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A New Trigonometric Hyperbolic Cryptology Algorithm of Natural Transformation

Ather Mehmood

Department of Mathematics and Statistics
University of Agriculture Faisalabad, Pakistan

Ghulam Farid

Department of Mathematics and Statistics
University of Agriculture Faisalabad, Pakistan

Aysha Nasir

Department of Mathematics and Statistics
University of Agriculture Faisalabad, Pakistan

Mehwish Batool

Department of Mathematics and Statistics
University of Agriculture Faisalabad, Pakistan

Abstract

In this paper, there is new term introduced of trigonometric hyperbolic cryptography with the help of Natural Transformation. Natural transformation of Trigonometric hyperbolic function helps us to safe and secure of sensitive information. Alert the numerals of plain text and apply transformation and get cipher text. After applying Natural Transformation generate the key algorithm and find the plain text again.

Keywords: Plain Text, Cipher text, Natural Transformation, Encryption Technique, Key Algorithm, Decryption Technique

1 Introduction

Natural Transformation used in cryptology process with trigonometric hyperbolic function. Without Mathematical Transformation the cryptography is meaningless. When we have plain text that is readable by humans alert the numbers of English alphabets from standard table and extended table of cryptography, write it in the form of series, apply Natural Transformation, due to linearity we find again numbers and take a mod 26 and mod 63 and again concern to tables of cryptography for get the cipher text that is not readable by any person. After converted the plain text in to cipher text we generate the key algorithm. With the help of key algorithm and apply inverse of Natural Transformation we get the plain text that is high level language we say this process as Mathematical cryptology. In basic terminology alert the numbers to alphabets of English for $a = 0, b = 1, c = 2$ and so on $z = 25$. And for Hard security and safeness used first sixty three positive integer numbers. Natural Transformation enabled us to proceed the process of Mathematical cryptology. Mathematical Natural transforms helps the people, companies, e-commerce sector, computer passwords, pay-TV and banks to feel secure for interchanging the data and secret information. Symbolic representation of Mathematical cryptology process as

$$E(P) = C \qquad \textbf{and also} \qquad D(C) = P.$$

For more understanding it also be written as

$$E^{-1}(C) = P \qquad \textbf{and} \qquad D^{-1}(P) = C.$$

2 Preliminary Notes / Materials and Methods

(i) P is the set of finite plain text,

(ii) C is the set of finite cipher text,

(iii) K is the set of finite keys,

(iv) $E =\{E_k : k \in K\}$ is belong to encryption function $E_k : P \to C$,

(v) $D = \{D_k : k \in K\}$ is belong to encryption function $D_k : C \to P$,

(vi) for all $e \in K \;\; \exists \;\; d \in K \;\; such that$ we write
$D_d(E_e(P)) = P \quad forall \;\; p \in P.$

2.1. The Natural Transformation: Natural Transformation is linear transformation. That is, if

If $'a'$ and $'b'$ are any constants and also $f(t)$ and $g(t)$ are functions, then

$$N\{af(t)+bg(t)\}=\int_0^\infty \{af(t)+bg(t)\}e^{-st}dt.$$

And also

$$N\{af(t)+bg(t)\}=aN\{f(t)\}+bN\{g(t)\}. \tag{1}$$

2.3 Some Standard Results:

We need the following standard results:

$$N(sinht)=\frac{au}{s^2-a^2u^2}. \tag{2}$$

$$N(t^\alpha)=\frac{\alpha!u^\alpha}{s^{\alpha+1}},\quad \alpha\in N. \tag{3}$$

$$N\left\{t^n\right\}=\frac{u^n n!}{s^{n+1}},\quad n\in N. \tag{4}$$

$$I_{(n,1)}=Q_{(n,1)}-26K_{(n,1)}\quad \textbf{for}\quad n=0,1,2,3..., \tag{5}$$

here

$$Q_{(n,1)}=2^{2n+1}(2n+2)(2n+3)I_{(n,0)}.$$

$$I_{(n,1)}=Q_{(n,1)}-63K_{(n,1)}\quad \textbf{for}\quad n=0,1,2,3..., \tag{6}$$

here

$$Q_{(n,1)}=2^{2n+1}(2n+2)(2n+3)I_{(n,0)}.$$

3 Results and Discussion

Encryption Technique

Theorem: The plain text in the form of $MATHEMATICS$ then the cipher text is $IAGKYAAYIES$ if $f(t)=t^2sinh(3t)$.

Proof:

Assign the number of alphabets from standard table of cryptography

12 0 19 7 4 12 0 19 8 2 18

Suppose that

$I_{(0,0)}=12$ $I_{(1,0)}=0$ $I_{(2,0)}=19$ $I_{(3,0)}=7$ $I_{(4,0)}=4$
$I_{(5,0)}=12$ $I_{(6,0)}=0$ $I_{(7,0)}=19$ $I_{(8,0)}=8$ $I_{(9,0)}=2$
$I_{(10,0)}=18$ $I_{(n,0)}=0$ for $n\geq 11$.

We have a result of Hyperbolic sin Function as

$$sinh(3t)=\frac{3t}{1!}+\frac{3^3t^3}{3!}+\frac{3^5t^5}{5!}\frac{3^7t^7}{7!}+...$$

Multiply by t^2 on both sides

$$t^2 sinh(3t) = \frac{3t^3}{1!} + \frac{3^3t^5}{3!} + \frac{3^5t^7}{5!}\frac{3^7t^9}{7!} + ... + \frac{3^{2\alpha+1}.t^{2\alpha+3}}{(2\alpha+1)!} + ...$$

Write numbers of alphabets in the form of coefficient of this series as

$$f(t) = \sum_{n=0}^{\infty} I_(n,0)\frac{3^{2\alpha+1}t^{2\alpha+3}}{(2\alpha+1)!}.$$

It can also be composed as

$$f(t) = I_{(0,0)}\frac{3t^3}{1!} + I_{(1,0)}\frac{3^3t^5}{3!} + I_{(2,0)}\frac{3^5t^7}{5!} + I_{(3,0)}\frac{3^7t^9}{7!} + I_{(4,0)}\frac{3^9t^{11}}{9!} + I_{(5,0)}\frac{3^{11}t^{13}}{11!} + I_{(6,0)}\frac{3^{13}t^{15}}{13!} + I_{(7,0)}\frac{3^{15}t^{17}}{15!} + I_{(8,0)}\frac{3^{17}t^{19}}{17!} + I_{(9,0)}\frac{3^{19}t^{21}}{19!} + I_{(10,0)}\frac{3^{21}t^{23}}{21!}.$$

It becomes as

$$f(t) = (12)\left(\frac{3t^3}{1!}\right) + (0)\left(\frac{3^3t^5}{3!}\right) + (19)\left(\frac{3^5t^7}{5!}\right) + (7)\left(\frac{3^7t^9}{7!}\right) + (4)\left(\frac{3^9t^{11}}{9!}\right) + (12)\left(\frac{3^{11}t^{13}}{11!}\right) + (0)\left(\frac{3^{13}t^{15}}{13!}\right) + (19)\left(\frac{3^{15}t^{17}}{15!}\right) + (8)\left(\frac{3^{17}t^{19}}{17!}\right) + (2)\left(\frac{3^{19}t^{21}}{19!}\right) + (18)\left(\frac{3^{21}t^{23}}{21!}\right).$$

Apply Natural Transformation on both sides

$$N\{f(t)\} = N\{\sum_{n=0}^{10}\frac{t^{n+1}}{n!}I_{(n,0)}\},$$

$$N\{f(t)\} = N\{(12)\left(\frac{3t^3}{1!}\right) + (0)\left(\frac{3^3t^5}{3!}\right) + (19)\left(\frac{3^5t^7}{5!}\right) + (7)\left(\frac{3^7t^9}{7!}\right) + (4)\left(\frac{3^9t^{11}}{9!}\right) + (12)\left(\frac{3^{11}t^{13}}{11!}\right) + (0)\left(\frac{3^{13}t^{15}}{13!}\right) + (19)\left(\frac{3^{15}t^{17}}{15!}\right) + (8)\left(\frac{3^{17}t^{19}}{17!}\right) + (2)\left(\frac{3^{19}t^{21}}{19!}\right) + (18)\left(\frac{3^{21}t^{23}}{21!}\right)\}.$$

By using (1) and (4), it becomes as

$$N\{f(t)\} = 36.\frac{u^3\ 3!}{s^4} + \frac{0}{3!}.\frac{u^5\ 5!}{s^6} + \frac{4617}{5!}.\frac{u^7\ 7!}{s^8} + \frac{15309}{7!}.\frac{u^9\ 9!}{s^{10}} + \frac{78732}{9!}.\frac{u^{11}\ 11!}{s^{12}} + \frac{2125764}{11!}.\frac{u^{13}\ 13!}{s^{14}} + \frac{0}{13!}.\frac{u^{15}\ 15!}{s^{16}} + \frac{272629233}{15!}.\frac{u^{17}\ 17!}{s^{18}} + \frac{1033121304}{17!}.\frac{u^{19}\ 19!}{s^{20}} + \frac{23244522934}{19!}.\frac{u^{21}\ 21!}{s^{22}} + \frac{188286357700}{21!}.\frac{u^{23}\ 23!}{s^{24}}.$$

It can also be written as

$$N\{f(t)\} = \frac{216u^3}{s^4} + \frac{0u^5}{s^6} + \frac{193914u^7}{s^8} + \frac{1174248u^9}{s^{10}} + \frac{8660520u^{11}}{s^{12}} + \frac{331619184u^{13}}{s^{14}} + \frac{0u^{15}}{s^{16}} + \frac{74155151380u^{17}}{s^{18}} + \frac{353327486000u^{19}}{s^{20}} + \frac{9762699631000u^{21}}{s^{22}} + \frac{95272897000000u^{23}}{s^{24}}$$

Here first step security process start if we put $u = 1$, it become as

$$N\{f(t)\} = \frac{216}{s^4} + \frac{0}{s^6} + \frac{193914}{s^8} + \frac{1174248}{s^{10}} + \frac{8660520}{s^{12}} + \frac{331619184}{s^{14}} + \frac{0}{s^{16}} + \frac{74155151380}{s^{18}} + \frac{353327486000}{s^{20}} + \frac{9762699631000}{s^{22}} + \frac{95272897000000}{s^{24}}$$

Now adjusting these values

216 0 193914 1174248 8660520 331619184 0

74155151380 353327486000 9762699631000 95272897000000
to mod 26, it becomes as
8 0 6 10 24 0 0 24 8 4 18
From standard table the cipher text is $IAGKYAAYIES$

3.1 Process of Generating the Key Algorithm

By using (5), the key algorithm is
8 0 7458 45163 333096 12754584 0
28521211970 13589518690 375488447300 3664342192000

3.2 Process of Decryption

Theorem: If the cipher text in the form of $IAGKYAAYIES$
then by inverse of Natural Transformation the plain text is $MATHEMATICS$
Proof:
By using (5), researchers have originated
216 0 193914 1174248 8660520 331619184 0
74155151380 353327486000 9762699631000 95272897000000
It can also be written as

$$\frac{a}{s^2-a^2u^2} = \frac{216}{s^4} + \frac{0}{s^6} + \frac{193914}{s^8} + \frac{1174248}{s^{10}} + \frac{8660520}{s^{12}} + \frac{331619184}{s^{14}} + \frac{0}{s^{16}} + \frac{74155151380}{s^{18}} + \frac{353327486000}{s^{20}} + \frac{9762699631000}{s^{22}} + \frac{95272897000000}{s^{24}}.$$

It can be composed as

$$\frac{au}{s^2-a^2u^2} = 36.\frac{u^3\ 3!}{s^4} + \frac{0}{3!}.\frac{u^5\ 5!}{s^6} + \frac{4617}{5!}.\frac{u^7\ 7!}{s^8} + \frac{15309}{7!}.\frac{u^9\ 9!}{s^{10}} + \frac{78732}{9!}.\frac{u^{11}\ 11!}{s^{12}} + \frac{2125764}{11!}.\frac{u^{13}\ 13!}{s^{14}} + \frac{0}{13!}.\frac{u^{15}\ 15!}{s^{16}} + \frac{272629233}{15!}.\frac{u^{17}\ 17!}{s^{18}} + \frac{1033121304}{17!}.\frac{u^{19}\ 19!}{s^{20}} + \frac{23244522934}{19!}.\frac{u^{21}\ 21!}{s^{22}} + \frac{188286357700}{21!}.\frac{u^{23}\ 23!}{s^{24}}.$$

Apply inverse Natural Transformation

$$N^{-1}\{\frac{au}{s^2-a^2u^2}\} = N^{-1}\{36.\frac{u^3\ 3!}{s^4} + \frac{0}{3!}.\frac{u^5\ 5!}{s^6} + \frac{4617}{5!}.\frac{u^7\ 7!}{s^8} + \frac{15309}{7!}.\frac{u^9\ 9!}{s^{10}} + \frac{78732}{9!}.\frac{u^{11}\ 11!}{s^{12}} + \frac{2125764}{11!}.\frac{u^{13}\ 13!}{s^{14}} + \frac{0}{13!}.\frac{u^{15}\ 15!}{s^{16}} + \frac{272629233}{15!}.\frac{u^{17}\ 17!}{s^{18}} + \frac{1033121304}{17!}.\frac{u^{19}\ 19!}{s^{20}} + \frac{23244522934}{19!}.\frac{u^{21}\ 21!}{s^{22}} + \frac{188286357700}{21!}.\frac{u^{23}\ 23!}{s^{24}}\}.$$

By using (2) and (4), it becomes as

$$t^2 sinh(3t) = (12)\left(\frac{3t^3}{1!}\right) + (0)\left(\frac{3^3t^5}{3!}\right) + (19)\left(\frac{3^5t^7}{5!}\right) + (7)\left(\frac{3^7t^9}{7!}\right) + (4)\left(\frac{3^9t^{11}}{9!}\right) + (12)\left(\frac{3^{11}t^{13}}{11!}\right) + (0)\left(\frac{3^{13}t^{15}}{13!}\right) + (19)\left(\frac{3^{15}t^{17}}{15!}\right) + (8)\left(\frac{3^{17}t^{19}}{17!}\right) + (2)\left(\frac{3^{19}t^{21}}{19!}\right) + (18)\left(\frac{3^{21}t^{23}}{21!}\right).$$

From the above expansion, we have the coefficients as
12 0 19 7 4 12 0 19 8 2

18
From standard table of alphabets $MATHEMATICS$

4 Extended Mathematical Cryptology of sin Hyperbolic Function

4.1 Encryption Process

Theorem: The plain text in the form of $I\ Love\ my\ Mentor$ by using Natural Transformation if $f(t) = t^3 sinh(2t)$, then cipher text is in the form of $hAAAA5RUXbehlost$
Proof:
Alert the number of alphabets from extended standard table
8 63 11 40 47 30 63 38 50 63 12 30 39 45 40 43
Suppose that
$I_{(0,0)} = 8$ $I_{(1,0)} = 63$ $I_{(2,0)} = 11$ $I_{(3,0)} = 40$ $I_{(4,0)} = 47$
$I_{(5,0)} = 30$ $I_{(6,0)} = 63$ $I_{(7,0)} = 38$ $I_{(8,0)} = 50$ $I_{(9,0)} = 63$
$I_{(10,0)} = 12$ $I_{(11,0)} = 30$ $I_{(12,0)} = 39$ $I_{(13,0)} = 45 I_{(14,0)} = 40$
$I_{(15,0)} = 43$ $I_{(n,0)} = 0$ for $n \geq 16$.
Now we have a function $f(t) = t^3 sinh(2t)$.
Write numbers of alphabets in the form of coefficient of this series, it become

$$f(t) = I_{(0,0)}\frac{2t^4}{1!} + I_{(1,0)}\frac{2^3t^6}{3!} + I_{(2,0)}\frac{2^5t^8}{5!} + I_{(3,0)}\frac{2^7t^{10}}{7!} + I_{(4,0)}\frac{2^9t^{12}}{9!} + I_{(5,0)}\frac{2^{11}t^{14}}{11!} + I_{(6,0)}\frac{2^{13}t^{16}}{13!} + I_{(7,0)}\frac{2^{15}t^{18}}{15!} + I_{(8,0)}\frac{2^{17}t^{20}}{17!} + I_{(9,0)}\frac{2^{19}t^{22}}{19!} + I_{(10,0)}\frac{2^{21}t^{24}}{21!} + I_{(11,0)}\frac{2^{23}t^{25}}{23!} + I_{(12,0)}\frac{2^{25}t^{27}}{25!} + I_{(13,0)}\frac{2^{27}t^{29}}{27!} + I_{(14,0)}\frac{2^{29}t^{31}}{29!} + I_{(15,0)}\frac{2^{31}t^{33}}{31!}.$$

It also becomes as

$$f(t) = (8)\left(\frac{2t^4}{1!}\right) + (63)\left(\frac{2^3t^6}{3!}\right) + (11)\left(\frac{2^5t^8}{5!}\right) + (40)\left(\frac{2^7t^{10}}{7!}\right) + (47)\left(\frac{2^9t^{12}}{9!}\right) + (30)\left(\frac{2^{11}t^{14}}{11!}\right) + (63)\left(\frac{2^{13}t^{16}}{13!}\right) + (38)\left(\frac{2^{15}t^{18}}{15!}\right) + (50)\left(\frac{2^{17}t^{20}}{17!}\right) + (63)\left(\frac{2^{19}t^{22}}{19!}\right) + (12)\left(\frac{2^{21}t^{24}}{21!}\right) + (30)\left(\frac{2^{23}t^{26}}{23!}\right) + (39)\left(\frac{2^{25}t^{28}}{25!}\right) + (45)\left(\frac{2^{27}t^{30}}{27!}\right) + (40)\left(\frac{2^{29}t^{32}}{29!}\right) + (43)\left(\frac{2^{31}t^{34}}{31!}\right).$$

Apply Natural Transformation on both sides

$$N\{f(t)\} = N\{(8)\left(\frac{2t^4}{1!}\right) + (63)\left(\frac{2^3t^6}{3!}\right) + (11)\left(\frac{2^5t^8}{5!}\right) + (40)\left(\frac{2^7t^{10}}{7!}\right) + (47)\left(\frac{2^9t^{12}}{9!}\right) + (30)\left(\frac{2^{11}t^{14}}{11!}\right) + (63)\left(\frac{2^{13}t^{16}}{13!}\right) + (38)\left(\frac{2^{15}t^{18}}{15!}\right) + (50)\left(\frac{2^{17}t^{20}}{17!}\right) + (63)\left(\frac{2^{19}t^{22}}{19!}\right) + (12)\left(\frac{2^{21}t^{24}}{21!}\right) + (30)\left(\frac{2^{23}t^{26}}{23!}\right) + (39)\left(\frac{2^{25}t^{28}}{25!}\right) + (45)\left(\frac{2^{27}t^{30}}{27!}\right) + (40)\left(\frac{2^{29}t^{32}}{29!}\right) + (43)\left(\frac{2^{31}t^{34}}{31!}\right)\}.$$

By using (1), (6), (4) and for u=1, we concluded that

$$N\{f(t)\} = \frac{384}{s^5} + \frac{60480}{s^7} + \frac{118272}{s^9} + \frac{3.6864e6}{s^{11}} + \frac{3.176448e7}{s^{13}} + \frac{1.3418496e8}{s^{15}} + \frac{1.73408256e9}{s^{17}}$$
$$+\frac{6.096420864e9}{s^{19}} + \frac{4.4826624e10}{s^{21}} + \frac{3.051985306e11}{s^{23}} + \frac{3.056137667e11}{s^{25}} + \frac{3.925868544e12}{s^{27}}$$
$$+\frac{2.57222907e13}{s^{29}} + \frac{1.471294734e14}{s^{31}} + \frac{6.390911336e14}{s^{33}} + \frac{3.315439874e15}{s^{35}}$$

Now adjusting these values
384 60480 118272 $3.6864e6$ $3.176448e7$ $1.3418496e8$
$1.73408256e9$ $6.096420864e9$ $4.4826624e10$ $3.051985306e11$
$3.056137667e11$ $3.925868544e12$ $2.57222907e13$ $1.471294734e14$
$6.390911336e14$ $3.315439874e15$
to mod 63, we selected the numbers as
6 0 21 18 6 0 0 9 18 40 53 18
27 42 32 47
the cipher text as $GAVSGAAJSo1Sbqgv$

4.2 Process of Generating the Key Algorithm

Researchers used (6) for constructing the key algorithm that is
6 960 1877 58514 504198 2129920 27525120 96768585
711533714 4844421120 4851012169 6.231537371e10 4.082703286e11
2.335388467e12 1.014430371e13 5.262602975e13

4.3 Process of Decryption

Researchers have cipher text that is $GAVSGAAJSo1Sbqgv$
By using table, (6) and with the help of inverse Natural Transformation the plain text is $I\ Love\ my\ Mentor$

5 Conclusion

A new Trigonometric hyperbolic cryptology scheme is introduced by using Natural Transformation and the key of mod 26 and mod 63. Natural Transformation facilitated us to safe way of communication through the meek channal that it may be on mannual or internet.

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