International Mathematical Forum, 5, 2010, no. 15, 703 - 707

Biminimal Structure Spaces

Chawalit Boonpok

Department of Mathematics
Faculty of Science
Mahasarakham University
Mahasarakham 44150, Thailand
chawalit.b@msu.ac.th

Abstract

The purpose of this paper is to introduce the concept of biminimal structure spaces and study some fundamental properties of $m^1_Xm^2_X$-closed sets and $m^1_Xm^2_X$-open sets in biminimal structure spaces.

Mathematics Subject Classification: 54A05, 54C08

Keywords: minimal structure, biminimal structure space, $m^1_Xm^2_X$-closed, $m^1_Xm^2_X$-open

1 Introduction

The concept of minimal structure (briefly m-structure) was introduced by V. Popa and T. Noiri [4] in 2000. Also they introduced the notion of m_X-open set and m_X-closed set and characterize those sets using m_X-closure and m_X-interior operators respectively. Further they introduced m-continuous functions and studied some of its basic properties. J.C. Kelly [1] introduce the notion of bitopological spaces. Such spaces are equipped with two arbitrary topologies. Furthermore, Kelly extended some of the standard results of separation axioms in a topological space to a bitopological space. Thereafter, a large number of papers have been written to generalize topological concepts to bitopological setting. In this paper, we introduce the concept of biminimal structure space and study $m^1_Xm^2_X$-closed sets and $m^1_Xm^2_X$-open sets in biminimal structure spaces.

2 Preliminaries

Definition 2.1. [3] Let X be a nonempty set and $P(X)$ the power set of X. A subfamily m_X of $P(X)$ is called a *minimal structure* (briefly *m-structure*) on X if $\emptyset \in m_X$ and $X \in m_X$

By (X, m_X), we denote a nonempty set X with an m-structure m_X on X and it is called an m-space. Each member of m_X is said to be m_X*-open* and the complement of an m_X-open set is said to be m_X*-closed*.

Definition 2.2. [3] Let X be a nonempty set and m_X an m-structure on X. For a subset A of X, the m_X*-closure* of A and the m_X*-interior* of A are defined as follows:

(1) $mCl(A) = \cap\{F : A \subseteq F, X - F \in m_X\}$,

(2) $mInt(A) = \cup\{U : U \subseteq A, U \in m_X\}$.

Lemma 2.3. [2] *Let X be a nonempty set and m_X a minimal structure on X. For subset A and B of X, the following properties hold:*

(1) $mCl(X - A) = X - mInt(A)$ and $mInt(X - A) = X - mCl(A)$,

(2) If $(X - A) \in m_X$, then $mCl(A) = A$ and if $A \in m_X$, then $mInt(A) = A$,

(3) $mCl(\emptyset) = \emptyset$, $mCl(X) = X$, $mInt(\emptyset) = \emptyset$ and $mInt(X) = X$,

(4) If $A \subseteq B$, then $mCl(A) \subseteq mCl(B)$ and $mInt(A) \subseteq mInt(B)$,

(5) $A \subseteq mCl(A)$ and $mInt(A) \subseteq A$,

(6) $mCl(mCl(A)) = mCl(A)$ and $mInt(mInt(A)) = mInt(A)$.

Lemma 2.4. [2] *Let X be a nonempty set with a minimal structure m_X and A a subset of X. Then $x \in mCl(A)$ if and only if $U \cap A \neq \emptyset$ for every $U \in m_X$ containing x.*

Definition 2.5. [2] An m-structure m_X on a nonempty set X is said to have *property B* if the union of any family of subsets belong to m_X belong to m_X.

Lemma 2.6. [3] *Let X be a nonempty set and m_X an m-structure on X sastisfying property B. For a subset A of X, the following properties hold:*

(1) $A \in m_X$ if and only if $mIntA = A$,

(2) If A is m_X-closed if and only if $mCl(A) = A$,

(3) $mInt(A) \in m_X$ and $mCl(A) \in m_X$-closed.

3 Biminimal Structure Spaces

In this section, we introduce the concept of biminimal structure spaces and study some properties of $m_X^1m_X^2$-closed sets and $m_X^1m_X^2$-open sets in biminimal structure spaces.

Definition 3.1. Let X be a nonempty set and m_X^1, m_X^2 be minimal structures on X. A triple (X, m_X^1, m_X^2) is called a *biminimal structure space* (briefly *bim-space*).

Let (X, m_X^1, m_X^2) be a biminimal structure space and A be a subset of X. The m_X-closure and m_X-interior of A with respect to m_X^i are denote by $mCl_i(A)$ and $mInt_i(A)$, respectively, for $i = 1, 2$.

Definition 3.2. A subset A of a biminimal structure space (X, m_X^1, m_X^2) is called $m_X^1m_X^2$*-closed* if $mCl_1(mCl_2(A)) = A$. The complement of $m_X^1m_X^2$-closed set is called $m_X^1m_X^2$*-open*.

Example 3.3. *Let $X = \{a, b\}$. Define m-structures m_X^1 and m_X^2 on X as follows: $m_X^1 = \{\emptyset, \{a\}, X\}$ and $m_X^2 = \{\emptyset, \{a\}, X\}$. Then $\{b\}$ is $m_X^1m_X^2$-closed.*

Let (X, m_X^1, m_X^2) be a biminimal structure space and A be a subset of X. Then A is $m_X^1m_X^2$-closed if and only if $mCl_1(A) = A$ and $mCl_2(A) = A$.

The following statement is evident:

Proposition 3.4. *Let m_X^1 and m_X^2 be m-structures on X satisfying property B. Then A is a $m_X^1m_X^2$-closed subset of a biminimal structure space (X, m_X^1, m_X^2) if and only if A is both m_X^1-closed and m_X^2-closed.*

Proposition 3.5. *Let (X, m_X^1, m_X^2) be a biminimal structure space. If A and B are $m_X^1m_X^2$-closed subsets of (X, m_X^1, m_X^2), then $A \cap B$ is $m_X^1m_X^2$-closed.*

Proof. Let A and B be $m_X^1m_X^2$-closed. Then $mCl_1(mCl_2(A)) = A$ and $mCl_1(mCl_2(A)) = A$. Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$, $mCl_1(mCl_2(A \cap B)) \subseteq mCl_1(mCl_2(A))$ and $mCl_1(mCl_2(A \cap B)) \subseteq mCl_1(mCl_2(B))$. Therefore, $mCl_1(mCl_2(A \cap B)) \subseteq mCl_1(mCl_2(A)) \cap mCl_1(mCl_2(B)) = A \cap B$. But $A \cap B \subseteq mCl_1(mCl_2(A \cap B))$. Consequently, $mCl_1(mCl_2(A \cap B)) = A \cap B$. Hence, $A \cap B$ is $m_X^1m_X^2$-closed. □

Remark 1. *The union of two $m_X^1m_X^2$-closed set is not a $m_X^1m_X^2$-closed set in general as can be seen from the following example.*

Example 3.6. *Let $X = \{1, 2, 3\}$. Define m-structures m_X^1 and m_X^2 on X as follows: $m_X^1 = \{\emptyset, \{1, 3\}, \{2, 3\}, X\}$ and $m_X^2 = \{\emptyset, \{1\}, \{2\}, \{1, 3\}, \{2, 3\}, X\}$. Then $\{1\}$ and $\{2\}$ are $m_X^1m_X^2$-closed but $\{1\} \cup \{2\} = \{1, 2\}$ is not $m_X^1m_X^2$-closed.*

Proposition 3.7. *Let (X, m_X^1, m_X^2) be a biminimal structure space. Then A is a $m_X^1 m_X^2$-open subset of (X, m_X^1, m_X^2) if and only if $A = mInt_1(mInt_2(A))$.*

Proof. Let A be a $m_X^1 m_X^2$-open subset of (X, m_X^1, m_X^2). Then $X - A$ is $m_X^1 m_X^2$-closed. Therefore, $mCl_1(mCl_2(X - A)) = X - A$. By Lemma 2.3(1), $X - mInt_1(mInt_2(A)) = X - A$.Consequently, $A = mInt_1(mInt_2(A))$.

Conversely, let $A = mInt_1(mInt_2(A))$. Therefore, $X - A = X - mInt_1(mInt_2(A))$. By Lemma 2.3(1), $X - A = mCl_1(mCl_2(X - A))$. Hence, $X - A$ is $m_X^1 m_X^2$-closed. Consequently, A is $m_X^1 m_X^2$-open. □

Proposition 3.8. *Let (X, m_X^1, m_X^2) be a biminimal structure space. If A and B are $m_X^1 m_X^2$-open subsets of (X, m_X^1, m_X^2), then $A \cup B$ is $m_X^1 m_X^2$-open.*

Proof. Let A and B be $m_X^1 m_X^2$-open. Then $mInt_1(mInt_2(A)) = A$ and $mInt_1(mInt_2(B)) = B$. Since $A \subseteq A \cup B$ and $B \subseteq A \cup B$, $mInt_1(mInt_2(A)) \subseteq mInt_1(mInt_2(A \cup B))$ and $mInt_1(mInt_2(B)) \subseteq mInt_1(mInt_2(A \cup B))$. Therefore, $A \cup B = mInt_1(mInt_2(A)) \cup mInt_1(mInt_2(B)) \subseteq mInt_1(mInt_2(A \cup B))$. But $mInt_1(mInt_2(A \cup B)) \subseteq A \cup B$. Consequently, $mInt_1(mInt_2(A \cup B)) = A \cup B$. Hence, $A \cup B$ is $m_X^1 m_X^2$-open. □

Remark 2. *The intersection of two $m_X^1 m_X^2$-open set is not a $m_X^1 m_X^2$-open set in general as can be seen from the following example.*

Example 3.9. *Let $X = \{1, 2, 3\}$. Define m-structures m_X^1 and m_X^2 on X as follows: $m_X^1 = \{\emptyset, \{1\}, \{2\}, \{1, 3\}, \{2, 3\}, X\}$ and $m_X^2 = \{\emptyset, \{1\}, \{2\}, \{1, 3\}, \{2, 3\}, X\}$. Then $\{1, 3\}$ and $\{2, 3\}$ are $m_X^1 m_X^2$-open but $\{1, 3\} \cap \{2, 3\} = \{3\}$ is not $m_X^1 m_X^2$-open.*

Definition 3.10. Let (X, m_X^1, m_X^2) be a biminimal structure space and Y be a subset of X. Define minimal structures m_Y^1 and m_Y^2 on Y as follows: $m_Y^1 = \{A \cap Y | A \in m_X^1\}$ and $m_Y^2 = \{B \cap Y | B \in m_X^2\}$. A triple (Y, m_Y^1, m_Y^2) is called a *biminimal structure subspace* (briefly *bim-subspace*) of (X, m_X^1, m_X^2).

Let (Y, m_Y^1, m_Y^2) be a biminimal structure subspace of (X, m_X^1, m_X^2) and let A be a subset of Y. The m_Y-closure and m_Y-interior of A with respect to m_Y^i are denote by $m_Y Cl_i(A)$ and $m_Y Int_i(A)$, respectively, for $i = 1, 2$. Then $m_Y Cl_1(A) = Y \cap mCl_1(A)$ and $m_Y Cl_2(A) = Y \cap mCl_2(A)$.

Proposition 3.11. *Let (Y, m_Y^1, m_Y^2) be a biminimal structure subspace of (X, m_X^1, m_X^2) and F be a subset of Y. If F is $m_X^1 m_X^2$-closed, then F is $m_Y^1 m_Y^2$-closed.*

Proof. Let F be $m_X^1 m_X^2$-closed. Then $mCl_1(mCl_2(F)) = F$. Therefore, $mCl_1(F) = F$ and $mCl_2(F) = F$. Hence, $Y \cap mCl_1(F) = F$ and $Y \cap mCl_2(F) = F$. Consequently, $m_Y Cl_1(m_Y Cl_2(F)) = F$. Hence, F is $m_Y^1 m_Y^2$-closed. □

References

[1] J.C. Kelly, *Bitopological spaces*, Pro. London Math. Soc. **3** (13)(1969), 71–79.

[2] H. Maki, K.C. Rao and A. Nagoor Gani, *On generalized semi-open and preopen sets*, Pure Appl. Math. Sci. **49** (1999), 17-29.

[3] T. Noiri and V. Popa, *A generalized of some forms of g-irresolute functions*, European J. of Pure and Appl. Math. **2** (4)(2009), 473–493.

[4] V. Popa and T. Noiri, *On M-continuous functions*, Anal. Univ. "Dunarea de Jos" Galati, Ser. Mat. Fiz. Mec. Teor.(2), **18** (23)(2000), 31–41.

Received: September, 2009