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Some Properties of Intuitionistic Nil Radicals of Intuitionistic Fuzzy Ideals

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Abstract

The notion of intuitionistic fuzzy sets was introduced by Atanassov as a generalization of the notion of fuzzy sets. In this paper, we consider the notion of intuitionistic nil radicals of intuitionistic fuzzy ideals in commutative rings some properties of such nil radicals.

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1 Introduction

Zadeh introduced the notion of a fuzzy subset of a non-empty set X, as a function from X to $[0, 1]$ in [11].
After the introduction of fuzzy sets by Zadeh, there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov [3] is one among them. For more details on intuitionistic fuzzy sets, we refer the reader to [3,4]. As applications of intuitionistic fuzzy sets, Davvaz et al. [6] applied the concept of an intuitionistic fuzzy set to H_v-modules. Dudek et al. [7] considered the intuitionistic fuzzification of the concept of sub-hyperquasigroups in a hyperquasigroup. Several authors studied intuitionistic fuzzy subrings/ideals in a ring (see [5,10]). Gupta and Kantroo [8] introduced the notion of fuzzy nil radical of an ideal of a commutative ring which has been successful in establishing the analogues of most of the fundamental ground results involving radicals in the fuzzy setting. Jun et al. [9] considered the intuitionistic nil radicals of intuitionistic fuzzy ideals and Euclidean intuitionistic fuzzy ideals in rings. In this paper, we apply the notion of intuitionistic nil radicals of intuitionistic fuzzy ideals in commutative rings is introduced, and related properties are investigated.

2 Fuzzy sets and intuitionistic fuzzy sets

Let R be a commutative ring, then a non-empty subset I of R is a ideal if and only if for all $a, b \in I$ and $r \in R$:
(i) $a - b \in I$;
(ii) $ar \in R$.

The concept of a fuzzy set in a non-empty set was introduced by Zadeh [11] in 1965. Let H be a non-empty set. A mapping $\mu : H \longrightarrow [0;1]$ is called a *fuzzy set* in H. The *complement* of μ, denoted by μ^c, is the fuzzy set in H given by $\mu^c(x) = 1 - \mu(x)$ for all $x \in H$.

Definition 2.1 An *intuitionistic fuzzy set* A in a non-empty set X is an object having the form

$$A = \{(x, \mu_A(x), \lambda_A(x)) | x \in X\},$$

where the functions $\mu_A : X \longrightarrow [0;1]$ and $\lambda_A : X \longrightarrow [0;1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\lambda_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$ for all $x \in X$. For the sake of simplicity, we shall use the symbol $A = (\mu_A, \lambda_A)$ for the intuitionistic fuzzy set $A = \{(x, \mu_A(x), \lambda_A(x)) | x \in X\}$.

Definition 2.2 (see [3]) Let $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ be intuitionistic fuzzy sets in X. Then
(1) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\lambda_A(x) \geq \lambda_B(x)$ for all $x \in X$,
(2) $A^c = \{(x, \lambda_A(x), \mu_A(x)) | x \in X\}$,
(3) $A \cap B = \{(x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\lambda_A(x), \lambda_B(x)\}) | x \in X\}$,
(4) $A \cup B = \{(x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\lambda_A(x), \lambda_B(x)\}) | x \in X\}$,
(5) $\heartsuit A = \{(x, \mu_A(x), \mu_A^c(x)) | x \in X\}$,
(6) $\diamondsuit A = \{(x, \lambda_A^c(x), \lambda_A(x)) | x \in X\}$.

Now, we define an intuitionistic fuzzy subring of a ring.

Definition 2.3 (see [5,10]) An intuitionistic fuzzy set $A = (\mu_A, \lambda_A)$ in a ring R is called an *intuitionistic fuzzy subring* of R if it satisfies the following conditions:
(i) $\mu_A(x - y) \geq \min\{\mu_A(x), \mu_A(y)\}$,
(ii) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$,
(iii) $\lambda_A(x - y) \leq \max\{\lambda_A(x), \lambda_A(y)\}$,
(iv) $\lambda_A(xy) \leq \max\{\lambda_A(x), \lambda_A(y)\}$,
for all $x, y \in R$.

Definition 2.4 (see [5,10]) An intuitionistic fuzzy set $A = (\mu_A, \lambda_A)$ in a ring R is called an *intuitionistic fuzzy left ideal* of R if it satisfies the following

conditions:
(i) $\mu_A(x-y) \geq \min\{\mu_A(x), \mu_A(y)\},\ \lambda_A(x-y) \leq \max\{\lambda_A(x), \lambda_A(y)\},\ \forall x, y \in R$,
(ii) $\mu_A(ax) \geq \mu_A(x),\ \lambda_A(ax) \geq \lambda_A(x),\ \forall a, x \in R$.
If we replace the condition (ii) by the following condition:
(ii)' $\mu_A(xa) \geq \mu_A(x),\ \lambda_A(xa) \geq \lambda_A(x),\ \forall a, x \in R$,
then $A = (\mu_A, \lambda_A)$ is called an intuitionistic fuzzy right ideal of R.
If $A = (\mu_A, \lambda_A)$ is both an intuitionistic fuzzy left and *intuitionistic fuzzy right ideal* of a ring R, then $A = (\mu_A, \lambda_A)$ is called an intuitionistic fuzzy ideal of R.

Note that $A = (\mu_A, \lambda_A)$ is an *intuitionistic fuzzy ideal* of R if and only if it satisfies:
(i) $\mu_A(x - y) \geq \min\{\mu_A(x), \mu_A(y)\},\ \lambda_A(x - y) \leq \max\{\lambda_A(x), \lambda_A(y)\}$,
(ii) $\mu_A(xy) \geq \max\{\mu_A(x), \mu_A(y)\},\ \lambda_A(xy) \leq \min\{\lambda_A(x), \lambda_A(y)\}$,
for all $x, y \in R$.

By Definition 2.4, we have the following Corollary:

Corollary 2.5 *Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy ideal of R. Then $\mu_A(x^n) \geq \mu_A(x)$ and $\lambda_A(x^n) \leq \lambda_A(x)$, for all $n \in N^*$, where N^* is the set of all nonzero natural numbers.*

3 Main Results

In this section let R denote a commutative ring unless otherwise specified.

Definition 3.1 Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy ideal of R. The intuitionistic nil radical of $A = (\mu_A, \lambda_A)$ is defined to be an intuitionistic fuzzy set $\sqrt{A} = (\mu_{\sqrt{A}}, \lambda_{\sqrt{A}})$ in R defined by

$$\mu_{\sqrt{A}}(x) = \sup_{n \geq 1} \mu_A(x^n), \quad \lambda_{\sqrt{A}}(x) = \inf_{n \geq 1} \lambda_A(x^n),$$

for all $x \in R$ and some $n \in N^*$.

Theorem 3.2 *(see [9]) Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy ideal of R. Then $\sqrt{A} = (\mu_{\sqrt{A}}, \lambda_{\sqrt{A}})$ is an intuitionistic fuzzy ideal of R.*

Corollary 3.3 *Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy ideal of R. Then $\sqrt{A} = (\mu_{\sqrt{A}}, \lambda_{\sqrt{A}})$ is an intuitionistic fuzzy subring of R.*

Proof. Let $x, y \in R$. Then by Theorem 3.2, $\sqrt{A} = (\mu_{\sqrt{A}}, \lambda_{\sqrt{A}})$ is an intuitionistic fuzzy ideal of R, thus by Definition 2.4, we have
(i) $\mu_{\sqrt{A}}(x - y) \geq \min\{\mu_{\sqrt{A}}(x), \mu_{\sqrt{A}}(y)\},\ \lambda_{\sqrt{A}}(x - y) \leq \max\{\lambda_{\sqrt{A}}(x), \lambda_{\sqrt{A}}(y)\}$,
(ii) $\mu_{\sqrt{A}}(xy) \geq \max\{\mu_{\sqrt{A}}(x), \mu_{\sqrt{A}}(y)\},\ \lambda_{\sqrt{A}}(xy) \leq \min\{\lambda_{\sqrt{A}}(x), \lambda_{\sqrt{A}}(y)\}$,
and thus the conditions (i) and (iii) of Definition 2.4, are valid. On the other

hand, by (ii) we have
$\mu_{\sqrt{A}}(xy) \geq \max\{\mu_{\sqrt{A}}(x), \mu_{\sqrt{A}}(y)\} \geq \min\{\mu_{\sqrt{A}}(x), \mu_{\sqrt{A}}(y)\}$,
$\lambda_{\sqrt{A}}(xy) \leq \min\{\lambda_{\sqrt{A}}(x), \lambda_{\sqrt{A}}(y)\} \leq \max\{\lambda_{\sqrt{A}}(x), \lambda_{\sqrt{A}}(y)\}$,
implying that the conditions (ii) and (iv) of Definition 2.3, are met. $\diamond$

Theorem 3.4 *Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy ideal of R. Then $\heartsuit\sqrt{A} = (\mu_{\sqrt{A}}, \mu^c_{\sqrt{A}})$ is an intuitionistic fuzzy ideal of R.*

Proof. Let $x, y \in R$. Then by Theorem 3.2, $\sqrt{A} = (\mu_{\sqrt{A}}, \lambda_{\sqrt{A}})$ is an intuitionistic fuzzy ideal of R. Thus
(i) $\mu_{\sqrt{A}}(x-y) \geq \min\{\mu_{\sqrt{A}}(x), \mu_{\sqrt{A}}(y)\}$, $\lambda_{\sqrt{A}}(x-y) \leq \max\{\lambda_{\sqrt{A}}(x), \lambda_{\sqrt{A}}(y)\}$,
(ii) $\mu_{\sqrt{A}}(xy) \geq \max\{\mu_{\sqrt{A}}(x), \mu_{\sqrt{A}}(y)\}$, $\lambda_{\sqrt{A}}(xy) \leq \min\{\lambda_{\sqrt{A}}(x), \lambda_{\sqrt{A}}(y)\}$.
It is sufficient to show that $\mu^c_{\sqrt{A}}$ satisfies the conditions
$\mu^c_{\sqrt{A}}(x-y) \leq \max\{\mu^c_{\sqrt{A}}(x), \mu^c_{\sqrt{A}}(y)\}$ and $\mu^c_{\sqrt{A}}(xy) \leq \min\{\mu^c_{\sqrt{A}}(x), \mu^c_{\sqrt{A}}(y)\}$.
For $x, y \in R$ we have

$$\mu_{\sqrt{A}}(x-y) \geq \min\{\mu_{\sqrt{A}}(x), \mu_{\sqrt{A}}(y)\}$$

and so

$$1 - \mu^c_{\sqrt{A}}(x-y) \geq \min\{1 - \mu^c_{\sqrt{A}}(x), 1 - \mu^c_{\sqrt{A}}(y)\}$$

which implies

$$\mu^c_{\sqrt{A}}(x-y) \leq 1 - \min\{1 - \mu^c_{\sqrt{A}}(x), 1 - \mu^c_{\sqrt{A}}(y)\}.$$

Therefore

$$\mu^c_{\sqrt{A}}(x-y) \leq \max\{\mu^c_{\sqrt{A}}(x), \mu^c_{\sqrt{A}}(y)\}.$$

Similarly, for $x, y \in R$ we have

$$\mu_{\sqrt{A}}(xy) \geq \max\{\mu_{\sqrt{A}}(x), \mu_{\sqrt{A}}(y)\}$$

and so

$$1 - \mu^c_{\sqrt{A}}(xy) \geq \max\{1 - \mu^c_{\sqrt{A}}(x), 1 - \mu^c_{\sqrt{A}}(y)\}$$

which implies

$$\mu^c_{\sqrt{A}}(xy) \leq 1 - \max\{1 - \mu^c_{\sqrt{A}}(x), 1 - \mu^c_{\sqrt{A}}(y)\}.$$

Therefore

$$\mu^c_{\sqrt{A}}(xy) \leq \min\{\mu^c_{\sqrt{A}}(x), \mu^c_{\sqrt{A}}(y)\},$$

implying that the conditions of Definition 2.4, are met. $\diamond$

Theorem 3.5 *Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy ideal of R. Then $\Diamond\sqrt{A} = (\lambda^c_{\sqrt{A}}, \lambda_{\sqrt{A}})$ is an intuitionistic fuzzy ideal of R.*

Proof. The proof is similar to the proof of Theorem 3.4. ⋄

Combining the above two Theorems it is not difficult to verify that the following two Corollaries are valid.

Corollary 3.6 *$A = (\mu_{\sqrt{A}}, \lambda_{\sqrt{A}})$ is an intuitionistic fuzzy ideal of R if and only if $\heartsuit\sqrt{A}$ and $\diamondsuit\sqrt{A}$ are intuitionistic fuzzy ideal of R.*

Corollary 3.7 *$A = (\mu_{\sqrt{A}}, \lambda_{\sqrt{A}})$ is an intuitionistic fuzzy ideal of R if and only if $(\lambda^c_{\sqrt{A}}, \mu^c_{\sqrt{A}})$ is an intuitionistic fuzzy ideal of R.*

Theorem 3.8 *(see [9]) Let $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ be intuitionistic fuzzy ideals of R. Then $\sqrt{A} \cap \sqrt{B} = \sqrt{A \cap B}$.*

Theorem 3.9 *Let $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ be intuitionistic fuzzy ideals of R. Then*

$$\sqrt{A \cap B} = \sqrt{A} \cap \sqrt{B} = (\min\{\mu_{\sqrt{A}}(x), \mu_{\sqrt{A}}(x)\}, \max\{\mu_{\sqrt{B}}(y), \mu_{\sqrt{B}}(y)\})$$

is an intuitionistic fuzzy ideal of R.

Proof. Since $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ are intuitionistic fuzzy ideals of R. Then by Theorem 3.2, $\sqrt{A}$ and $\sqrt{B}$ are an intuitionistic fuzzy ideal of R, thus
(i) $\mu_{\sqrt{A}}(x-y) \geq \min\{\mu_{\sqrt{A}}(x), \mu_{\sqrt{A}}(y)\},\ \lambda_{\sqrt{A}}(x-y) \leq \max\{\lambda_{\sqrt{A}}(x), \lambda_A(y)\}$,
(ii) $\mu_{\sqrt{A}}(xy) \geq \max\{\mu_{\sqrt{A}}(x), \mu_{\sqrt{A}}(y)\},\ \lambda_{\sqrt{A}}(xy) \leq \min\{\lambda_{\sqrt{A}}(x), \lambda_{\sqrt{A}}(y)\}$,
(iii) $\mu_{\sqrt{B}}(x-y) \geq \min\{\mu_{\sqrt{B}}(x), \mu_{\sqrt{B}}(y)\},\ \lambda_{\sqrt{B}}(x-y) \leq \max\{\lambda_{\sqrt{B}}(x), \lambda_B(y)\}$,
(iv) $\mu_{\sqrt{B}}(xy) \geq \max\{\mu_{\sqrt{B}}(x), \mu_{\sqrt{B}}(y)\},\ \lambda_{\sqrt{B}}(xy) \leq \min\{\lambda_{\sqrt{B}}(x), \lambda_{\sqrt{B}}(y)\}$,
for all $x, y \in R$. Then we have

$$\begin{aligned}
&\min\{\mu_{\sqrt{A\cap B}}(x), \mu_{\sqrt{A\cap B}}(y)\} \\
&\quad = \min\{\min\{\mu_{\sqrt{A}}(x), \mu_{\sqrt{A}}(x)\}, \min\{\mu_{\sqrt{B}}(y), \mu_{\sqrt{B}}(y)\}\} && \text{(by Theorem 3.8)} \\
&\quad \leq \min\{\mu_{\sqrt{A}}(x-y), \mu_{\sqrt{B}}(x-y)\} && \text{(by (i) and (iii))} \\
&\quad = \mu_{\sqrt{A\cap B}}(x-y) && \text{(by Theorem 3.8)}
\end{aligned}$$

Also, we have

$$\begin{aligned}
&\max\{\mu_{\sqrt{A\cap B}}(x), \mu_{\sqrt{A\cap B}}(y)\} \\
&\quad = \max\{\min\{\mu_{\sqrt{A}}(x), \mu_{\sqrt{B}}(x)\}, \min\{\mu_{\sqrt{A}}(y), \mu_{\sqrt{B}}(y)\}\} && \text{(by Theorem 3.8)} \\
&\quad = \min\{\max\{\mu_{\sqrt{A}}(x), \mu_{\sqrt{A}}(y)\}, \max\{\mu_{\sqrt{B}}(x), \mu_{\sqrt{B}}(y)\}\} \\
&\quad \leq \min\{\mu_{\sqrt{A}}(xy), \mu_{\sqrt{B}}(xy)\} && \text{(by (ii) and (iv))} \\
&\quad = \mu_{\sqrt{A\cap B}}(xy) && \text{(by Theorem 3.8)}
\end{aligned}$$

Similarly, we have $\lambda_{\sqrt{A\cap B}}(x-y) \leq \max\{\lambda_{\sqrt{A\cap B}}(x), \lambda_{A\cap B}(y)\}$, and $\lambda_{\sqrt{A\cap B}}(xy) \leq \min\{\lambda_{\sqrt{A\cap B}}(x), \lambda_{\sqrt{A\cap B}}(y)\}$. Thus the conditions of Definition 2.4 are satisfied. ⋄

Example 3.10 *Let $A = 2\mathbf{Z}$ and $R = \mathbf{Z}$, where $\mathbf{Z}$ is the set of all integer numbers. Define fuzzy sets μ_A and λ_A in R by*

$$\mu_A(x) = \begin{cases} 0.7 & \text{if} \quad x \in A \\ 0.5 & \text{otherwise,} \end{cases} \qquad \lambda_A(x) = \begin{cases} 0.2 & \text{if} \quad x \in A \\ 0.4 & \text{otherwise.} \end{cases}$$

Then $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of R, also by Theorem 3.2, $\sqrt{A} = (\mu_{\sqrt{A}}, \lambda_{\sqrt{A}})$ is an intuitionistic fuzzy ideal of R.

Definition 3.11 Let f be a mapping from a set X to a set Y. Let μ be a fuzzy set in X and λ be a fuzzy set in Y. Then the *inverse image* $f^{-1}(\lambda)$ of λ is a fuzzy set in X defined by $f^{-1}(\lambda)(x) = \lambda(f(x))$ for all $x \in X$.
The *image* $f(\mu)$ of μ is the fuzzy set in Y defined by

$$f(\mu_A)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

for all $y \in Y$.

Definition 3.12 For any $t \in [0, 1]$ and fuzzy set μ in X, the set

$$U(\mu; t) = \{x \in X | \mu(x) \geq t\} \text{ (resp. } L(\mu; t) = \{x \in X | \mu(x) \leq t\})$$

is called an *upper* (respectively, *lower*) *t-level cut* of μ.

Definition 3.13 A fuzzy set μ in a set X is said to have *sup property* (respectively, *inf property*) if for every non-empty subset S of X, there exists $x_0 \in S$ (respectively, $x_1 \in S$) such that $\mu(x_0) = \sup_{x \in S} \mu(x)$ (respectively, $\mu(x_1) = \inf_{x \in S} \mu(x)$.)

Theorem 3.14 *Let $f : R \longrightarrow S$ be a ring homomorphism and surjection. Let $A = (\mu_A, \lambda_A)$ be intuitionistic fuzzy ideals of R, such that $\mu_{\sqrt{A}}$, $\lambda_{\sqrt{A}}$ and μ_A have sup property and λ_A has inf property, then*
(i) $U(f(\lambda_{\sqrt{A}}); t) \subseteq f(U(\lambda_A; t))$;
(ii) $L(f(\mu_{\sqrt{A}}); t) \subseteq f(L(\mu_A; t))$;
(iii) $U(f(\mu_{\sqrt{A}}); t) \supseteq f(U(\mu_A; t))$;
(iv) $L(f(\lambda_{\sqrt{A}}); t) \supseteq f(L(\lambda_A; t))$,
for every $t \in [0, 1]$.

Proof. (i) We have

$$\begin{aligned} y \in U(f(\lambda_{\sqrt{A}}); t) &\Longrightarrow f(\lambda_{\sqrt{A}})(y) \geq t \\ &\Longrightarrow \sup_{x \in f^{-1}(y)} \lambda_{\sqrt{A}}(x) \geq t \\ &\Longrightarrow \exists x_0 \in f^{-1}(y),\ \lambda_{\sqrt{A}}(x_0) \geq t \\ &\Longrightarrow f(x_0) = y,\ \inf_{n \geq 1} \lambda_A(x_0^n) \geq t \\ &\Longrightarrow f(x_0) = y,\ \exists n_0 \in N^*,\ \lambda_A(x_0^{n_0}) \geq t \\ &\Longrightarrow f(x_0) = y,\ \lambda_A(x_0) \geq t \qquad \text{(by Corollary 2.5)} \\ &\Longrightarrow f(x_0) = y,\ x_0 \in U(\lambda_A; t) \\ &\Longrightarrow y \in f(U(\lambda_A; t)). \end{aligned}$$

(ii) We have

$$\begin{aligned} y \in L(f(\mu_{\sqrt{A}});t) &\Longrightarrow f(\mu_{\sqrt{A}})(y) \leq t \\ &\Longrightarrow \sup_{x\in f^{-1}(y)} \mu_{\sqrt{A}}(x) \leq t \\ &\Longrightarrow \mu_{\sqrt{A}}(x) \leq t,\ \forall x \in f^{-1}(y) \\ &\Longrightarrow \sup_{n\geq 1}\mu_A(x^n) \leq t,\ \forall x \in f^{-1}(y) \\ &\Longrightarrow \mu_A(x^n) \leq t,\ \forall n \in N^*,\ \forall x \in f^{-1}(y) \\ &\Longrightarrow \mu_A(x) \leq t,\ \forall x \in f^{-1}(y) \\ &\Longrightarrow x \in L(\mu_A;t),\ \forall x \in f^{-1}(y) \\ &\Longrightarrow y \in f(L(\mu_A;t)). \end{aligned}$$

The proofs of (iii) and (iv) are similar to those of (i) and (ii). $\diamond$

Theorem 3.15 *Let $f : R \longrightarrow S$ be a ring homomorphism. Let $B = (\mu_B, \lambda_B)$ be intuitionistic fuzzy ideals of S, such that $\mu_{\sqrt{A}}$, $\lambda_{\sqrt{A}}$ and μ_A have sup property and λ_A has inf property, then*
(i) $U(f^{-1}(\lambda_{\sqrt{B}});t) \subseteq f^{-1}(U(\lambda_B;t))$;
(ii) $L(f^{-1}(\mu_{\sqrt{B}});t) \subseteq f^{-1}(L(\mu_B;t))$;
(iii) $U(f^{-1}(\mu_{\sqrt{B}});t) \supseteq f^{-1}(U(\mu_B;t))$;
(iv) $L(f^{-1}(\lambda_{\sqrt{B}});t) \supseteq f^{-1}(L(\lambda_B;t))$,
for every $t \in [0,1]$.

Proof. (i) We have

$$\begin{aligned} x_0 \in U(f^{-1}(\lambda_{\sqrt{B}});t) &\Longrightarrow f^{-1}(\lambda_{\sqrt{B}})(x_0) \geq t \\ &\Longrightarrow \lambda_{\sqrt{B}}(f(x_0)) \geq t \\ &\Longrightarrow \inf_{n\geq 1}\lambda_B(f^n(x_0)) \geq t \\ &\Longrightarrow \lambda_B(f^n(x_0)) \geq t,\ \forall n \in N^* \\ &\Longrightarrow \lambda_B(f(x_0)) \geq t \\ &\Longrightarrow f(x_0) \in U(\lambda_B;t) \\ &\Longrightarrow x_0 \in f^{-1}(U(\lambda_B;t)). \end{aligned}$$

(ii) We have

$$\begin{aligned} x_0 \in L(f^{-1}(\mu_{\sqrt{B}});t) &\Longrightarrow f^{-1}(\mu_{\sqrt{B}})(x_0) \leq t \\ &\Longrightarrow \mu_{\sqrt{B}}(f(x_0)) \leq t \\ &\Longrightarrow \sup_{n\geq 1}\mu_B(f^n(x_0)) \leq t \\ &\Longrightarrow \mu_B(f^n(x_0)) \leq t,\ \forall n \in N^* \\ &\Longrightarrow \mu_B(f(x_0)) \leq t \\ &\Longrightarrow f(x_0) \in (L(\mu_B;t)) \\ &\Longrightarrow x_0 \in f^{-1}(L(\mu_B;t)). \end{aligned}$$

The proofs of (iii) and (iv) are similar to those of (i) and (ii). $\diamond$

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