Int. Journal of Math. Analysis Vol. 6, 2012, no.47, 2325-2333

sb^*-closed sets in Topological spaces

A.Poongothai

Department of Science and Humanities
Karpagam College of Engineering
Coimbatore -32,India
poongo_pup@yahoo.co.in

R.Parimelazhagan

Department of Science and Humanities
Karpagam College of Engineering
Coimbatore -32, India
pari_kce@yahoo.com

Abstract

In this paper, we introduce the concept of a strongly b^* - closed set (breifly sb^*-closed set) in a topological space (X, τ) and investigate the relation between the associated topologies.

Mathematics Subject Classification: 54A05.
Keywords and Phrases: g-closed sets, α- closed sets, αg- closed sets, w- closed sets, b - closed sets, g^* - closed sets, sb^*- closed sets.

1. Introduction

Levine[4] introduced and investigated the concept of generalized closed sets in topological spaces. After Levine's work many authors defined and investigated various forms of stronger and weaker form of closed sets. Noiri [9]introduced a new concept called strongly θ- closed sets which is stronger than closed sets.Regular open sets and strongly regular open sets have been introduced and investigated by Stone[10] and Tong[12] respectively. Complements of regular open sets and strongly regular open sets are called regular closed sets and strongly regular closed sets.

In this paper, we introduce the notion of strongly b^*-closed sets and obtain some characterizations of these classes and several properties. Also we investigate the relationships with other classes of closed sets in topological spaces.

2. Preliminaries

Let (X, τ) be a topological space and A be a subset. The closure of A and interior of A are denoted by $cl(A)$ and $int(A)$ respectively. In this section, we recall some definitions of open sets in topological spaces.

Definition 2.1[5]: A subset A of a topological space (X, τ) is called a semiopen set if $A \subseteq cl(int(A))$ and semiclosed if $int(cl(A)) \subseteq A$.

Definition 2.2[8]: A subset A of a topological space (X, τ) is called α-open set if $A \subseteq int(cl(int(A)))$ and α-closed if $cl(int(cl(A))) \subseteq A$.

Definition 2.3[4]: A subset A of a topological space (X, τ) is called a g-closed set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

Definition 2.4[2]:A subset A of a topological space (X, τ) is called a semipre open set (β- open set) if $A \subseteq cl(int(cl(A)))$ and semi preclosed or β-closed set if $int(cl(int(A))) \subseteq A$.

Definition 2.5[3]: A subset A of a topological space (X, τ) is called semi generalized closed set (briefly sg-closed)if $scl(A) \subseteq U$ whenever $A \subseteq U$, U is semi open in X.

Definition 2.6 [6]: A subset A of a topological space (X, τ) is called α-generalized closed set (briefly αg-closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

Definition 2.7 [1]: A subset A of a topological space (X, τ) is called b-open set if $A \subseteq (cl(int(A)) \cup int(cl(A)))$.

Definition 2.8[7]: A subset A of a topological space (X, τ) is called weakly generalized closed set (briefly wg-closed) if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

Definition 2.9[11]: A subset A of a topological space (X, τ) is called w-closed set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X.

Definition 2.10[13]: A subset A of a topological space (X, τ) is called g^* closed set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g- open in X.

Definition 2.11: A subset A of a topological space (X, τ) is called b^{**} open set if $A \subseteq (int(cl(int(A)))) \cup (cl(int(cl(A))))$.

3. strongly b^* - closed sets

In this section, we introduce and study the concept of strongly b^* -closed set in topological spaces.

Definition 3.1: A subset A of a topological space (X, τ) is called a strongly b^*- closed set (briefly sb^*- closed) if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is b open in X.

Theorem 3.2: Every sb^*- closed set is b closed.

Proof: Assume that A is a sb^*- closed set in X and let U be an open set such that $A \subseteq U$. Since every open set is b open set and A is sb^*- closed set, $cl(int(A)) \subseteq (cl(int(A))) \cup (int(cl(A))) \subseteq U$. Therefore A is b closed set in X.

Remark 3.3: The converse of the above theorem need not be true as seen from the following example.

Example 3.4: Consider $X = \{a, b, c\}$ with $\tau = \{X, \varphi, \{a\}, \{c\}, \{a, c\}\}$. In this topological space, the subset $A = \{c\}$ is b closed but not sb^* - closed set.

Theorem 3.5: A set A is sb^*-closed set iff $cl(int(A)) - A$ contains no non-empty b closed sets.

Proof: Necessity: Suppose that F is a non empty b closed subset of $cl(int(A))$ such that $F \subseteq cl(int(A)) - A$. Then $F \subseteq cl(int(A)) \cap A^c$. Therefore $F \subseteq cl(int(A))$ and $F \subseteq A^c$. Since F^c is b open set and A is sb^* - closed set, $cl(int(A)) \subseteq F^c$. Thus $F \subseteq (cl(int(A)))^c$. Therefore $F \subseteq [cl(int(A))] \cap [cl(int(A))]^c = \varphi$. Therefore $F = \varphi$. This implies that $cl(int(A)) - A$ contains no non empty b closed sets.

Sufficiency: Consider $A \subseteq U$ is b open set. Suppose that $cl(int(A))$ is not contained in U, then $[cl(int(A))]^c$ is a non empty b closed set and contained in $cl(int(A)) - A$, which is a contradiction. Therefore $cl(int(A)) \subseteq U$ and hence A is sb^*- closed set.

Theorem 3.6: Let $B \subseteq Y \subseteq X$, if B is sb^*- closed set relative to Y and Y is open and sb^* - closed set in (X, τ) then B is sb^* - closed set in (X, τ).

Proof: Let U be a b open set in (X, τ) such that $B \subseteq U$. Given that $B \subseteq Y \subseteq X$. Therefore $B \subseteq Y$ and $B \subseteq U$. This implies that $B \subseteq$

$Y \cap U$. Since B is sb^*-closed set relative to Y, then $cl(int(B)) \subseteq U$. $Y \cap cl(int(B)) \subseteq Y \cap U$ implies that $Y \cap cl(int(B)) \subseteq U$. Thus $[Y \cap cl(int(B))] \cup [cl(int(B))]^c \subseteq U \cup (cl(int(B))^c$. This implies that $(Y \cup (cl(int(B)))^c) \cap (cl(int(B)) \cup (cl(int(B)))^c \subseteq U \cup (cl(int(B)))^c$. Therefore $(Y \cup (cl(int(B)))^c) \subseteq U \cup (cl(int(B)))^c$. Since Y is sb^*-closed set in X, $cl(int(Y)) \subseteq U \cup (cl(int(B)))^c$. Also $B \subseteq Y$ implies that $cl(int(B)) \subseteq cl(int(Y))$. Thus $cl(int(B)) \subseteq cl(int(Y)) \subseteq U \cup [cl(int(B))]^c$. Therefore $cl(int(B)) \subseteq U$. Since $cl(int(B))$ is not contained in $(cl(int(B)))^c$,B is sb^*-closed set relative to X.

Theorem 3.7: Let $A \subseteq Y \subseteq X$ and suppose that A is sb^*-closed set in X then A is sb^* - closed set relative to Y.

Proof: Assume that $A \subseteq Y \subseteq X$ and A is sb^*-closed set in X. To show that A is sb^* -closed set relative to Y, let $A \subseteq Y \cap U$ where U is b open in X. Since A is sb^*-closed set in X, $A \subseteq U$ implies $cl(int(A)) \subseteq U$.That is $Y \cap cl(int(A)) \subseteq Y \cap U$.Therefore $cl(int(A)) \subseteq U$. Thus A is sb^*-closed set relative to Y.

Theorem 3.8: If A is a sb^*-closed set and $A \subseteq B \subseteq cl(int(A))$ then B is a sb^*-closed set.

Proof: Let A be a sb^*-closed set, such that $A \subseteq B \subseteq cl(int(A))$. Let U be a b open set of X such that $B \subseteq U$. Since A is sb^*-closed set, we have $cl(int(A)) \subseteq U$ whenever $A \subseteq U$. Since $A \subseteq B$ and $B \subseteq cl(int(A))$ then $cl(int(B)) \subseteq cl(int(cl(int(A)))) \subseteq cl(int(A)) \subseteq U$. Therefore $cl(int(B)) \subseteq U$. Thus B is sb^*-closed set in X.

Theorem 3.9: The intersection of a sb^*-closed set and a closed set is a sb^*-closed set.

Proof: Let A be a sb^*-closed set and F be a closed set. Since A is sb^*-closed set, $cl(int(A)) \subseteq U$ whenever $A \subseteq U$, where U is a b open set. To show that $A \cap F$ is sb^*-closed set, it is enough to show that $cl(int(A \cap F)) \subseteq U$ whenever $A \cap F \subseteq U$, where U is b open set.Let $G = X - F$ then $A \subseteq U \cup G$. Since G is open set, $U \cup G$ is b open set and A is sb^* closed set, $cl(int(A)) \subseteq U \cup G$. Now $cl(int(A \cap F)) \subseteq cl(int(A)) \cap cl(int(F)) \subseteq cl(int(A)) \cap F \subseteq (U \cup G) \cap F \subseteq (U \cap F) \cup (G \cap F) \subseteq (U \cap F) \cup \varphi \subseteq U$.
This implies that $A \cap F$ is sb^* -closed set.

Theorem 3.10: If A and B are two sb^* -closed sets defined for a non empty set X, then their intersection $A \cap B$ is sb^*-closed set in X.

Proof: Let A and B are sb^*-closed sets and also consider U be a b open set in X such that $A \cap B \subseteq U$. Now $cl(int(A \cap B)) \subseteq cl(int(A)) \cap cl(int(B)) \subseteq U$. Hence $A \cap B$ is sb^* - closed set.

Remark 3.11: The Union of two sb^*-closed sets need not be sb^*-closed set.

Example 3.12: Let $X = \{a, b, c\}$ with $A = \{X, \varphi, \{a\}, \{c\}, \{a, c\}\}$ and $B = \{X, \varphi, \{a\}, \{b\}, \{a, b\}\}$ then $A \cup B$ is not a sb^*-closed set since $\{b, c\}$ does not belong to $A \cup B$.

Theorem 3.13: Every closed set is strongly b^* - closed set.

Remark 3.14: The converse of the above theorem need not be true as seen from the following example.

Example 3.15: Consider $X = \{a, b, c\}$ with $\tau = \{X, \varphi, \{b\}\}$. In this topological space,the subset $A = \{a\}$ is sb^*- closed but not a closed set.

Theorem 3.16: Every g^* closed set is sb^* -closed set.

Remark 3.17: The converse of the above theorem need not be true as seen from the following example.

Example 3.18: Consider $X = \{a, b, c\}$ with $\tau = \{X, \varphi, \{a, c\}\}$.In this topological space, the subset $A = \{c\}$ is sb^*- closed but not g^* closed set.

Theorem 3.19: Every sb^*- closed set is b^{**} closed set.

Remark 3.20: The converse of the above theorem need not be true as seen from the following example.

Example 3.21: Consider $X = \{a, b, c\}$ with $\tau = \{X, \varphi, \{a\}, \{c\}, \{a, c\}\}$. In this topological space, the subset $A = \{a\}$ is b^{**} closed but not sb^* -closed set.

Theorem 3.22: Every α-closed set is sb^*-closed set and not conversely.

Proof: Suppose A be a α closed set in X. Let U be an open set in X such that $A \subseteq U$. Since A is α -closed set, $\alpha cl(A) \subseteq U$. Now $\alpha cl(A) \subseteq cl(int(A)) \subseteq U$. Therefore A is sb^*-closed set in X.

Example 3.23: Consider $X = \{a, b, c\}$ with $\tau = \{X, \varphi, \{b\}, \{a, c\}\}$.In this topological space , the subset $A = \{a, b\}$ is sb^*- closed set but not α -closed set.

Theorem 3.24: Every sb^*-closed set is wg- closed set.

Remark 3.25: The converse of the above theorem need not be true as seen from the following example.

Example 3.26: Consider $X = \{a, b, c\}$ with $\tau = \{X, \varphi, \{a\}, \{a, b\}\}$.In this topological space, the subset $A = \{a, c\}$ is wg- closed but not sb^* - closed set.

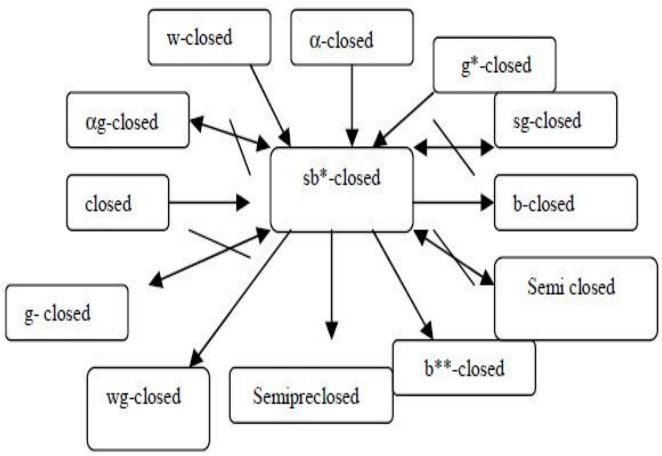
Theorem 3.27: If a subset A of a topological space X is w closed then it is sb^* - closed set.

Proof : Let A be a w-closed set in X. This implies that $int(A) \subseteq U$, U is semiopen. Since every semi open set is b open set and $cl(int(A)) \subseteq cl(A) \subseteq U$, A is sb^*-closed set.

Remark 3.28: The converse of the above theorem need not be true as seen from the following example.

Example 3.29: Consider $X = \{a, b, c\}$ with $\tau = \{X, \varphi, \{a\}\}$.In this topological space, the subset $A = \{a\}$ is sb^*- closed set but not w - closed set.

Remark 3.30: The following are the implications of sb^*-closed sets and none of them is reversible.



4. sb^* - closed set is independent of other closed sets

In this section, we introduce the independency of sb^*-closed sets with some other closed sets.

Remark 4.1: The following example shows that the concept of g -closed and sb^*-closed sets are independent.

Example 4.2: Consider $X = \{a, b, c\}$ with the topology $\tau_1 = \{X, \varphi, \{b\}\}$. In this topological space, the subset $A = \{a, b\}$ is g closed set but not sb^* -closed set. For the topology $\tau_2 = \{X, \varphi, \{a, c\}\}$, the subset $B = \{a\}$ is sb^*-closed set but not g closed set.

Remark 4.3: The following example shows that the concept of αg- closed and sb^*-closed sets are independent.

Example 4.4: Consider $X = \{a, b, c\}$ with the topology $\tau_1 = \{X, \varphi, \{a\}\}$. In this topological space, the subset $A = \{a\}$ is sb^*- closed set but not αg -closed set. For the topology $\tau_2 = \{X, \varphi, \{b\}\}$,the subset $B = \{a, b\}$ is αg-closed set but not sb^*- closed set.

Remark 4.5: The following example shows that the concept of semi closed and sb^*-closed sets are independent.

Example 4.6: Consider $X = \{a, b, c\}$ with the topology $\tau_1 = \{X, \varphi, \{a, c\}\}$.In this topological space, the subset $A = \{b, c\}$ is sb^*- closed set but not semi closed set. For the topology $\tau_2 = \{X, \varphi, \{a\}, \{c\}, \{a, c\}\}$, the subset $B = \{c\}$ is semi closed set but not sb^*- closed set.

Remark 4.7: The following example shows that the concept of sg- closed and sb^*-closed sets are independent.

Example 4.8: Consider $X = \{a, b, c\}$ with the topology $\tau_1 = \{X, \varphi, \{a, c\}\}$.In this topological space, the subset $A = \{a\}$ is sb^*- closed set but not sg - closed set. For the topology $\tau_2 = \{X, \varphi, \{a\}, \{c\}, \{a, c\}\}$, the subset $B = \{c\}$ is sg-closed set but not sb^*- closed set.

References

1. Andrijevic.D, On b-open sets, Mat. Vesink, 48 (1996), 59 - 64.

2. Andrijevic.D, Semi-preopen sets, Mat. Vesink, 38(1) (1986), 24 - 32.

3. Bhattacharya.P and Lahiri.B.K, Semi-generalised closed sets in topology, Indian J. Math., 29(3) (1987), 375 - 382.

4. Levine.N, Generalised closed sets in Topology, Rend. Circ. Mat. Palerno , 19(2) (1970), 89 - 96.

5. Levine.N, Semi-open sets and semi-continuity in Topological spaces, Amer. Math. Monthly, 70 (1963), 36 - 41

6. Maki.H, Devi.R and Balachandran.K, Generalised α-closed sets in topology, Bull. Fukuoka. Univ. Ed, III, 42 (1993), 13 - 21.

7. Nagaveni.N Studies on generalisations of homeomorphisms in topological spaces,Ph.D.Thesis, Bharathiar University,Coimbatore(1999).

8. Njastad.O, On some classes of nearly open sets, Pacific J. Math., 15 (1965), 961 - 970.

9. Noiri. T and Ahmad. B, On semi-weakly continuous functions, Kyungpook Math. J.,25(1985),123-126.

10. Stone.M, Application of the theory of Boolean rings to general topology, Trans.Amer.Maths.Soc.,41(1937),374-481.

11. Sundaram.P and Shiek John.M., On $w$closed sets in Topology,Acta ciencia Indica., 4 (2000), 389 - 392.

12. Tong.J, Weak almost continuous mappings and weak nearly compact spaces, Boll. Un. Mat.Ital.,6(1982),385-391

13. Veerakumar.M.K.R.S., Betweeen closed sets and $g$closed sets., Mem.Fac.Sci.Kochi.Univ.Ser A, Math 21 (2000),1-19.