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Neumann Problem for Coupled Diffusion Systems with Localized Nonlinear Reactions

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Abstract

This paper deals with the Neumann problem for coupled diffusion systems with localized nonlinear reactions. We give the blow-up conditions and the asymptotic behavior of the blow-up solution.

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1 Introduction and main results

We consider positive solutions to the system of diffusion equations coupled with localized source

$$u_{it} = \Delta u_i + u_{i+1}^{p_i}(x_0, t) \ \ (i = 1, 2, ..., k), \ \ u_{k+1} := u_1, \ x \in \Omega, \ t > 0, \qquad (1.1)$$

with homogeneous Neumann boundary values

$$\frac{\partial u_i}{\partial \nu} = 0 \ \ (i = 1, 2, ..., k), \ \ x \in \partial\Omega, \ t > 0, \qquad (1.2)$$

and nontrivial, nonnegative and bounded initial data

$$u_i(x, 0) = u_{i0}(x) \quad (i = 1, 2, ..., k), \quad x \in \Omega, \ t > 0, \qquad (1.3)$$

where $p_i > 0$ $(i = 1, 2..., k)$. $\Omega \in R^N$ is a bounded domain with smooth boundary $\partial\Omega$, and ν is the outward unit normal vector on the boundary $\partial\Omega$.

Equations (1.1) describe a physical phenomenon where the reaction in a dynamical system is driven by the temperature at a single point (see [1, 4, 7]).

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The uncoupled single equation of (1.1) and some it's variants were studied by some authors (see [2, 5]). In [2], Chadam et al. studied the single more general equation

$$u_t = \Delta u + f(u(x_0(t), t)), \quad x \in \Omega,\ t > 0,$$

with Neumann boundary conditions. They gave the blow-up conditions, and proved that the blow-up set is the whole region $\overline{\Omega}$ if a solution blows up at finite time T. They also showed that $u(x_0, t) \le (\frac{2}{(p-1)(T-t)})^{\frac{1}{p-1}}$ for $f(s) = s^p, (p > 1)$.

We remark that a lot of work have been done in the past few years on the blow-up problems for coupled systems (see [3, 6-9]). In the special cases $k = 2$ of (1.1)-(1.3), the blow-up rate and blow-up set were studied in [9]. There are some works on the blow-up rate for a general semilinear diffusion system

$$u_{it} = \Delta u_i + u_{i+1}^{p_i} \ \ (i = 1, 2, ...k), \quad u_{k+1} := u_1, \quad (x, t) \in \Omega \times (0, +\infty)$$

with homogeneous Dirichlet boundary conditions, where $\Omega \in \mathbb{R}^N$ or $\Omega = \mathbb{R}^N$(see [3, 8] and references therein).

Motivated by the above mentioned works, the aim of this paper is to present the asymptotic behavior of the blow-up solution to the system (1.1)-(1.3). Now, we introduce some useful symbols to state our results. Let $\alpha := (\alpha_1, \alpha_2, ..., \alpha_k)^T$ be the solution of the following linear algebraic system

$$A\alpha := \begin{pmatrix} 1 & -p_1 & 0 & ... & 0 & 0 \\ 0 & 1 & -p_2 & ... & 0 & 0 \\ ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & 1 & -p_{k-1} \\ -p_k & 0 & 0 & ... & 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ ... \\ \alpha_{k-1} \\ \alpha_k \end{pmatrix} = - \begin{pmatrix} 1 \\ 1 \\ ... \\ 1 \\ 1 \end{pmatrix}. \tag{1.4}$$

A series of standard computations yield $detA = 1 - \prod_{i=1}^k p_i$. We shall see that $detA = 0$ is the critical global existence curve. A direct computation also shows that

$$\alpha_i = \frac{1 + p_i + \sum_{j=i+1}^{k+i-2} p_i...p_j}{\prod_{i=1}^k p_i - 1} \quad (i = 1, 2, ..., k). \tag{1.5}$$

From (1.4), we see that

$$\alpha_i + 1 = p_i \alpha_{i+1} \ \ (i = 1, 2, ..., k), \quad \alpha_{k+1} := \alpha_1. \tag{1.6}$$

Now we state the main results of this paper.

Theorem 1.1 *(i) If $\prod_{i=1}^k p_i \le 1$ (i.e. $detA \ge 0$), then every solution of the system (1.1)-(1.3) is global in time; (ii) If $\prod_{i=1}^k p_i > 1$ (i.e. $detA < 0$), then all solutions of (1.1)-(1.3) blow up in finite time.*

Theorem 1.2 *Let $(u_1, u_2, ..., u_k)$ be the solution of (1.1)-(1.3) in $\Omega \times (0,T)$, which blows up in finite time T. Then there exist positive constants C_i such that* $\lim_{t\to T} u_i(x,t)(T-t)^{\alpha_i} \le C_i$ $(i = 1, 2, ..., k)$, $(x,t) \in \Omega \times (0,T)$, *uniformly in* $\overline{\Omega}$.

Finally, we give a brief line of the rest of this paper. In Section 2, we give the blow-up conditions and prove Theorem 1.1. The proof of Theorem 1.2 is the subject of Section 3.

2 Blow up in finite time

In this section, we characterize when all solutions to the problem (1.1)-(1.3) are global in time or blow up. Now, we start our arguments with the maximum principle that will be used in the sequel.

Lemma 2.1 *Let $(u_1, u_2, ..., u_k)$ be a classical solution of the problem*

$$\begin{array}{lll} u_{it} - \Delta u_i \ge c_i(x,t)u_{i+1}(x_0,t) \ (i = 1,2,...,k),\ u_{k+1} := u_1, & (x,t) \in \Omega \times (0,T), \\ \frac{\partial u_i}{\partial \nu} = 0 \qquad\qquad (i = 1,2,...,k), & (x,t) \in \partial\Omega \times (0,T), \\ u_i(x,0) \ge 0 \qquad\qquad (i = 1,2,...,k), & x \in \Omega. \end{array} \tag{2.1}$$

If $0 \le c_i(x,t) < C_i$, *then* $u_i(x,t) \ge 0$ $(i = 1, 2, ..., k)$, *for all* $(x,t) \in \overline{\Omega} \times [0,T)$.

Proof. Set $w_i = e^{-Kt}u_i$, where $K = \sum_{i=1}^k C_i$. We claim $w_i \ge 0$ on $\overline{\Omega} \times [0,T']$ for any $T' < T$. In fact, if $min(w_l, w_2, ..., w_k)(\overline{x},\overline{t}) < 0$ for some $(\overline{x},\overline{t}) \in \overline{\Omega} \times [0,T']$, without loss of generality, we assume that $min(w_1, w_2, ..., w_k)(x,t)$ takes negative minimum at $(\overline{x},\overline{t})$ and $w_1(\overline{x},\overline{t}) \le w_i(\overline{x},\overline{t}), i = 2, ..., k$. Using the first inequality in (2.1), we find that

$$w_{1t} - \Delta w_1 \ge -Kw_1(x,t) + c_1(x,t)w_2(x_0,t), \quad (x,t) \in \Omega \times [0,T']. \tag{2.2}$$

If $(\overline{x},\overline{t}) \in \Omega \times (0,T']$, then we have

$$\begin{aligned} w_{1t}(\overline{x},\overline{t}) - \Delta w_1(\overline{x},\overline{t}) &\ge -Kw_1(\overline{x},\overline{t}) + c_1(\overline{x},\overline{t})w_2(x_0,\overline{t}) \\ &\ge -Kw_1(\overline{x},\overline{t}) + C_1 w_1(\overline{x},\overline{t}) > 0, \end{aligned} \tag{2.3}$$

here we use $0 \le c_1(\overline{x},\overline{t}) \le C_1$ and $w_2(x_0,\overline{t}) \ge min(w_1, w_2, ..., w_k)(x_0,\overline{t}) \ge min(w_1, w_2, ..., w_k)(\overline{x},\overline{t}) = w_1(\overline{x},\overline{t})$. On the other hand, $w_1(x,t)$ attains negative minimum at $(\overline{x},\overline{t})$, so $w_{1t}(\overline{x},\overline{t}) - \Delta w_1(\overline{x},\overline{t}) \le 0$, which leads to a contradiction to inequality (2.3).

If $(\overline{x},\overline{t}) \in \partial\Omega \times (0,T']$, we have $\frac{w_1}{\partial \nu}(\overline{x},\overline{t}) = 0$. In this case, we may choose a small $\epsilon > 0$ satisfying $\epsilon < -(K - C_1)w_1(\overline{x},\overline{t})$, and find a point $x_\epsilon \in \Omega$, sufficiently close to $\overline{x}$, such that

$$w_1(x_\epsilon,\overline{t}) \le w_1(\overline{x},\overline{t}) + \frac{\epsilon}{3K}, \quad w_{1t}(x_\epsilon,\overline{t}) \le \frac{\epsilon}{3}, \quad -\Delta w_1(x_\epsilon,\overline{t}) \le \frac{\epsilon}{3}.$$

Then combining these inequalities with (2.2), we obtain $c_1(x_\epsilon,\overline{t})w_2(x_0,\overline{t}) \leq w_{1t}(x_\epsilon,\overline{t}) - \Delta w_1(x_\epsilon,\overline{t}) + Kw_1(x_\epsilon,\overline{t}) \leq \epsilon + Kw_1(\overline{x},\overline{t})$. It follows that $\epsilon \geq -Kw_1(\overline{x},\overline{t}) + c_1(x_\epsilon,\overline{t})w_2(x_0,\overline{t}) \geq -(K-C_1)w_1(\overline{x},\overline{t})$, which contradicts our choice of ϵ. Thus, $min(w_1,w_2,...,w_k) \geq 0$ on $\overline{\Omega}\times[0,T)$ and $u_i(x,t)\geq 0$ $(i=1,2,...,k)$ on $\overline{\Omega}\times[0,T)$. □

Proof of Theorem 1.1(i). For $\prod_{i=1}^k p_i < 1$, we follow from (1.5) that $-\alpha_i > 1$. Construct $\overline{u}_i(x,t) = (M+t)^{-\alpha_i}$ $(i=1,2,...,k)$, where $M>0$ is to be determined later. Define $\overline{u}_{k+1} = \overline{u}_1$ and $\alpha_{k+1}=\alpha_1$. It will be obtained from (1.6) that $\overline{u}_{it} - \Delta\overline{u}_i = -\alpha_i(M+t)^{-\alpha_i-1} > (M+t)^{-p_i\alpha_{i+1}} = \overline{u}_{i+1}^{p_i}$ $(i=1,2,...,k)$. It is easy to see that $\frac{\partial \overline{u_i}}{\partial\nu} = 0,\ x\in\partial\Omega,\ t\geq 0$, and $\overline{u}_i(x,0)\geq u_{i0}(x)$ $(i=1,2,...,k)$, where we take M sufficiently large. It follows from Lemma 2.1 that $(\overline{u}_1,\overline{u}_2,...,\overline{u}_k)\geq(u_1,u_2,...,u_k)$, which implies $(u_1,u_2,...,u_k)$ globally exists.

For $\prod_{i=1}^k p_i = 1$, set $\overline{u}_i = Ae^{L_i t}$ $(i=1,2,...,k)$, where $A>0,\ L_i$ are to be determined. A simple computation shows $\overline{u}_{it} - \Delta\overline{u}_i = AL_ie^{L_it} \geq A^{p_i}e^{p_iL_{i+1}} = \overline{u}_{i+1}^{p_i}$ $(i=1,2,...,k)$, where $\overline{u}_{k+1} := \overline{u}_1, L_{k+1} := L_1$, if we choose L_i large enough and $L_i = p_iL_{i+1}$. In the case of $i=1$, we must confirm $L_1 = p_1L_2 = p_1p_2L_3 = ... = L_1\prod_{i=1}^k p_i$. Noticing $\frac{\partial\overline{u_i}}{\partial\nu} = 0,\ x\in\partial\Omega,\ t\geq 0$, and $\overline{u}_i(x,0)\geq u_{i0}(x)$ $(i=1,2,...,k)$ for A sufficiently large, by Lemma 2.1, we have $(\overline{u}_1,\overline{u}_2,...,\overline{u}_k)\geq(u_1,u_2,...,u_k)$, which implies $(u_1,u_2,...,u_k)$ globally exists. We have proved Theorem 1.1(i) for system (1.1)-(1.3). □

Proof of Theorem 1.1(ii). Since the initial data are nontrivial, without loss of generality we assume that $u_{10}(x)\geq 0$ and $u_{10}(x)\not\equiv 0$. Let v be the solution of $v_t - \Delta v = 0$ in $\Omega\times(0,\infty)$ with $v(x,0)=u_{10}(x)$. It follows from the maximum principle for the heat equation that $u_1\geq v$ as long as u_1 exists. Moreover, $v(x,t)>0$ in $\Omega\times(0,\infty)$. On the other hand, u_k satisfies $u_{kt}-\Delta u_k = u_1^{p_k}(x_0,t)\geq v^{p_k}(x_0,t) > 0$ in $\Omega\times(0,T)$ with $u_{k0}(x)\geq 0$. So $u_k(x,t)>0$ in $\Omega\times(0,T)$. Similarly, we have $u_i(x,t)>0$ $(i=2,3...,k-1)$ in $\Omega\times(0,T)$. Therefore, without loss of generality, we may assume $u_{i0}(x)>0$ $(i=1,2...,k)$ for $x\in\overline{\Omega}$.

Now, we prove the non-existence of global solutions by constructing a blow-up subsolution of the system (1.1)-(1.3). Consider the ODE system

$$\begin{aligned} f_i'(t) &= f_{i+1}^{p_i} \quad (i=1,2,...,k), \quad f_{k+1} := f_1, \quad t>0, \\ f_i(0) &= a_i > 0, \end{aligned}$$

where $a_i = min_{\overline{\Omega}} u_{i0}(x)$. By Lemma 2.1, we have $(f_1,f_2,...,f_k)\geq(u_1,u_2,...,u_k)$.

We claim that exist a constants $\delta>0$ such that $\left(\prod_{i=1}^k f_i\right)'(t)\geq\delta\left(\prod_{i=1}^k f_i(t)\right)^m$, where $m = 1+\frac{1}{\sum_{i=1}^k\alpha_i}$ (the claim is to be proved later).

For $\prod_{i=1}^k p_i > 1$, (1.5) implies that $m>1$. Noting $\left(\prod_{i=1}^k f_i\right)(0) = \prod_{i=1}^k a_i > 0$, it follow that $\left(\prod_{i=1}^k f_i\right)(t)$ blows up in finite time. So dose $(u_1,...,u_k)$, which implies that Theorem 1.1(ii) holds.

Now we prove the claim. We denote first by

$$\widetilde{A} := \begin{pmatrix} p_1 & -1 & 0 & ... & 0 & 0 \\ 0 & p_2 & -1 & ... & 0 & 0 \\ ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & p_{k-1} & -1 \\ -1 & 0 & 0 & ... & 0 & p_k \end{pmatrix}.$$

and then observe that the linear algebraic system $\widetilde{A}(\frac{1}{q_1}, \frac{1}{q_2}, ..., \frac{1}{q_{k-1}}, \frac{1}{q_k})^T = (m-1, m-1, ..., m-1, m-1)^T$ exists a unique solution $(\frac{1}{q_1}, \frac{1}{q_2}, ..., \frac{1}{q_{k-1}}, \frac{1}{q_k})^T$ with $\frac{1}{q_1} = \frac{m-1}{det\widetilde{A}}[1 + p_k + p_k p_{k-1} + ... + p_k p_{k-1} p_{k-2} ... p_2]$. By using $p_i \frac{1}{q_i} - \frac{1}{q_{i+1}} = m-1$, we can deduce the other $\frac{1}{q_i}$ $(i = 2, 3, ..., k)$. After a series of computation, we get $\frac{1}{q_i} > 0$ and $\sum_{i=1}^{k} \frac{1}{q_i} = 1$ which imply that $0 < \frac{1}{q_i} < 1$ or equivalently $\mathrm{q_i} > 1$ $(\mathrm{i} = 1, 2, ..., \mathrm{k})$. Then we use Hölder's inequality to obtain

$$\begin{aligned} \left(\prod_{i=1}^{k} f_i(t)\right)^m &= f_2^{p_1\frac{1}{q_1}+1-\frac{1}{q_2}} f_3^{p_2\frac{1}{q_2}+1-\frac{1}{q_3}} f_4^{p_3\frac{1}{q_3}+1-\frac{1}{q_4}} ... f_1^{p_k\frac{1}{q_k}+1-\frac{1}{q_1}} \\ &= (f_2^{p_1+1} f_3 f_4 ... f_k)^{\frac{1}{q_1}} (f_1 f_3^{p_2+1} f_4 ... f_k)^{\frac{1}{q_2}} ... (f_1^{p_k+1} f_2 f_3 f_4 ... f_{k-1})^{\frac{1}{q_k}} \\ &\leq C(f_2^{p_1+1} f_3 f_4 ... f_k + f_1 f_3^{p_2+1} f_4 ... f_k + ... + f_1^{p_k+1} f_2 f_3 f_4 ... f_{k-1}). \end{aligned}$$

Combining the above inequality with $f_i'(t) = f_{i+1}^{p_i}$ $(i = 1, 2, ..., k)$, $f_{k+1} := f_1$, we have proved our claim. □

3 Asymptotic behavior

In this section, we concern with the asymptotic behavior of blow-up solution near the blow-up time and the set of blow-up points. We first give an important lemma.

Lemma 3.1 *Let $w \in C^{2,1}(\overline{\Omega} \times (0, T))$ be a solution of the problem*

$$\begin{aligned} &w_t - \Delta w = g(t), && (x, t) \in \Omega \times (0, T), \\ &\frac{\partial w}{\partial \nu} = 0, && (x, t) \in \partial\Omega \times (0, T), \\ &w(x, 0) = w_0(x) \geq 0, && x \in \Omega. \end{aligned}$$

Then we have $\lim_{t\to T} ||w(., t)||_\infty = +\infty$ if and only if $\int_0^T g(s)ds = +\infty$. Furthermore, if $\lim_{t\to T} ||w(., t)||_\infty = +\infty$, then $\lim_{t\to T} \frac{w(x,t)}{G(t)} = \lim_{t\to T} \frac{||w(.,t)||_\infty}{G(t)} = 1$, uniformly in $\overline{\Omega}$, where $G(t) = \int_0^t g(s)ds$.

Proof. Let $G(x, y; t-\tau)$ be Green's function associated with the heat operator $\frac{\partial}{\partial t} - \Delta$. Note that the function $G(x, y; t-\tau)$ possesses the properties $G(x, y; t-\tau) \geq 0$, $\int_\Omega G(x, y; t)dy = 1$. Then we have

$$\begin{aligned} w(x, t) &= \int_\Omega G(x, y; t) w_0(y) dy + \int_0^t \int_\Omega G(x, y; t-\tau) g(\tau) dy d\tau \\ &= \int_\Omega G(x, y; t) w_0(y) dy + \int_0^t g(\tau) d\tau, \quad (x, t) \in \Omega \times (0, T). \end{aligned}$$

It follows that $\int_0^t g(\tau)d\tau \le w(x,t) \le ||w_0||_\infty + \int_0^t g(\tau)d\tau,\ (x,t) \in \Omega \times (0,T)$. From the inequality, we see that Lemma 3.1 holds. □

Lemma 3.2 *Let $(u_1, u_2, ..., u_k)$ be a solution to the problem (1.1)-(1.3). If $(u_1, u_2, ..., u_k)$ blows up in finite time T, then u_i $(i = 1, 2, ..., k)$ blow up simultaneously.*

Proof. Suppose on the contrary that u_i $(i = 1, 2, ..., k)$ do not blow up simultaneously in finite time T. Without loss of generality, we may assume that u_1 blows up in finite time T and u_2 is bounded on $\overline{\Omega} \times [0, T]$. That is, there exists a constant $C > 0$ such that $0 \le u_2 \le C$ for all $(x,t) \in \overline{\Omega} \times [0,T]$. From the Lemma 3.1, we know that u_1 is bounded on $\overline{\Omega} \times [0, T]$, which is a contradiction. □

Remark. Lemmas 3.1 and 3.2 imply that the blow-up set of a blow-up solution of (1.1)-(1.3) is the whole region $\overline{\Omega}$.

In the following, we intend to prove the result of Theorems 1.2. For convenience, from now on we write $g_i(t) = u_{i+1}^{p_i}(x_0,t),\ G_i(t) = \int_0^t g_i(s)ds$ $(i = 1, 2, ..., k)$, $u_{k+1} := u_1$, and use the notation $f(t) \sim g(t)$ for $\lim_{t\to T} \frac{f(t)}{g(t)} = 1$. By Lemma 3.1, as $t \to T$, we have

$$G_i(t)' = g_i(t) = u_{i+1}^{p_i}(x_0,t) \sim G_{i+1}^{p_i}(t)\ \ (i = 1, 2, ..., k),\quad u_{k+1} := u_1,\quad G_{k+1}(t) := G_1(t).$$

Lemma 3.3 *If $(u_1, u_2, ..., u_k)$ is the solution of (1.1)-(1.3) with blow-up time T, then there exist positive constants C_i such that $\lim_{t\to T} G_i(t)(T-t)^{\alpha_i} \le C_i$ $(i = 1, 2, ..., k)$ uniformly in $\overline{\Omega}$.*

Proof. By similar arguments in the proof of Theorem 1.1(ii), we have $\left(\prod_{i=1}^k G_i\right)'(t) \ge \lambda \left(\prod_{i=1}^k G_i(t)\right)^{1+\frac{1}{\Sigma_{i=1}^k \alpha_i}}$, as $t \to T$, where λ is a positive constant. We get by integrating

$$\prod_{i=1}^k G_i(t) \le C(T-t)^{-\Sigma_i^k \alpha_i} \quad \text{as} \quad t \to T. \tag{3.1}$$

With (3.1) at hand, we claim there exist $C_i > 0$ such that $G_i(t) \le C_i(T-t)^{-\alpha_i}$ $(i = 1, 2, ..., k)$, as $t \to T$. In fact, we only need to show the case $i = 1$. On the contrary, noting $G_i(t)' \sim G_{i+1}^{p_i}(t)$, if there exist two sequences $\{t_n\}$, $(0 < t_n < T)$, and $\{c_n\}$ with $t_n \to T^-$ and $c_n \to \infty$ as $n \to \infty$ such that $G_1(t_n) \ge c_n(T-t_n)^{-\alpha_1}$, for large n, we have

$$\begin{aligned}
G_k(t) &\ge G_k(t_n) + c\int_{t_n}^t G_1^{p_k}(s)ds \ge cG_1^{p_k}(t_n)(t-t_n) \ge cc_n^{p_k}(T-t_n)^{-p_k\alpha_1}(t-t_n),\\
G_{k-1}(t) &\ge G_{k-1}(t_n) + c\int_{t_n}^t G_k^{p_{k-1}}(s)ds \ge c^{1+p_{k-1}}c_n^{p_{k-1}p_k}(T-t_n)^{-p_{k-1}p_k\alpha_1}\int_{t_n}^t (s-t_n)^{p_{k-1}}ds\\
&= \frac{1}{p_{k-1}+1}c^{1+p_{k-1}}c_n^{p_{k-1}p_k}(T-t_n)^{-p_{k-1}p_k\alpha_1}(t-t_n)^{p_{k-1}+1},\\
&\cdots\cdots\\
G_1(t) &\ge e(p_1, ..., p_{k-1})c_n^{\prod_{i=1}^k p_i}(T-t_n)^{-\alpha_i\prod_{i=1}^k p_i}(t-t_n)^{1+\sum_{l=1}^{k-1}\prod_{i=1}^l p_i}
\end{aligned}$$

where c is a suitable constant and $e(p_1, ..., p_{k-1})$ is a constant depending only on p_i $(i = 1, 2, ..., k-1)$. Multiplying the above inequalities, we get

$$\prod_{i=1}^{k} G_i(t) \geq e c_n^{\theta} (T - t_n)^{-p\alpha_1} (t - t_n)^q, \tag{3.2}$$

where $e, \theta > 0$ is are constants depending only on p_i, and

$$p = \sum_{l=0}^{k-1} \prod_{i=k-l}^{k} p_i, \quad q = 1 + (1 + p_{k-1}) + ... + (p_1 p_2 ... p_{k-1} + p_1 p_2 ... p_{k-2} + ... + p_1 + 1).$$

Taking $t = t' := \frac{T+t_n}{2}$ in (3.2), we have $t' \to T$ as $n \to \infty$ and

$$\prod_{i=1}^{k} G_i(t') \geq e c_n^{\theta} 2^{-q} (T - t_n)^{-p\alpha_1 + q} \geq e c_n^{\theta} 2^{-p\alpha_1} (T - t')^{-p\alpha_1 + q}. \tag{3.3}$$

Using the definitions of α_i; p; q, after a series of complicated but standard computations, we have $-p\alpha_1 + q = -\sum_{i=1}^{k} \alpha_i$, which implies inequality (3.3) contradicts (3.1) since $c_n \to \infty$ as $n \to \infty$. So the claim $G_1(t) \leq C_1(T - t)^{-\alpha_1}$ as $t \to T$ is true. Proceeding in a similar way with the other (in)equalities, we conclude $G_i(t) \leq C_i(T - t)^{-\alpha_i}$ $(i = 2, ..., k)$, as $t \to T$.□

We obtain the conclusion of the Theorem 1.2 by combining Lemmas 3.1 and 3.3. □

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