + 1)Az + \lambda}{1 + Az}$$

then $f(z) \in M_p^s$. The result is sharp for the function given by (2.6).

Theorem 2.3 *Let $f(z) \in M_p$ satisfies for $z \in U$, $f(z) \neq 0$ and for $-1 < A \leq 1$, $-1 \leq B < 1$, $A \neq B$, $0 < \gamma \leq 1$, $\lambda > 0$*

$$z^p f(z)\left[\lambda z^p f(z) + p + \frac{zf'(z)}{f(z)}\right] \prec h(z) \tag{2.8}$$

where

$$h(z) = \lambda\left(\frac{1 + Az}{1 + Bz}\right)^{2\gamma} + \frac{\gamma(A - B)z}{(1 + Az)^{1-\gamma}(1 + Bz)^{1+\gamma}}$$

then $f(z) \in M_p^{cs}(\gamma, A, B)$ and the result is sharp with extremal function

$$f(z) = z^{-p}\exp\left\{\gamma\int_0^z \frac{(A - B)}{(1 + At)(1 + Bt)}dt\right\}. \tag{2.9}$$

Proof. Theorem can be easily proved on the simillar lines of the proof of Theorem 2.1 for $p(z) = z^p f(z)$. ■

Theorem 2.4 *Let $f(z) \in M_p$ satisfies for $z \in U$, $f(z) \neq 0$ and for $-1 < A \leq 1$, $-1 \leq B < 1$, $A \neq B$, $0 < \gamma \leq 1$, $\lambda > 0$*

$$\lambda z^p f(z) + p + \frac{zf'(z)}{f(z)} \prec h(z) \tag{2.10}$$

where

$$h(z) = \lambda\left(\frac{1 + Az}{1 + Bz}\right)^{\gamma} + \frac{\gamma(A - B)z}{(1 + Az)(1 + Bz)}$$

then $f(z) \in M_p^{cs}(\gamma, A, B)$ and the result is sharp with extremal function given by (2.9).

Proof. Theorem can be easily proved on the simillar lines of the proof of Theorem 2.2 for $p(z) = z^p f(z)$. ■

Theorem 2.5 *Let $f(z) \in M_p$ satisfies for $z \in U$, $f'(z) \neq 0$ and for $-1 < A \leq 1$, $-1 \leq B < 1$, $A \neq B$, $0 < \gamma \leq 1$, $\lambda > 0$*

$$\frac{z^{p+1}f'(z)}{p}\left[\frac{\lambda}{p}z^{p+1}f'(z) - (p+1) + \frac{zf''(z)}{f'(z)}\right] \prec h(z) \tag{2.11}$$

where

$$h(z) = \lambda\left(\frac{1+Az}{1+Bz}\right)^{2\gamma} + \frac{\gamma(A-B)z}{(1+Az)^{1-\gamma}(1+Bz)^{1+\gamma}}$$

then $f(z) \in M_p^{cc}(\gamma, A, B)$ and the result is sharp with extremal function

$$f(z) = -p\int_0^z t^{-p-1}\left(\frac{1+At}{1+Bt}\right)^{\gamma} dt. \tag{2.12}$$

Proof. Theorem can be easily proved on the simillar lines of the proof of Theorem 2.1 for $p(z) = \frac{-z^{p+1}f(z)}{p}$. ■

Theorem 2.6 *Let $f(z) \in M_p$ satisfies for $z \in U$, $f'(z) \neq 0$ and for $-1 < A \leq 1$, $-1 \leq B < 1$, $A \neq B$, $0 < \gamma \leq 1$, $\lambda > 0$*

$$(p+1) - \frac{zf''(z)}{f'(z)} - \frac{\lambda}{p}z^{p+1}f'(z) \prec h(z) \tag{2.13}$$

where

$$h(z) = \lambda\left(\frac{1+Az}{1+Bz}\right)^{\gamma} + \frac{\gamma(A-B)z}{(1+Az)(1+Bz)}$$

then $f(z) \in M_p^{cc}(\gamma, A, B)$ and the result is sharp with extremal function give by (2.12).

Proof. Theorem can be easily proved on the simillar lines of the proof of Theorem 2.2 for $p(z) = \frac{-z^{p+1}f(z)}{p}$. ■

Remark 2.2 *Consequences of Theorem 2.3, 2.4, 2.5 and **2.6** can be obtained similar to Remark 2.1 and Corollary 2.1, 2.2, 2.3 and **2.4**.*

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