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Common Fixed Point Theorem for T-Hardy-Rogers Contraction Mapping in a Cone Metric Space

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Abstract. We prove common fixed point theorem for a Banach pair of mappings satisfying T-Hardy-Rogers type contraction condition in the setting of cone metric space. Our results shows a new direction to the literature of common fixed point theorems related to T-contraction mappings, Banach pair of mappings and cone metric space.

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1. Introduction

Since the Banach's contraction principle, several type of contraction mappings on metric space have appeared. Rhoades [24] made a comparison of various different types of contraction mappings. Recently, a new generalization of contraction mappings acting on complete metric spaces is introduced by Beiranvand et al. [4] called T-contraction and T-contractive mappings which are depending on another function.

Recently, Huang and Zhang [9] introduced the concept of cone metric space by replacing the set of real numbers by an ordered Banach space and obtained some fixed point theorems for mappings satisfying different contractive conditions. The category of cone metric spaces is larger than metric spaces and there are different types of cones. Subsequently, many authors Abbas and Jungck [1], Abbas and Rhoades [2], Ilic and Rakocevic [12], Jungck et al [13], Kadelburg et al [15], Raja and Vezapour [20] have generalized the results of Huang and Zhang [9] and studied the existence of common fixed points of a pair of self mappings satisfying a contractive type condition in the framework of normal cone metric spaces. Recently, Morales and Rojas [17], [18] have extended the concept of T-contraction mappings to cone metric space by proving fixed point theorems for T-Kannan, T-Zamfirescu, T-weakly contraction mappings.

On the other hand, Subrahmanyam [25] obtained the fixed point of a continuous Banach operator of type k in a complete metric space. Recently, Chen and Li [7] extended the concept of Banach operator of type k to Banach operator pair and proved various best approximation results using common fixed point theorems for f-nonexpansive mappings, where f is a self mapping of the subset M of a metric space X. Hussain [10], Al-thagafi and Shahzad [3] generalizing the results of Chen and Li [7], proved various common fixed point theorems and invariant approximation results for generalized nonexpansive Banach operator pair of mappings.

This new class of noncommuting mappings is different from the class of noncommuting mappings (viz. R-weakly commuting , R-subweakly commuting, compatible, weakly compatible, C_q-commuting etc.) existing in the literature so far. Hence the concept of Banach operator pair is of basic importance for study of common fixed points in best approximation.

The aim of this paper is to prove fixed point theorem for an extended Hardy and Rogers type T-contraction mapping in a cone metric space. If in addition, the pair of mappings is a Banach pair, then we have obtained a common fixed point. Our results generalize recent theorems existing in the literature of T-contraction mappings and cone metric space.

2. Definitions and Preliminaries

We recall some definitions and other results that will be needed in the sequel.

Definition 2.1. *A self mapping T of a metric space (X, d) is said to be a contraction mapping, if there exists a real number $0 \leq k < 1$ such that for all $x, y \in X$,*

$$d(Tx, Ty) \leq kd(x, y). \tag{2.1}$$

The following two new definitions are recently introduced by Beiranvand et al.[4]

Definition 2.2. [4] *Let T and f be two self mappings of a metric space (X, d). The self mapping f of X is said to be T-contraction, if there exists a real number $0 \leq k < 1$ such that*

$$d(Tfx, Tfy) \leq kd(Tx, Ty) \tag{2.2}$$

for all $x, y \in X$.

If $T = I$, the identity mapping, then the Definition(2.2) reduces to Banach contraction mapping.

The following example shows that a T-contraction mapping need not be a contraction mapping.

Example 2.1. *Let $X = [1, \infty)$ be with the usual metric. Define two mappings $T, f : X \to X$ as $Tx = \frac{1}{2x} + 2$ and $fx = 3x$. Obviously, f is not contraction but f is T-contraction which is seen from the following:*

$$|Tfx - Tfy| = |\frac{1}{6x} - \frac{1}{6y}| = \frac{1}{3}|Tx - Ty|.$$

Definition 2.3. [5] *Let T and f be two self mappings of a metric space (X, d). The self mapping f of X is said to be T-contractive, if for every $x, y \in X$ such that $Tx \neq Ty$ and*

$$d(Tfx, Tfy) < d(Tx, Ty).$$

It is obvious that every T-contraction mapping is T-contractive but the converse need not be true.

Example 2.2. *Let $X = [1, \infty)$ be with the usual metric. Define two mappings $T, f : X \to X$ as $Tx = x$ and $fx = \sqrt{x}$. Obviously, f is not T-contraction but f is T-contractive.*

Definition 2.4. [4] *Let T be a self mapping of a metric space (X, d). Then*

1. *the mapping T is said to be sequentially convergent, if the sequence $\{y_n\}$ in X is convergent whenever $\{Ty_n\}$ is convergent.*
2. *the mapping T is said to be subsequentially convergent, if $\{y_n\}$ has a convergent subsequence whenever $\{Ty_n\}$ is convergent.*

The following theorem has been proved by Beiranvand et al. [4].

Theorem 2.1. [4] *Let (X, d) be a complete metric space and $T : X \to X$ be a one-to-one, continuous and subsequentially convergent mapping. Then every T-contraction and continuous self mapping $f : X \to X$ has a unique fixed point in X. Also, if T is sequentially convergent, then for each $x_0 \in X$, the sequence of iterates $\{f^n x_0\}$ converges to the fixed point.*

The following is the definition introduced by Subrahmanyam [25].

Definition 2.5. [25] *Let T be a self mapping of a normed space X. Then T is called a Banach operator of type k if*

$$\|T^2x - Tx\| \leq k\|Tx - x\|$$

for some $k \geq 0$ and for all $x \in X$.

Extending the concept of Subrahmanyam [25] Chen and Li [7] introduced the following definition in the setup of normed linear space.

Definition 2.6. [7] *Let T and f be two self mappings of a nonempty subset M of a normed linear space X. Then (T, f) is a Banach operator pair, if any one of the following conditions is satisfied:*

1. $T[F(f)] \subseteq F(f)$ *(i.e)* $F(f)$ *is* T*-invariant.*
2. $fTx = Tx$ *for each* $x \in F(f)$.
3. $fTx = Tfx$ *for each* $x \in F(f)$.
4. $\|Tfx - fx\| \leq k\|fx - x\|$ *for some* $k \geq 0$.

Definition 2.7. [9] *Let* E *be a real Banach space. A subset* P *of* E *is called a cone if and only if*

1. P *is nonempty, closed and* $P \neq \{0\}$;
2. $\alpha, \beta \in R$, $\alpha, \beta \geq 0$ *and* $x, y \in P \Rightarrow \alpha x + \beta y \in P$
3. $x \in P$ *and* $-x \in P$ *(i.e)* $P \bigcap (-P) = \{0\}$.

For a given cone $P \subseteq E$, *a partial ordering is defined as* $\leq$ *on* E *with respect to* P *by* $x \leq y$, *if and only if* $y - x \in P$. *It is denoted as* $x < y$ *to indicate that* $x \leq y$ *but* $x \neq y$, *while* $x \ll y$ *will stand for* $y - x \in intP$, *where* $int\ P$ *denotes the interior of* P.

The cone $P \subset E$ *is called normal, if there is a number* $K > 0$ *such that for all* $x, y \in E$, $0 \leq x \leq y$ *implies*

$$\|x\| \leq K\|y\| \tag{2.3}$$

The least positive number K *satisfying (2.3) is called the normal constant of* P. *There are non normal cones also.*

The definition of a Cone metric space given by Huang and Zhang [9] is as follows:

Definition 2.8. [9] *Let* X *be a nonempty set. Suppose* E *is a real Banach space,* P *is a Cone with* $int\ P \neq \emptyset$ *and* $\leq$ *is a partial ordering with respect to* P.

If the mapping $d : X \times X \to E$ *satisfies:*

1. $0 \leq d(x, y)$ *for all* $x, y \in X$ *and* $d(x, y) = 0$ *if and only if* $x = y$;
2. $d(x, y) = d(y, x)$ *for all* $x, y \in X$;
3. $d(x, y) \leq d(x, z) + d(y, z)$ *for all* $x, y, z \in X$;

then d *is called a cone metric on* X *and* (X, d) *is called a cone metric space.*

Example 2.3. [9] *Let* $E = \mathbb{R}^2$, $P = \{(x, y) \in E : x, y \geq 0\} \subset R^2, X = \mathbb{R}$ *and* $d : X \times X \to E$ *such that* $d(x, y) = (\mid x - y \mid, \alpha \mid x - y \mid)$, *where* $\alpha \geq 0$ *is a constant. Then* (X, d) *is a Cone metric space.*

Definition 2.9. [17] *Let (X, d) be a cone metric space and $\{x_n\}$ be a sequence in X. Then,*

1. *$\{x_n\}$ converges to $x \in X$, if for every $c \in E$ with $0 \ll c$, there is $n_0 \in N$, the set of all natural numbers such that for all $n \geq n_0$,*

$$d(x_n, x) \ll c.$$

It is denoted by $lim_{n\to\infty} x_n = x$ or $x_n \to x$, $(n \to \infty)$.

2. *If for any $c \in E$, there is a number $n_0 \in N$ such that for all $m, n \geq n_0$*

$$d(x_n, x_m) \ll c,$$

then $\{x_n\}$ is called a Cauchy sequence in X ;

3. *(X, d) is a complete cone metric space, if every Cauchy sequence in X is convergent.*
4. *A self mapping $T : X \to X$ is said to be continuous at a point $x \in X$, if $lim_{n\to\infty} x_n = x$ implies that $lim_{n\to\infty} Tx_n = Tx$ for every $\{x_n\}$ in X;*

The following two lemmas of Huang and Zhang [9] will be required in the sequel.

Lemma 2.1. [9] *Let (X, d) be a cone metric space and P be a normal cone with normal constant K. A sequence $\{x_n\}$ in X converges to x if and only if $d(x_n, x) \to 0$ as $n \to \infty$.*

Lemma 2.2. [9] *Let (X, d) be a cone metric space and P be a normal cone with normal constant K. A sequence $\{x_n\}$ in X is a Cauchy sequence if and only if $d(x_n, x_m) \to 0$ as $n, m \to \infty$.*

The following corollary of Rezapour [22] will be needed in the sequel.

Corollary 2.1. [22] *Let $a, b, c, u \in E$, the real Banach space.*

(i) *If $a \leq b$ and $b \ll c$, then $a \ll c$.*
(ii) *If $a \ll b$ and $b \ll c$, then $a \ll c$.*
(ii) *If $0 \leq u \ll c$ for each $c \in$ int P, then $u = 0$.*

Remark 2.1. [13] *If $c \in$ int P, $0 \leq a_n$ and $a_n \to 0$, then there exists n_0 such that for all $n > n_0$, it follows that $a_n \ll c$.*

3. Main Results

Theorem 3.1. *Let T and f be two continuous self mappings of a complete cone metric space (X, d). Assume that T is a injective mapping and P is a normal cone with normal constant. If the mappings T and f satisfy*

$$d(Tfx, Tfy) \leq a_1 d(Tx, Ty) + a_2 d(Tx, Tfx) + a_3 d(Ty, Tfy) + a_4 d(Tx, Tfy) + a_5 d(Ty, Tfx) \tag{3.1}$$

for all $x, y \in X$, where $a_i, i = 1, 2, 3, 4, 5$ are all nonnegative constants such that $a_1 + a_2 + a_3 + a_4 + a_5 < 1$, then f has a unique fixed point in X. Moreover, if (T, f) is a Banach pair, then T and f have a unique common fixed point in X.

Proof. Let $x_0 \in X$ be arbitrary. Define a sequence $\{x_n\}$ in X such that $x_{n+1} = fx_n$ for each $n = 0, 1, 2, ...\infty$. Consider,

$$\begin{aligned} d(Tx_n, Tx_{n+1}) &= d(Tfx_{n-1}, Tfx_n) \\ &\leq a_1 d(Tx_{n-1}, Tx_n) + a_2 d(Tx_{n-1}, Tfx_{n-1}) + a_3 d(Tx_n, Tfx_n) \\ &\quad + a_4 d(Tx_{n-1}, Tfx_n) + a_5 d(Tx_n, Tfx_{n-1}) \\ &\leq a_1 d(Tx_{n-1}, Tx_n) + a_2 d(Tx_{n-1}, Tx_n) + a_3 d(Tx_n, Tx_{n+1}) \\ &\quad + a_4 d(Tx_{n-1}, Tx_{n+1}) + a_5 d(Tx_n, Tx_n) \end{aligned}$$

$$d(Tx_n, Tx_{n+1}) \leq (a_1 + a_2 + a_4) d(Tx_{n-1}, Tx_n) + (a_3 + a_4) d(Tx_n, Tx_{n+1}) \tag{3.2}$$

Next, consider

$$\begin{aligned} d(Tx_{n+1}, Tx_n) &= d(Tfx_n, Tfx_{n-1}) \\ &\leq a_1 d(Tx_n, Tx_{n-1}) + a_2 d(Tx_n, Tfx_n) + a_3 d(Tx_{n-1}, Tfx_{n-1}) \\ &\quad + a_4 d(Tx_n, Tfx_{n-1}) + a_5 d(Tx_{n-1}, Tfx_n) \\ &\leq a_1 d(Tx_n, Tx_{n-1}) + a_2 d(Tx_n, Tx_{n+1}) + a_3 d(Tx_{n-1}, Tx_n) \\ &\quad + a_4 d(Tx_n, Tx_n) + a_5 d(Tx_{n-1}, Tx_{n+1}) \end{aligned}$$

$$d(Tx_{n+1}, Tx_n) \leq (a_1 + a_3 + a_5) d(Tx_{n-1}, Tx_n) + (a_2 + a_5) d(Tx_n, Tx_{n+1}). \tag{3.3}$$

Adding inequalities (3.2) and (3.3),

$$\begin{aligned} 2d(Tx_n, Tx_{n+1}) &\leq (2a_1 + a_2 + a_3 + a_4 + a_5)d(Tx_n, Tx_{n-1}) \\ &\quad +(a_3 + a_4 + a_2 + a_5)d(Tx_n, Tx_{n+1}) \\ d(Tx_n, Tx_{n+1}) &\leq \frac{(2a_1 + a_2 + a_3 + a_4 + a_5)}{(2 - a_2 - a_3 - a_4 - a_5)}d(Tx_n, Tx_{n-1}) = kd(Tx_n, Tx_{n-1}), \end{aligned}$$

where $k = \frac{2a_1+a_2+a_3+a_4+a_5}{2-a_2-a_3-a_4-a_5} < 1$ as $a_1+a_2+a_3+a_4+a_5 < 1$. Proceeding further,

$$d(Tx_n, Tx_{n+1}) \leq k^n d(Tx_0, Tx_1). \tag{3.4}$$

Next, to claim that $\{Tx_n\}$ is a Cauchy sequence. Consider $m, n \in N$ such that $m > n$,

$$\begin{aligned} d(Tx_n, Tx_m) &\leq d(Tx_n, Tx_{n+1}) + d(Tx_{n+1}, Tx_{n+2}) + ...d(Tx_{m-1}, Tx_m) \\ &\leq (k^n + k^{n+1} + ... + k^{m-1})d(Tx_1, Tx_0) \\ &\leq (k^n + k^{n+1} + ...)d(Tx_1, Tx_0) \\ &= \frac{k^n}{1-k}d(Tx_0, Tx_1). \end{aligned}$$

From (2.3), it follows that

$$\|d(Tx_m, Tx_n)\| \leq \frac{k^n}{1-k}\|d(Tx_0, Tx_1)\|. \tag{3.5}$$

Since $k \in (0, 1)$, $k^n \to 0$ as $n \to \infty$. Therefore $\|d(Tx_m, Tx_n)\| \to 0$ as $m, n \to \infty$. Thus $\{Tx_n\}$ is a Cauchy sequence in X. As X is a complete cone metric space, there exists $z \in X$ such that $lim_{n\to\infty}Tx_n = z$.

Since T is subsequentially convergent, $\{x_n\}$ has a convergent subsequence $\{x_m\}$ such that $lim_{m\to\infty}x_m = u$. As T is continuous,

$$lim_{m\to\infty}Tx_m = Tu. \tag{3.6}$$

By the uniqueness of the limit, $z = Tu$. Since f is continuous, $lim_{m\to\infty}fx_m = fu$. Again as T is continuous, $lim_{m\to\infty}Tfx_m = Tfu$. Therefore

$$lim_{m\to\infty}Tx_{m+1} = Tfu. \tag{3.7}$$

Now consider,

$$\begin{aligned}
d(Tfu,Tu) &\leq d(Tfu,Tx_m)+d(Tx_m,Tu)\\
&= d(Tfu,Tfx_{m-1})+d(Tx_m,Tu)\\
&\leq a_1d(Tu,Tx_{m-1})+a_2d(Tu,Tfu)+a_3d(Tx_{m-1},Tfx_{m-1})\\
&\quad +a_4d(Tu,Tfx_{m-1})+a_5d(Tx_{m-1},Tfu)+d(Tx_m,Tu)\\
&= a_1d(Tu,Tx_{m-1})+a_2d(Tu,Tfu)+a_3d(Tx_{m-1},Tx_m)\\
&\quad +a_4d(Tu,Tx_m)+a_5d(Tx_{m-1},Tfu)+d(Tx_m,Tu)\\
&\leq \frac{a_1}{1-a_2}d(Tu,Tx_{m-1})+\frac{a_3}{1-a_2}d(Tx_{m-1},Tx_m)\\
&\quad +\frac{1+a_4}{1-a_2}d(Tx_m,Tu)+\frac{a_5}{1-a_2}d(Tx_{m-1},Tfu)\\
&\leq \frac{a_1}{1-a_2}[d(Tu,Tx_m)+d(Tx_m,Tx_{m-1})]\\
&\quad +\frac{a_3}{1-a_2}d(Tx_{m-1},Tx_m)+\frac{1+a_4}{1-a_2}d(Tx_m,Tu)\\
&\quad +\frac{a_5}{1-a_2}[d(Tx_{m-1},Tx_m)+d(Tx_m,Tu)+d(Tu,Tfu)]\\
(1-\frac{a_5}{1-a_2})d(Tu,Tfu) &\leq \frac{1+a_1+a_4+a_5}{1-a_2}d(Tu,Tx_m)+\frac{a_1+a_3+a_5}{1-a_2}d(Tx_{m-1},Tx_m).
\end{aligned}$$

Therefore,

$$d(Tu,Tfu) \leq \frac{1+a_1+a_4+a_5}{1-a_2-a_5}d(Tu,Tx_m)+\frac{a_1+a_3+a_5}{1-a_2-a_5}d(Tx_{m-1},Tx_m) \tag{3.8}$$

Let $0 \ll c$ be arbitrary. By (3.6) $d(Tu,Tx_m) \ll \frac{c(1-a_2-a_5)}{2(1+a_1+a_4+a_5)}$. Similarly by (3.7), it follows that $d(Tx_{m-1},Tx_m) \ll \frac{c(1-a_2-a_5)}{2(a_1+a_3+a_5)}$.

Then, (3.8) becomes

$$d(Tu,Tfu) \ll \frac{c}{2}+\frac{c}{2}=c$$

Thus $d(Tu,Tfu) \ll c$ for each $c \in intP$. Now, Using Corollary 2.1(iii), it follows that $d(Tu,Tfu)=\mathbf{0}$ which implies that $Tu=Tfu$. As T is injective, $u=fu$. Thus u is the fixed point of f.

To Prove Uniqueness: If w is another fixed point of f, then $w = fw$.

$$\begin{aligned} d(Tu, Tw) = d(Tfu, Tfw) &\leq a_1 d(Tu, Tw) + a_2 d(Tu, Tfu) + a_3 d(Tw, Tfw) \\ &\quad + a_4 d(Tu, Tfw) + a_5 d(Tw, Tfu) \\ &\leq a_1 d(Tu, Tw) + a_4 d(Tu, Tw) + a_5 d(Tw, Tu) \\ &= (a_1 + a_4 + a_5) d(Tu, Tw) \\ &\leq (a_1 + a_2 + a_3 + a_4 + a_5) d(Tu, Tw) \\ &< d(Tu, Tw) \text{ as } a_1 + a_2 + a_3 + a_4 + a_5 < 1, \end{aligned}$$

a contradiction. Hence $d(Tu, Tw) = 0$ which implies $Tu = Tw$. As T is injective, $u = w$ is the unique fixed point of f.

As (T, f) is a Banach pair, T and f commutes at the fixed point of f which implies that $Tfu = fTu$ for $u \in F(f)$. (i.e) $Tu = fTu$ which implies that Tu is another fixed point of f. By uniqueness of fixed point of f, $u = Tu$. Hence $u = fu = Tu$ is the unique common fixed point of f and T in X. □

The following corollary extends the main result of Beiranvand et al. [4] to cone metric space.

Corollary 3.1. *Let T and f be two continuous self mappings of a complete cone metric space (X, d). Assume that T be injective and P be a normal cone with normal constant. If the mappings T and f satisfy*

$$d(Tfx, Tfy) \leq kd(Tx, Ty)$$

for all $x, y \in X$, for some $k < 1$, then f has a unique fixed point in X.

Proof. The proof of this Corollary follows by taking $k = a_1$, $a_2 = a_3 = a_4 = a_5 = 0$ in Theorem 3.1. Then $k = a_1 \leq a_1 + a_2 + a_3 + a_4 + a_5 < 1$. □

The following Corollary is Corollary 2.4 of Abbas and Rhoades [2].

Corollary 3.2. [2] *Let f be continuous self mapping of a complete cone metric space (X, d). Assume that P is a normal cone with normal constant. If the mapping f satisfy*

$$\begin{aligned} d(fx, fy) &\leq a_1 d(x, y) + a_2 d(x, fx) + a_3 d(y, fy) \\ &\quad + a_4 d(x, fy) + a_5 d(y, fx) \end{aligned}$$

for all $x, y \in X$, where $a_i, i = 1, 2, 3, 4, 5$ are all nonnegative constants such that $a_1 + a_2 + a_3 + a_4 + a_5 < 1$, then f has a unique fixed point in X

Proof. The proof of this Corollary follows by taking $T = I$, the identity mapping in Theorem 3.1. □

The following corollary is obtained which is the main result of Hardy and Rogers [8] in the setup of cone metric space.

Corollary 3.3. *Let f be self mapping of a complete cone metric space (X, d) satisfying*

$$d(fx, fy) \leq a_1 d(x, y) + a_2 d(x, fx) + a_3 d(y, fy) + a_4 d(x, fy) + a_5 d(y, fx)$$

for all $x, y \in X$, where $a_i, i = 1, 2, 3, 4, 5$ are all nonnegative constants such that $a_1 + a_2 + a_3 + a_4 + a_5 < 1$, then f has a unique fixed point in X.

Proof. The proof of this Corollary follows by taking $Tx = x$ for all $x \in X$ or $T = I$, the identity mapping in Theorem 3.1. □

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