International Mathematical Forum, 5, 2010, no. 29, 1417 - 1428

Congestion with Fuzzy Data; An Application of DEA

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Abstract

In most applications of Data Envelopment Analysis (DEA), the models presented are designed to obtain a measure of efficiency. Therefore, DEA is a mathematical programming approach that uses the production frontiers to assess relative efficiency. DEA based congestion in input occurs whenever reducing one or some inputs can increase one or some outputs and increasing one or some inputs decreases some outputs without improving other inputs or outputs.
In this paper we provide an extension to the DEA based congestion concept for applying an empirical application with fuzzy data.

Keywords: Data Envelopment Analysis, Efficiency, Benchmark

1 Introduction

Data Envelopment Analysis (DEA) is a non-parametric method for evaluating the relative efficiency of Decision Making Units (DMUs) of multiple inputs and multiple outputs. The original DEA models (Charnes et al (1978), Banker et al. (1984), Charnes et al. (1985)) assume that inputs and outputs are measured by exact values and the value of efficiency be assessed on base relationship between the evaluated unit and its projection point on efficient frontier.

The original DEA-based congestion assumes that input and outputs are measured by exact values on a ratio scale. However, this assumption may not be true, in the sense that some inputs and outputs may be only known as in forms of bounded or fuzzy data. This paper is organized as follows: in section 2 we give a concept of congestion, section 3 introduces fuzzy data also an application example is given in section 4, and in section 5, conclusion is put forward.

2 Preliminaries

The most frequently used DEA model is the CCR model, name after Charnes, Cooper and Rhodes (1978) and BCC model, name after Banker, Charnes, Cooper (1984). Assume n decision making units $DMU_j,\ j \in \{1,..,n\}$ that each using m inputs produce s outputs. Also assume that $X_j = (x_{1j}, ..., x_{mj})$ and $Y_j = (y_{1j}, ..., y_{sj})$ be inputs and outputs vectors DMU_j , where, $X_j \geq 0, X_j \neq 0, Y_j \geq 0, Y_j \neq 0$. The relative efficiency score of the $DMU_o, o \in \{1, ..., n\}$ is obtained from the following model which is called output-oriented $\epsilon - BCC$ envelopment model.

$$
\begin{array}{lll}
max & \phi + \epsilon(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+) & \\
s.t. & \Sigma_{j=1}^{n}\lambda_j x_{ij} + s_i^- = x_{io}, & i = 1, ..., m \\
& \Sigma_{j=1}^{n}\lambda_j y_{rj} - s_r^+ = \phi y_{ro}, & r = 1, ..., s \\
& \Sigma_{j=1}^{n}\lambda_j = 1, & \\
& \lambda_j,\ s_i^-,\ s_r^+ \ \geq 0, & j = 1, ..., n,\ i = 1, ..., m,\ r = 1, ..., s.
\end{array}
\qquad (1)
$$

Where, $\epsilon > 0$ is a non-Archimedean element defined to be smaller than any positive real number. This means that ϵ is not a real number. If ϕ^* be the optimal value of evaluating DMU_o, then DMU_o is called (strong) efficient if and only if $\phi^* = 1$ and all slack variables be zero in all optimal solutions of model (1).

2.1 Congestion

Congestion accrues whenever reducing some inputs can increase outputs, or increasing some inputs can decrease outputs. The first approach due to Fare et al. (FGL, 1985) and the latter approach, due to Cooper et al. (CTT, 1996) was extended by Brockett et al. (BCSW, 1998), to treat tradeoff possibilities between employment and output in Chinese production when congestion is present.

Cooper et al. (2002) introduced the one-model approach for evaluating congestion as follows:

$$\begin{array}{lll} max & z = \phi + \epsilon(\sum_{r=1}^{s} s_r^+ - \epsilon \sum_{i=1}^{m} s_i^{-c}) & \\ s.t. & \Sigma_{j=1}^{n}\lambda_j x_{ij} + s_i^{-c} = x_{io}, & i = 1, ..., m \\ & \Sigma_{j=1}^{n}\lambda_j y_{rj} - s_r^+ = \phi y_{ro}, & r = 1, ..., s \\ & \Sigma_{j=1}^{n}\lambda_j = 1, & \\ & \lambda_j,\ s_i^{-c},\ s_r^+ \ \geq 0, & j = 1, ..., n,\ i = 1, ..., m,\ r = 1, ..., s. \end{array} \qquad (2)$$

Note that, in fact, model (2) is composed of three models. Suppose that in evaluating DMU_o, $(\phi^*, \lambda^*, S^{+*}, S^{-c*})$ is an optimal solution of model (2), then S^{-c*} is the congestion amount it (Fig. 1.).

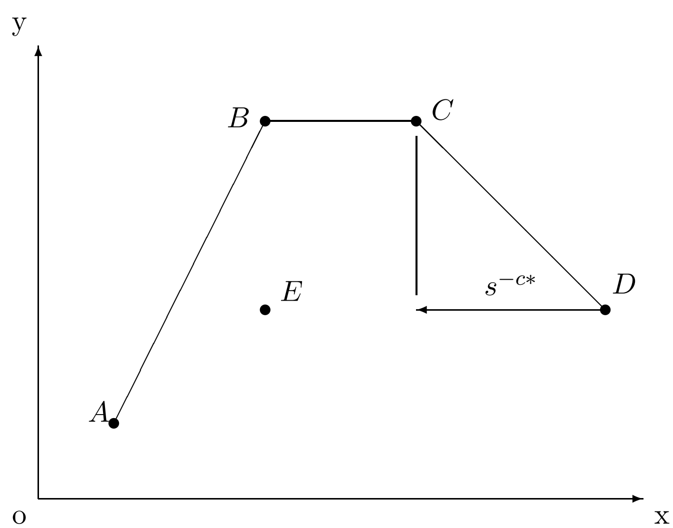

Fig. 1. The present of congestion

3 Fuzzy Linear Programming

3.1 Preliminaries

3.1.1 Definition

An ordered pair of functions $\tilde{u} = (\underline{u}(r), \overline{u}(r))$, $0 \leq r \leq 1$, is called a fuzzy number if and only if it satisfied in the following requirements.

1. $\underline{u}(r)$ is a bounded left continues non-decreasing function over [0,1].
2. $\overline{u}(r)$ is a bounded left continues non-increasing function over [0,1].
3. $\underline{u}(r)$ and $\overline{u}(r)$ are right continues in 0.
4. $\underline{u}(r) \leq \overline{u}(r)$, $0 \leq r \leq 1$.

A crisp number α is simply represented by $\underline{u}(r) = \overline{u}(r) = \alpha$, $0 \leq r \leq 1$.

3.1.2 Definition

Let $\tilde{a} = (a^l, a^u, \alpha, \beta)$ denoted the trapezoidal fuzzy number, where $[a^l - \alpha, a^u + \beta]$ is the support of $\tilde{a}$ and $[a^l, a^u]$ is modal set.

3.1.1 Remark

We denote the set of trapezoidal fuzzy numbers by $F(R)$. If $a = a^l = a^u$ then we obtain a triangular fuzzy number, and we show it as follows:

$\tilde{a} = (\underline{a}(r), \overline{a}(r)) = (a + \alpha(r-1), a + \beta(1-r))$

3.1.1 Theorem

If $\tilde{a} = (\underline{a}(r), \overline{a}(r)), \tilde{b} = (\underline{b}(r), \overline{b}(r))$ be two fuzzy numbers, $k \in R$ then:

1. $\tilde{a} = \tilde{b}$ if and only if $\underline{a}(r) = \underline{b}(r)$ and $\overline{a}(r) = \overline{b}(r)$.
2. $\tilde{a} + \tilde{b} = (\underline{a}(r) + \underline{b}(r), \overline{a}(r) + \overline{b}(r))$.
3. $k\tilde{a} = \begin{cases} (k\underline{a}(r), k\overline{a}(r)), & k>0 \\ (k\overline{a}(r), k\underline{a}(r)), & k<0 \end{cases}$

Several methods for solving fuzzy linear programming problems have represented, Fang (1999) , Lai and Hwang (1992), Maleki et al. (2000). One of the most convenient of these methods is based on the concept of comparison of fuzzy numbers by use of ranking functions. In fact, an efficient approach for ordering the elements of $F(R)$ is to define a ranking function $\tau : F(R) \rightarrow R$ which maps each fuzzy number into the line, where a natural order exist. We define orders on $F(R)$ by

1. $\tilde{a} \succeq \tilde{b}$ if and only if $\tau(\tilde{a}) \geq \tau(\tilde{b})$.
2. if $\tilde{a} \succ \tilde{b}$ and only if $\tau(\tilde{a}) > \tau(\tilde{b})$.
3. if $\tilde{a} \simeq \tilde{b}$ and only if $\tau(\tilde{a}) = \tau(\tilde{b})$.

Where $\tilde{a}$ and $\tilde{b}$ are in $F(R)$. The following lemma is now immediate.

3.1.1. Lemma

Let τ be any linear ranking function. Then

1. $\tilde{a} \succeq \tilde{b}$ iff $\tilde{a} - \tilde{b} \succeq 0$ iff $-\tilde{b} \succeq -\tilde{a}$.

2. $\tilde{a} \succeq \tilde{b}$ and $\tilde{c} \succeq \tilde{d}$ iff $\tilde{a} + \tilde{c} \succeq \tilde{b} + \tilde{d}$.

We restrict our attention to linear ranking function τ : $\tau(k\tilde{a} + \tilde{b}) = k\tau(\tilde{a}) + \tau(\tilde{b})$

For any $\tilde{a}$ and $\tilde{b}$ belong to $F(R)$ and any $k \in R$.

Here, we introduce a linear ranking function is similar to the ranking function adopted by Maleki (FJMS, 2002). For a fuzzy number $\tilde{a} = (\underline{a}(r), \overline{a}(r))$, we use ranking function as follows:

$\tau(\tilde{a}) = \frac{1}{2}\int_0^1(\underline{a}(r) + \overline{a}(r))dr$. This reduces to $\tau(\tilde{a}) = \frac{1}{2}(a^l + a^u + \frac{1}{2}(\beta - \alpha))$.

3.1.2 Remark

Suppose that $\tilde{a} = (a^l, a^u, \alpha, \beta)$, $\tilde{b} = (b^l, b^u, \gamma, \delta)$ be two fuzzy numbers. Then

$\tilde{a} \succeq \tilde{b} \Leftrightarrow (a^l + a^u + \frac{1}{2}(\beta - \alpha)) \geq (b^l + b^u + \frac{1}{2}(\delta - \gamma))$.

3.2 Fuzzy Linear programming Problem

Authors who use ranking function for comparison of fuzzy linear programming problems usually define a crisp model which is equivalent to the fuzzy linear programming and then use optimal solution of this model as the optimal solution of fuzzy linear programming problems. We now define fuzzy linear programming problem and the corresponding crisp model.

3.2.1 Definition

A fuzzy linear programming problem (FLP) is as follows:

$$\begin{aligned} &min \quad z \simeq cx \\ &S.t. \quad \tilde{A}x \simeq \tilde{b} \\ &\qquad\quad x \succeq 0 \end{aligned}$$

Where " $\simeq$ " and " $\succeq$ " mean equality and inequality with respect to the ranking function τ , and $\tilde{A} = [\tilde{a}_{ij}]_{mn}$, $\tilde{c} = (\tilde{c_1}, ..., \tilde{c_n})$, $\tilde{b} = (\tilde{b_1}, ..., \tilde{b_m})$, $x = (x_1, ..., x_n)$, and $\tilde{a_{ij}}, \tilde{b_i}, \tilde{c_j} \in F(R)$, $x_j \in R$. for $i = 1, ..., m$, $j = 1, ..., n$.

3.2.2 Definition

Any x which satisfies the set of constraints of FLP is called a feasible solution. Let $\tilde{S}$ be the set of all feasible solution of FLP. We say that $x^* \in \tilde{S}$ is an optimal feasible solution for FLP iff $\tilde{c}x^* \preceq \tilde{c}x$ for all $x \in \tilde{S}$.

3.2.3 Definition

We say that the real number a is corresponds to the fuzzy number $\tilde{a}$, with respect to a given linear ranking function τ , if $a = \tau(\tilde{a})$. However, the following theorem shows that any FLP can be converted to a linear programming problem.

3.2.1 Theorem

The following linear programming problem (LP) and the FLP are equivalent:

$$\begin{aligned} min \quad & z \simeq cx \\ S.t. \quad & \tilde{A}x \simeq \tilde{b} \\ & x \succeq 0 \end{aligned}$$

$$\begin{aligned} min \quad & z = cx \\ S.t. \quad & \tilde{A}x = \tilde{b} \\ & x \geq 0 \end{aligned}$$

Where $a_{ij},\ b_i,\ c_j$ are real numbers corresponding to the fuzzy numbers $\tilde{a_{ij}}, \tilde{b_i}, \tilde{c_j}$ with respect to a given linear ranking function τ , respectively.

3.3 Fuzzy dea (FDEA) and congestion with fuzzy data

Data envelopment analysis is a widely applied approach for measuring the relative efficiencies of a set of decision making units which use multiple inputs to produce multiple outputs. When some observations are fuzzy, the efficiencies become fuzzy as well. Fuzzy DEA (FDEA) models take the form of fuzzy linear programming model. The following formulated model with fuzzy coefficients is in output-oriented envelopment form for evaluating congestion:

$$\begin{aligned} max \quad & z = \phi + \epsilon(\textstyle\sum_{r=1}^{s} s_r^{+} - \epsilon \sum_{i=1}^{m} s_i^{-c}) \\ s.t. \quad & \Sigma_{j=1}^{n}\lambda_j \tilde{x_{ij}} + s_i^{-c} \simeq \tilde{x_{io}}, \qquad i = 1, ..., m \\ & \Sigma_{j=1}^{n}\lambda_j \tilde{y_{rj}} - s_r^{+} \simeq \phi\tilde{y_{ro}}, \qquad r = 1, ..., s \\ & \Sigma_{j=1}^{n}\lambda_j \simeq \tilde{1}, \\ & \lambda_j,\ s_i^{-c},\ s_r^{+}\ \geq 0, \qquad j = 1, ..., n,\ i = 1, ..., m,\ r = 1, ..., s. \end{aligned} \tag{3}$$

We know the fuzzy model cannot be solved by a crisp model because coefficients in the fuzzy model are fuzzy sets. By considerate the ranking function it is seen that every optimal feasible solution of FDEA model is an optimal feasible solution of DEA model. Hence we consider below model (4), where $x_{ij},\ y_{rj}$ are real number corresponding to the fuzzy number $\tilde{x}_{ij}$, $\tilde{y}_{rj}$ in above model (3) with respect to a given linear ranking function τ , respectively.

$$\begin{aligned} max \quad & z = \phi + \epsilon(\sum_{r=1}^{s} s_r^+ - \epsilon \sum_{i=1}^{m} s_i^{-c}) \\ s.t. \quad & \Sigma_{j=1}^{n}\lambda_j x_{ij} + s_i^{-c} = x_{io}, \qquad i = 1, ..., m \\ & \Sigma_{j=1}^{n}\lambda_j y_{rj} - s_r^+ = \phi y_{ro}, \qquad r = 1, ..., s \\ & \Sigma_{j=1}^{n}\lambda_j = 1, \\ & \lambda_j,\ s_i^{-c},\ s_r^+ \geq 0, \qquad j = 1, ..., n,\ i = 1, ..., m,\ r = 1, ..., s. \end{aligned} \tag{4}$$

4 An application example

Consider twenty branches of Tehran Social Security Insurance Organization at this section. Each branch uses four inputs on order to produce four outputs. Tables of inputs and outputs are given in below.

Table 1. The labels of inputs and outputs

	Inputs	Outputs
1	The number of personals	The total number of insured persons
2	The total number of computes	The number of insured persons agreements
3	The area of the branch	The total number of life-pension receivers
4	A administrative expenses	The receipt total sum (Income)

The set of data are given as follows. The triangular fuzzy inputs and outputs are given in Tables 2 , 3 respectively. It is assumed that “M” as number middle, “U” as number up and “L” as number low:

Table 2. The triangular fuzzy inputs for 20 branches of Tehran Social Security Insurance Organization

DMU_s	Im1	Iu1	Il1	Im2	Iu2	Il2	Im3	IU3	IL3	Im4	Iu4	IL4
1	98	100	94	84	86	83	4000	4000	4000	61196520.56	103656656	14730450
2	78	81	75	95	95	94	2565	2565	2565	66287094.89	95701909	41144517
3	78	80	77	88	89	87	1343	1343	1343	47612874.67	69423818	28792550
4	92	94	91	94	94	93	1500	1500	1500	349278138.6	2592824900	24277018
5	91	92	89	87	87	87	1680	1680	1680	68356757.22	80729482	43355800
6	103	105	102	98	98	97	3750	3750	3750	75508661.33	139483237	8425500
7	96	100	94	94	97	91	3313	3313	3313	114264317.2	171152176	78011675
8	86	90	83	94	95	92	1500	1500	1500	74950922.22	135858469	14969393
9	106	112	102	95	101	92	1600	1600	1600	106720450.6	182951858	61024310
10	107	111	103	98	100	97	1725	1725	1725	66010355.44	94511449	21625962
11	96	101	93	80	82	79	1920	1920	1920	86613046.78	124984300	39664990
12	78	79	76	92	93	91	4433	4433	4433	69942333.33	139888000	23106000
13	103	105	102	105	106	105	2500	2500	2500	78657408.78	144462162	17756770
14	87	90	82	100	102	95	2800	2800	2800	78951533.89	168961589	30082208
15	80	82	77	93	95	92	1630	1630	1630	69126959.33	158581843	46388243
16	90	91	89	86	86	85	1127	1127	1127	146206909	827138457	35631132
17	90	93	84	105	106	104	3400	3400	3400	107289969.8	256783575	33983780
18	106	116	94	95	96	95	1304	1304	1304	165532950.4	389185592	37568447
19	100	103	97	101	102	98	4206	4206	4206	68355245.33	100855390	5504769
20	85	88	82	100	101	100	1340	1340	1340	92342642.33	136748402	52060934

Table 3. The triangular fuzzy outputs for 20 branches of Tehran Social Security Insurance Organization

DMU_s	Om1	Ou1	Ol1	Om2	Ou2	Ol2	Om3	Ou3	Ol3	Om4	Ou4	Ol4
1	5703	5848	5583	40.8	58	30	1269	1350	1117	175	219	145
2	3687	3711	3674	18.5	35	0	8543.3	8776	8385	313.1	486	175
3	3868	3944	3800	20.2	32	11	6594.7	6603	6588	274.8	585	113
4	3593	3665	3546	32.4	59	10	10516.5	10821	8083	248.5	341	128
5	5445	5608	5292	30.1	44	9	9684.6	9955	9493	221.6	299	101
6	7227	7857	7025	11.7	19	0	8022	8752	7536	329	615	82
7	3662	3953	3258	101.2	129	47	14513.3	15264	13121	264.6	392	154
8	4636	5002	4290	17.1	27	11	1622.5	1661	1563	226.1	634	54
9	8606	8785	8453	71.4	111	43	10645	11080	10206	297.8	616	179
10	4724	4780	4692	26.2	36	9	6824.3	7472	6608	215.7	342	117
11	3897	4429	3155	184.1	242	81	12226	12582	11996	178.3	286	37
12	3821	3962	2716	21.7	31	11	7561.7	7731	7422	157.4	184	124
13	5834	6054	5614	45.2	77	30	7584	7936	7380	269.5	430	185
14	8847	9146	8042	40	66	28	661.2	707	630	136.2	302	51
15	5049	5072	5021	14.3	24	6	10264.1	10293	10247	125.1	295	28
16	4790	4985	4016	25	37	15	7491.5	7786	7302	188.5	286	85
17	5958	8222	5292	19.8	29	14	4952.7	5205	4740	157.3	240	109
18	8307	8911	7734	23.4	33	13	4917	5151	4745	141.5	247	72
19	5102	8633	4615	17.5	25	13	1528.3	1636	825	217.7	477	129
20	2965	3303	2797	77.3	325	23	14766.3	15125	14473	306.2	411	190

After using of ranking function τ , the data is given as crisp in Table 4.

Table 4. The crisp inputs and outputs for 20 branches of Tehran Social Security Insurance Organization

DMUs	I1	I2	I3	I4	O1	O2	O3	O4
1	98.2	84.3	4000	61196521	57.02	40.7	1268	174
2	78.5	95	2565	66287095	3687.5	18.4	8543	313
3	78.2	88	1344	47612873	3869	20.5	6594	274.8
4	91.8	94.2	1502	349278137	3593.5	32.6	10516	248.5
5	90.5	87.3	1682	68356758	5445.9	30.9	9684	221.6
6	103	97.5	3750	75508662	7226.5	11.6	8025	329.5
7	96	94.3	3312	114264318	3661	101.2	14514	264.6
8	86.8	95	1501	7495092	4635	17.5	1623	226.8
9	105	95	1600	106720451	8605	71.4	10644	297.8
10	107.2	97.5	1726	6601034	4724.4	26.6	6824	215.2
11	96.5	80.5	1925	86613046	3897.8	184.2	12226	178.5
12	78	92	4435	69942335	3822.5	21.7	7564	158.4
13	103.6	104	2500	78657407	5833	45.8	7583	266.5
14	87.8	100.5	2800	78951533	8848.5	40.5	662	137.2
15	80.5	92.8	1632	69126958	5048	14.0	10265	124.1
16	91	86.8	1125	146206908	4792	25.5	7492	188.5
17	90.2	105	3402	107289969	5956	19.0	4953	156.5
18	105	95	1304	165532951	8305	23.8	4917	142.8
19	99	101.2	4205	68355245	5100	17.8	1529	212.5
20	85.8	100	1341	92342642	2963	77.5	14764	305.4

Eventually, with using model (4), we obtain the congestion value for inefficient four units 4, 13, 17 and 19 which is given in Table 5.

Table 5. The results of assessment congestion in input-oriented for inefficient units

Units	Objective value	s_1^{-c}	s_2^{-c}	s_3^{-c}	s_4^{-c}
4	1.1360	3.8056	0	0	265800662.3
13	1.1027	47.168	8.5575	0	0
17	1.2570	0	8.1795	1204.1025	23820768.94
19	1.3785	0	4.5675	1316.308	0

5 Conclusion

In this research twenty branches of Tehran Social Security Insurance Organization of Iran were selected. The results show that some of the branches as 4, 13, 17 and 19 had congestion. For example, as showed in Table 5, the unit 4 has congestion in input indexes, the number of personals and administrative expenses, but it does not congestion in the other indexes, that is, the number of computes and the area of the branch.

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Received: August, 2009