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An Introduction to Quasi N-Open Sets and Associated Compactness Concepts in Bitopological Spaces

Doris Doda [*] and Adea Bajri [**]

[*] Barleti University, Tirana, Albania
[**] Turgut Ozal High School, Tirana, Albania

Abstract

In this paper, we investigate three distinct varieties of N-open sets in the context of bitopological spaces and analyze their influence on various compactness notions. Bitopological spaces, which are endowed with two distinct topologies, offer a rich framework for examining the interplay between these topologies through different forms of open sets. Our study focuses on N-open sets, a class of sets that generalize traditional open sets and provide new avenues for exploring compactness. We define and explore three specific types of N-open sets and introduce corresponding concepts of compactness that arise from their use. These concepts are examined in depth, highlighting their theoretical underpinnings and practical implications. By providing formal definitions and rigorous proofs, we demonstrate how these new notions of compactness extend classical results and reveal novel insights into the structure of bitopological spaces.[4]
Furthermore, we explore potential applications of these new compactness concepts in various mathematical and practical contexts. By expanding the definitions and studying their implications, our work aims to enhance the understanding of how

these new forms of compactness interact with the structure of bitopological spaces. This study not only contributes to the theoretical development of compactness in bitopological spaces but also suggests avenues for future research and applications in areas where bitopological spaces are utilized.
Overall, our paper provides a thorough investigation into the role of N-open sets in defining and understanding compactness, offering new insights into their theoretical and practical significance within the broader field of topology.

Keywords: ω-open set, N-open set, Compactness, Countably Compact, Continuous Function

Introduction

Let (X, τ) represent a topological space, and let A be a subset of X. A point x∈X is referred to as an infinite point (or condensation point) of A if, for every open set U ∈τ containing x, the intersection U∩A is infinite (or uncountable). The set A is defined as N-closed [2,3] (or ω-closed [7]) if it includes all its infinite points (or condensation points). Conversely, the complement of an N-closed (or ω-closed) set is termed N-open [2, 3] (or ω-open [8]). For a topological space (X, τ), we denote the collection of ω-open sets by τω and the collection of N-open sets by τN. It is known that τN is situated between τ and τω. Additionally, a set A is N-open (or ω-open) if and only if, for every x∈A, there exists an open set O∈τ and a finite (or countable) set G such that x∈O−G⊆A. Using ω-open sets, Al-Omari and Noorani in [2,3] provided various characterizations of compact and strongly compact topological spaces.
Expanding on classical topological spaces, Kelly introduced bitopological spaces in [9] as an ordered triple (X,τ,σ) consisting of a set X and two topologies τ and σ. Datta, in [6], introduced the concept of quasi-open sets within bitopological spaces. Subsequent research in [1—5, 11-16, 18-20] has explored various modifications of quasi-open sets to develop new bitopological concepts. In this study, we define and examine quasi N-open sets as a novel class of sets in bitopological spaces and utilize them to establish new compactness concepts. We also provide characterizations related to compact bitopological spaces.

1. An introduction to the Types of N-Open Sets in Bitopological Spaces

Definition 1.1. [6] Let (X,τ,σ) be a bitopological space.

(a) The least upper bound topology on X is the smallest topology on X that includes both τ and σ.
(b) A set A⊆X is called *semi-open* (or s-open) if it is open in the least upper bound topology on X.
When τ and σ are two topologies on X, the least upper bound topology on X is denoted by ⟨τ,σ⟩.

Proposition 1.2. [5] Consider two topologies τ and σ on a set X. A subset A⊆X is said to be s-open if and only if, for every point x∈A, there exist open sets U in τ and V in σ such that x∈U∩V⊆A.

Proof:
(⇒) We suppose that A is s-open.
By the definition of s-open, a subset A⊆X is s-open if and only if for every point x∈A, two topologies τ and σ on a set X, there exist open sets U∈τ and V∈σ such that x∈U∩V⊆A.
A is s-open if for every x∈A, there exist open sets U∈τ and V∈σ such that x∈U∩V⊆A.
This condition is directly satisfied by the assumption, so the implication is straightforward. For each x∈A, you can find such open sets U and V because A is s-open by assumption.
(⇐) We suppose that for every point x∈A, there exist open sets U∈τ and V∈σ such that x∈U∩V⊆A.
We want to show that A is s-open under this condition. Let x∈A. By assumption, there exist open sets U in τ and V in σ such that x∈U∩V⊆A.
Since U is open in τ and V is open in σ, their intersection U∩V is open in the topology induced by the product τ and σ.
Therefore, U∩V is a neighborhood of x contained within A. This implies that x has a neighborhood contained in A, satisfying the condition for A to be s-open.
Both directions of the proof hold, so we conclude that a subset A⊆X is s-open if and only if for every point x ∈A, there exist open sets U in τ and V in σ such that x∈U∩V⊆A. This completes the proof of Proposition 1.2.

Definition 1.3. [10] A set A⊆X is said to be *u-open* if A belongs to the union of the topologies τ and σ.
The collection of all u-open sets in (X,τ,σ) is denoted by u(τ,σ).

Definition 1.4. [6] A set A ⊆X is called *quasi-open (or q-open)* if for every point x∈A, there exists an open set U_x∈τ such that x∈U_x⊆A or an open set V_x∈σ such

that x∈V_x⊆A. Equivalently, A is q-open if and only if A can be expressed as B∪C, where B∈τ and C∈σ. A set A⊆X is q-closed if X−A is q-open.
The collection of all q-open sets in (X,τ,σ) is denoted by q(τ,σ).

Proposition 1.5. [6] states the following in a bitopological space (X,τ,σ):
(a) The union topology u(τ,σ) is a subset of the q(τ,σ)-open sets, which in turn is a subset of the join ⟨τ,σ⟩. Generally, q(τ,σ) is not equal to ⟨τ,σ⟩, and it is also not always equal to the union of τ and σ.
(b) The q(τ,σ)-open sets are closed under arbitrary unions, but this does not necessarily imply that q(τ,σ) forms a topology on X.
(c) The intersection of any collection of q-closed sets is itself q-closed.

Definition 1.6. [5] Let (X,τ,σ) be a bitopological space and let A⊆X.
(a) A is called u-ω -open if A is in the union of τ_ω and σ_ω. Equivalently, A is u-ω-open if A∈u (τ_ω, σ_ω).
(b) A is u-ω-closed if X−A is u-ω-open in (X,τ,σ).
(c)A is s-ω-open if it is an open set in the least upper bound topology on X that is generated by τ_ω and σ_ω.

Definition 1.7. [4] Let (X,τ,σ) be a bitopological space and A⊆X.
(a) A set A is termed u-N-open if A belongs to the union of τ_N and σ_N. Equivalently, A is u-N-open if and only if A∈u (τ_N, σ_N).
(b) A set A is called u-N-closed if X−A is u-N-open in (X,τ,σ).
(c) A set A is referred to as s-N-open if it is an open set in the least upper bound topology on X derived from τ_N and σ_N.
In the bitopological space (X,τ,σ), the collection of all u-N-open sets is denoted by u-N(τ,σ), while the collection of all N-open sets in the topological space (X,⟨τ,σ⟩) is denoted by ⟨τ,σ⟩N

Theorem 1.8.[4]
(a) Every u-open set in a bitopological space is also u-N-open.
(b) Every u-N-open set in a bitopological space is also u-ω-open.

Proof:
(a) Let (X,τ,σ) be a bitopological space and let A be a u-open set. Since A is in u(τ,σ), and given that τ⊆τ_N and σ⊆σ_N, it follows that u(τ,σ)⊆u(τ_N ,σ_N). Thus, A∈u(τ_N, σ_N), which means A is u-N-open.
(b) Let A be a u-N-open set. Since A∈u (τ_N , σ_N) and τ_N ⊆τ_ω and σ_N⊆σ_ω, we have u (τ_N, σ_N) ⊆u (τ_ω,σ_ω). Therefore, A∈u (τ_ω,σ_ω), making A u-ω-open.

Example 1.9. [4] Consider (R,τ,σ) where τ is the left ray topology and σ is the indiscrete topology. It is evident that R−Q is u-ω-open but not u-N-open, and R − {1} is u-N-open but not u-open.

Theorem 1.10. [4] Let (X,τ,σ) be a bitopological space. Then ⟨τ,σ⟩N=⟨τ,σ⟩N=⟨τ_N ,σ_N⟩.

Proof: Let A∈⟨τ,σ⟩N. This implies A is N-open in the topology ⟨τ,σ⟩. Thus, for every x∈A, there exist H and F such that H∈⟨τ,σ⟩ and F is a finite subset with x∈H−F⊆A. Since H∈⟨τ,σ⟩, there are U∈τ and V∈σ such that x∈U∩V⊆H. Thus, U−F∈τ_N and V−F∈σ_N , and x ∈(U−F) ∩(V−F) ⊆(U∩V) −F⊆H−F⊆A. Therefore, A ∈⟨τ_N, σ_N⟩.
Conversely, if A ∈⟨τ_N, σ_N⟩, then for every x∈A, there exist W∈τ_N, and G∈σ_N such that x∈W∩G⊆A. Since W and G can be expressed as U−F and V−M respectively, where U∈τ and V∈σ, with finite F and M, we have U∩V∈⟨τ,σ⟩ and x∈(U∩V)−(F∪M)⊆W∩G⊆A. Thus, A∈⟨τ,σ⟩N.

Theorem 1.11. [4] Let (X,τ,σ) be a bitopological space. Then u-N(τ,σ)⊆⟨τ,σ⟩N.

Proof: Since u-N(τ,σ)=τ_N∪σ_N and τ_N∪σ_N⊆⟨τ_N,σ_N⟩, and by Theorem 1.10, ⟨τ_N,σ_N ⟩=⟨τ,σ⟩N, it follows that u-N(τ,σ)⊆⟨τ,σ⟩N.

Example 1.12. [4] In the bitopological space (R, τ_{lr}, τ_{rr}), let A= (6,9). Here, A∈⟨τ,σ⟩ and A⊆⟨τ,σ⟩N, but A∉τ_N∪σ_N, hence A∉u-N(τ,σ).
As defined in [5], a set A⊆(X,τ,σ) is termed q-ω-open if for every point x∈A, there exists U_x∈σ_N such that x∈U_x⊆A or V_x∈σ_ω such that x∈V_x⊆A. Equivalently, A is q-ω-open if and only if A∈q(τ_ω, σ_ω). A set A⊆X is q-ω-closed if X−A is q-ω-open. The collection of all q-ω-open sets in (X,τ,σ) is denoted by q-ω(τ,σ).

Definition 1.13. [4] In a bitopological space (X,τ,σ), a set A⊆X is called q-N-open if for every point x∈A, there exists U_x∈τ_N such that x∈U_x⊆A or V_x∈σ_N such that x∈V_x⊆A. Alternatively, A is q-N-open if and only if A∈q(τ_N,σ_N). A set A is q-N-closed if its complement X−A is q-N-open. The collection of all q-N-open sets in (X,τ,σ) is denoted by q-N(τ,σ).

Theorem 1.14. [4] For a bitopological space (X,τ,σ) and a set A⊆X, the following are equivalent:
(a) A is q-N-open.

(b) For each x∈A, there exists U∈u(τ,σ) and a finite set F⊆X such that x∈U−F⊆A.

Proof: **(a) ⇒ (b):**
Assume A is q-N-open. By definition, this means that for each point x∈A, there exist sets B∈τ_N and C∈σ_N such that A=B∪C.
For each x∈A, assume without loss of generality that x∈B. By the definition of τ_N, there exists U∈τ and a finite set F⊆X such that x∈U−F⊆B.
Since B⊆A, we have x∈U−F⊆A.
Because U∈τ and F is finite, U−F is a subset of some U′∈u(τ,σ) (where u(τ,σ) denotes the topology generated by τ and σ).
Therefore, x∈U′−F⊆A, which satisfies the condition for A to be q-N-open.
Thus, (a) ⇒ (b) is proved.
(b) ⇒ (a):
Assume that for each x∈A, there exists an open set U∈u(τ,σ) and a finite set F⊆X such that x∈U−F⊆A.
Define B_x=U−F for each x∈A, where U and F satisfy the conditions of statement ((b).
Notice that B_x⊆A and each B_x is q-N-open because U is an open set in the topology u(τ,σ) and F is finite.
Then $= \bigcup_{x\in A} B_x$, which is a union of q-N-open sets and thus q-N-open itself.
Thus, (b) ⇒ (a) is proved.

Theorem 1.15. [4] For a bitopological space (X,τ,σ):
(a) u-N(τ,σ)⊆q-N(τ,σ).
(b) q(τ,σ)⊆q-N(τ,σ).
(c) q-N(τ,σ)⊆⟨τ,σ⟩N.
(d) {∅, X} ⊆q-N(τ,σ).
(e) The family q-N(τ,σ) is closed under arbitrary unions.
(f) The family of all q-N-closed sets is closed under arbitrary intersections.
(g) q-N(τ,σ)⊆qN(τ,σ)⊆q-ω(τ,σ).

Proof:
(a) Since u(τ,σ)⊆q(τ,σ) and u-N(τ,σ)=u(τ_N, σ_N), it follows that u(τ_N, σ_N)⊆q(τ, σ_N)= qu (τ_N, ,σ_N)⊆q(τ_N, σ_N)=q-N(τ,σ). Hence, u-N (τ, σ) ⊆qN(τ,σ)⊆q-N(τ,σ).
(b) Let A∈q(τ,σ). Then A=B∪C where B∈τ and C∈σ. Since τ⊆τ_N and σ⊆σ_N, B∈τ_N and C∈σ_N. Therefore, A∈q(τ_N, σ_N) =q-N(τ,σ).
(c) Since q(τ,σ)⊆⟨τ,σ⟩, it follows that q-N(τ,σ)=q(τ_N,σ_N)⊆⟨τ_N,σ_N⟩. By Theorem 1.10, ⟨τ_N, σ_N⟩=⟨τ,σ⟩N, hence q-N(τ,σ)⊆⟨τ,σ⟩N.

(d) Since {∅, X} ⊆u {∅, X} ⊆u-N(τ,σ) and by part (a), {∅,X}⊆q-N(τ,σ).

(e) Since q-N(τ,σ)=q(τ_N,σ_N) and q(τ_N,σ_N) is closed under arbitrary unions, q-N(τ,σ) is also closed under arbitrary unions.

(f) Let {Aα: α∈Δ} be a collection of q-N-closed sets. The intersection $\bigcap_{\alpha\in\Delta} A_\alpha$ is q-N-closed because the complement of this intersection is q-N-open by the definition of q-N-closed sets.

(g) This follows from the definitions and Theorem 1.8.

Examples: [4]

- Example 1.16. In the bitopological space (R,τ_{lr}, τ_{rr}), the set A= (−∞,0) ∪ (1, ∞) is q-N-open but not u-N-open.
- Example 1.17. In the same space, the set A= (−∞,1) − {0} is q-N-open but not in q(τ,σ).
- Example 1.18. For A= (0,2) in (R,τ_{lr}, τ_{rr}), A∈⟨τ_{lr}, τ_{rr}⟩⊆⟨τ_{lr}, τ_{rr}⟩N, but A is not q-N-open.
- Example 1.19. In (R, τ_{lr}, τ_{rr}), A= (−∞,2) and B= (0, ∞) are both q-N-open, but their intersection A∩B = (0,2) is not q-N-open.

Theorem 1.20. [4] In a bitopological space (X, τ, σ), q (τ, σ) forms a topology on X if and only if q (τ, σ) =⟨τ, σ⟩.

Theorem 1.21. [4] In a bitopological space (X, τ, σ), q-N (τ, σ) is a topology on X if and only if q-N (τ, σ) =⟨τ, σ⟩N.

Proof:

(⇒) Assume q-N (τ, σ) is a topology. By Theorem 1.15(c), we need to show ⟨τ,σ⟩N ⊆q-N(τ,σ). If A∈⟨τ,σ⟩N, then there exists U_x∈⟨τ,σ⟩ and a finite set Fx⊆X such that x∈U_x− F_x⊆A. Since U_x∈⟨τ,σ⟩, by Proposition 1.2, there exist H_x∈ τ and G_x ∈ σ such that

$x \in H_x \cap G_x \subseteq A$. Since q-N (τ, σ) is a topology on X, $H_x \in \tau \subseteq q$-N (τ, σ) and $G_x \in \sigma \subseteq q$-N ($\tau$, σ), then $H_x \cap G_x \in q$-N (τ, σ) and so $(H_x \cap G_x) - F_x \in q$-N ($\tau$, σ). By Theorem 1.15 (e), it follows that A=$\bigcup\{(H_x \cap G_x) - F_x: x \in A\}$is q-N-open.

(⇐)This is because h_τ, σ_{iN} is a topology on X.

2. Analysis of Compactness in Bitological spaces

Definition 2.1. [4] In a bitopological space (X,τ,σ), a cover U is called:

(a) τσ-open if U⊆u(τ,σ).
(b) p-open if it is τσ-open and contains at least one nonempty member of τ and at least one nonempty member of σ.

Definition 2.2. [4] In a bitopological space (X,τ,σ), the space is called: (a) s-compact if every τσ-open cover of (X,τ,σ) has a finite subcover. (b) p-compact if every p-open cover of (X,τ,σ) has a finite subcover.

Theorem 2.3. [4] For a bitopological space (X,τ,σ), (X,τN,σN) is s-compact if and only if every cover of X consisting of elements from the set A={W−F: W∈u(τ,σ) and F is a finite set} has a finite subcover.

Proof:
(⇒)Assume (X, τ_N, σ_N) is s-compact. If H is a cover of X with H⊆A, then H is a $\tau_N\sigma_N$-an open cover of (X, τ_N, σ_N). Since (X, τ_N, σ_N) is s-compact, a finite subcover from H that covers X exists.
(⇐)If H = {H_α: α∈Δ} is a τ_N σ_N -open cover of (X, τ_N, σ_N), then each H_α∈u (τ_N, σ_N) and $H_\alpha \in \tau_N \cup \sigma_N$) . Thus, Hα can be expressed as $H_\alpha = C_\alpha - F_\alpha$ where C_α∈u(τ,σ) and F_α is finite. Hence, H_α∈A. Given H⊆A, the assumption ensures that H has a finite subcover. Therefore, (X, τ_N, σ_N) is s-compact.

Theorem 2.4. For a bitopological space (X,τ,σ), the following statements are equivalent: (a) (X,τ,σ) is s-compact. (b) (X, τ_N, σ_N) is s-compact. (c) Every cover of X consisting of elements from q−N(τ,σ) has a finite subcover. (d) Every cover of X consisting of elements from q(τ,σ) has a finite subcover.

Proof:
(a)⇒(b): Suppose (X,τ,σ) is s-compact. By Theorem 2.3, if A= {W−F: W∈u(τ,σ) and F is finite}, then any cover of X by elements of A has a finite subcover. Thus, (X, τ_N, σ_N) being s-compact implies it satisfies the condition.
(b) ⇒(c): Suppose (X, τ_N, σ_N) is s-compact. Let H = {H_α: α∈Δ} be a cover of X by elements of q−N(τ,σ). Each H_α can be written as $H_\alpha = A_\alpha \cup B_\alpha$ where $A_\alpha \in \tau_N$ and $B_\alpha \in \sigma_N$. Since {$A_\alpha \cup B_\alpha$: α∈Δ} covers X and{A_α,B_α:α∈Δ} ⊆ u (τ_N, σ_N), by assumption, there is a finite subcover {$A_\alpha \cup B_\alpha$: α∈Δ_0}. Thus, {H_α: α∈Δ_0} is a finite subcover.
(c) ⇒ (d): Suppose every cover of X by elements of q−N(τ,σ) has a finite subcover. Since q(τ,σ)⊆q−N(τ,σ), any cover of X by elements of q(τ,σ) also has a finite subcover.

(d) ⇒ (a): Since u(τ,σ)⊆q(τ,σ), if every cover by elements of q(τ,σ) has a finite subcover, then every cover by elements of u(τ,σ) (which is a subset of q(τ,σ)) also has a finite subcover. Thus, (X,τ,σ) is s-compact.

Theorem 2.5. [4] For a bitopological space (X,τ,σ), the following statements are equivalent: (a) (X,τ,σ) is p-compact. (b) (X, τ_N, σ_N) is p-compact.

Proof:
(a) ⇒ (b): Let (X,τ,σ) be p-compact. If H={H_α: α∈Δ} is a p-open cover of (X, τ_N, σ_N), then H is a cover where each Hα∈τ_N or σ_N.Since {H_α} can be expressed as $H_\alpha = C_\alpha - F_\alpha$ where C_α∈u(τ,σ) and F_α is finite, H is a cover by elements of p-open sets, ensuring a finite subcover. Thus, (X, τ_N, σ_N) is p-compact.
(b) ⇒ (a): If (X, τ_N, σ_N) is p-compact, then any p-open cover of (X,τ,σ) (which is also a cover for (X,τ_N, σ_N) will have a finite subcover.

Theorem 2.6. [4] In an s-compact bitopological space (X,τ,σ), if A is a q−N-closed subset, then A is an s-compact subset of (X,τ,σ).

Proof: Suppose (X,τ,σ) is s-compact. Let A be a q−N-closed subset. If H is a τσ-open cover of A, then H⊆q−N(τ,σ). Since X−A is q−N(τ,σ)-open, H∪{X−A} is a cover of X by elements of q−N(τ,σ). By s-compactness, there is a finite subcover of H∪{X−A}, implying A is s-compact.

Corollary 2.7. [4] In an s-compact bitopological space (X,τ,σ), if A is a u−N-closed subset, then A is an s-compact subset of (X,τ,σ).

Proof: This follows directly from Theorem 2.6.

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