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b^*-Closed Sets in Topological Spaces

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Abstract

In this paper, the authors introduce and study the concept of a new class of closed sets called b^*-closed sets (briefly b^*-closed set). Also we investigate some of their properties.

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1. Introduction

Levine[12] introduced the concept of generalized closed sets(briefly g-closed) in topological spaces and a class of topological spaces called $T_{1/2}$ spaces. Arya and Nour[4], Bhattacharya and Lahiri[5], Levine[11], Mashhour[15], Njastad[16] and Andrijevic[2,3] introduced and investigated generalized semi-open sets, semi generalized open sets, generalized open sets, semi-open sets, pre-open sets and α - open sets, semi pre-open sets and b-open sets which are some of the weak forms of open sets and the complements of these sets are called the same types of closed sets. Tong[18,19] has introduced A-sets, B-sets and t-sets. A-sets and B-sets are also weak forms of open sets whereas t-sets is a weak form of a closed sets. Ganster and Reilly[10] have introduced locally closed sets, which are weaker than both open and closed sets. Cameron[6] has introduced regular semi-open sets which are weaker than regular open sets. Ahmad Al-Omari[1] introduced generalized b-closed sets and studied the

properties.
In this paper we investigate the behaviour of b^*-closed sets and its various characterisation are studied

2. Preliminaries

Before entering into our work we recall the following definitions which are due to Levine.
Definition 2.1 [15]: A subset A of a topological space (X, τ) is called a pre-open set if $A \subseteq int(cl(A))$ and pre-closed set if $cl(int(A)) \subseteq A$.

Definition 2.2 [11]: A subset A of a topological space (X, τ) is called a semi-open set if $A \subseteq cl(int(A))$ and semi closed set if $int(cl(A)) \subseteq A$.

Definition 2.3 [16]: A subset A of a topological space (X, τ) is called an α-open set if $A \subseteq int(cl(int(A)))$ and an α -closed set if $cl(int(cl(A))) \subseteq A$.

Definition 2.4 [2]: A subset A of a topological space (X, τ) is called a semi-preopen set (β-open set) if $A \subseteq cl(int(cl(A)))$ and semi-preclosed set if $int(cl(int(A))) \subseteq A$.

Definition 2.5 [1]: A subset A of a topological space (X, τ) is called a generalized b-closed set (simply gb-closed) if $bcl(A) \subseteq U$,whenever $A \subseteq U$ and U is open in X.

Definition 2.6 [12]: A subset A of a topological space (X, τ) is called a generalized closed set(briefly g-closed) if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in X.

Definition 2.7 [9]: A subset of A topological space (X, τ) is called a θgeneralized closed set (briefly θg-closed) if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ).

Definition 2.8 [13]: A subset A of a topological space (X, τ) is called a generalized α -closed (briefly $g\alpha$ -closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in (X, τ).

Definition 2.9 [5]: A subset A of a topological space (X, τ) is called a semi-generalized closed set, (briefly sg-closed) if $scl(A) \subseteq U$, whenever $A \subseteq U$, U is semi-open in (X, τ).

Definition 2.10 [7]: A subset A of a topological space (X, τ) is called a generalized semi-preclosed set(briefly gsp-closed) if $spcl(A) \subseteq U$, whenever $A \subseteq U$

and U is open in X.

Definition 2.11 [20]: A subset A of a topological space (X, τ) is called a generalized* closed set(briefly g^*-closed) if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is g-open in X.

Definition 2.12 [8]: A subset of A topological space (X, τ) is called a δgeneralized closed set (briefly δg-closed) if $cl_\delta(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ).

Definition 2.13 [17]: A subset A of a topological space (X, τ) is called a Strongly generalized semi-closed set(briefly g^*s-closed) if $scl(A) \subseteq U$, whenever $A \subseteq U$ and U is gs-open in X.

Definition 2.14 [14]: A subset A of a topological space (X, τ) is called a generalized pre-closed set(briefly gp-closed) if $pcl(A) \subseteq U$, whenever $A \subseteq U$ and U is g-open in X.

Definition 2.15 [3]: A subset A of a topological space (X, τ) is called a b-open set if $A \subseteq cl(int(A)) \cup int(cl(A))$ and b-closed set if $cl(int(A)) \cup int(cl(A)) \subseteq A$.

3.Some basic properties of b^*-closed sets

In this section we introduce the concept of b^*-closed sets in topological space.

Definition 3.1: A subset A of a topological space (X, τ) is called a b^* closed set if $int(cl(A)) \subseteq U$, whenever $A \subseteq U$ and U is b-open.

Theorem 3.2: If a subset A of a topological space X is b-closed then it is b^*-closed set but not conversely.

Proof: Suppose A is b-closed, let G be an open set containing A in X, then $cl(A) \subset G$. Now $int(cl(A)) \subset cl(A) \subset G$. Thus A is b^* closed set.

Remark 3.3: The converse of the theorem[3.2] need not be true as seen from the following example.

Example 3.4: Let X={a,b,c} with the topology $\tau = \{X, \phi, \{a\}, \{a, b\}\}$. In this topological space the subset $\{b\}$ is b^* closed but not b closed.

Theorem 3.5: If a subset A of a topological space is both open and b^* closed then it is closed.

Proof: Suppose a subset A of X is both open and b^* closed. Now $int(cl(A)) \subseteq cl(A) \subseteq A$ (ie) $cl(A) \subseteq A$ .Therefore A is closed in X.

Theorem 3.6: A set A is b^* closed if and only if $int(cl(A)) - A$ contains no non empty closed set.

Proof: Necessity: Suppose that F is a non empty closed subset of $int(cl(A))$ .Now $int(cl(A)) - A \subseteq A$ implies $int(cl(A)) \cap A^c \subseteq F$,since $int(cl(A)) - A = (int(cl(A)) \cap A^c$ .Thus $int(cl(A)) \subseteq F$. Now $A^c \subseteq F$ implies $F^c \subseteq A$. Here F^c is open and A is b^*-closed ,we have $F^c \subseteq int(cl(A))$. Thus $F \subseteq [int(cl(A))]^c$. Hence $int(cl(A)) \cap [int(cl(A))]^c \subseteq F = \phi$ (i.e) $F = \phi$ implies $int(cl(A)) - A$ contains no nonempty closed set.

Sufficiency: Let $G \subseteq A$, G is b-open. Suppose that $int(cl(A))$ is not contained in G,then $int(cl(A)) \cap G^c$ is a non empty closed set of $int(cl(A)) - A$ which is contradiction .Therefore $G \subseteq int(cl(A))$ and hence A is b^* -closed.

Corollary 3.7: Let A be a gb-closed set then A is b^*-closed if and only if $int(cl(A)) - A$ is closed.

Proof: Let A be gb-closed set. If A is b^*-closed, then we have $int(cl(A)) - A = \phi$ which is closed set. Conversely, let $int(cl(A)) - A$ be closed. Then by theorem[3.6] $int(cl(A)) - A$ doesnot contain any non-empty closed subset and since $int(cl(A))$ is closed subset of itself. Then $int(cl(A)) - A = \phi$. This implies that $A = int(cl(A))$ and so A is b^*-closed set.

Theorem 3.8: Suppose that $B \subseteq A \subseteq X$,$B$ is b^*-closed set relative to A and that A is both b-open and b^*-closed subset of X, then B is b^*-closed set relative to X.

Proof: Let $G \subseteq B$ and G be a b-open set in X. But given that $B \subseteq A \subseteq X$, therefore $B \subseteq A$ and $G \subseteq B$. This implies $A \cap G \subseteq B$. Since B is b^*-closed relative to A, $A \cap G \subseteq int(cl(A))$.(i.e) $A \cap G \subseteq A \cap int(cl(A))$ implies $G \subseteq A \cap int(cl(A))$. Thus $G \cup [int(cl(B))]^c \subseteq A \cap int(cl(B)) \cup [int(cl(B))]^c$ implies $G \cup [int(cl(B))]^c \subseteq A \cup [int(cl(B))]^c$. Since A is b^*-closed in X, we have $G \cup [int(cl(B))]^c \subseteq int(cl(A))$. Also $B \subseteq A$ implies $int(cl(A)) \subseteq int(cl(B))$. Thus $G \cup [int(cl(B))]^c \subseteq int(cl(A)) \subseteq int(cl(B))$. Therefore $G \subseteq int(cl(B))$, since $int(cl(B))$ is not contained in $[int(cl(B))]^c$. Thus B is b^*-closed set relative to X.

Theorem 3.9: Let $A \subseteq Y \subseteq X$ and supposed that A is b^*-closed in X then A is b^*-closed relative to Y.

Proof: Given that $A \subseteq Y \subseteq X$ and A is b^*-closed in X.To show that A is b^*-closed relative to Y. Let $Y \cap G \subseteq A$ where G is b-open in X. Since A is b^*-closed in X, $G \subseteq A$ implies $G \subseteq int(cl(A))$ (i.e) $Y \cap G \subseteq Y \cap int(cl(A))$ where $Y \cap int(cl(A))$ is interior of closure of A in Y. Thus A is b^*-closed relative to Y.

Theorem 3.10: If a Subset A of a topological space X is pre-closed then it is b^*-closed but not conversely.

Proof: Suppose A is pre-closed,G be a b-open set containing A. All A is pre-closed $A \subseteq int(cl(A))$. Thus A is b^*-closed in X.

Remark 3.11: The converse of the theorem[3.10] need not be true as shown by the following example.

Example 3.12: Let X={a,b,c} with the topology $\tau = \{X, \phi, \{a\}\}$. In this topological space the subset {a,b} is b^*-closed but not pre-closed.

Theorem 3.13: If a subset A of a topological space X is nowhere dense then it is b^*-closed but not conversely.

Proof: Suppose a subset A is nowhere dense then $int(cl(A)) = \phi$.It is obvious that $A \subseteq cl(A)$ and also $int(A) \subseteq int(cl(A))$.As A is nowhere dense, $int(A) = \phi$ which implies $int(cl(A)) = \phi$. Thus A is b^*-closed in X.

Remark 3.14: The converse of the theorem[3.13] neednot be true from the following example.

Example 3.15: Consider X={a,b,c} with the topology $\tau = \{X, \phi, \{a\}, \{a, c\}\}$. In this topological space the subset $\{a\}$ is b^*-closed but not nowhere dense.

4. b^*-open sets

Definition 4.1: A subset A of a topological space X is called b^*-open set if it's complement A^c is b^*-closed.

Theorem 4.2: If a subset A of a topological space X is g-open then it is b^*-open butnot conversely,

Proof: Let A is a g-open set in X. Then A^c is b^*-closed. Therefore A is b^*-open in X.

Remark 4.3: The converse of the theorem[4.2] need not be true as seen from the following example.

Example 4.4: Let X={a,b,c} with the topology $\tau = \{X, \phi, \{a\}, \{a, b\}\}$. In this topological space the subset {a,c}is b^*-open but not g-open.

Theorem 4.5: A subset A of a topological space X is b^*-open if and only if $F \subseteq cl(int(A))$ whenever F is closed and $F \subseteq A$.

Proof: Assume that A is b^*-open. Then A^c is b^*-closed. Let F be a closed set in X contained in A. Then F^c is an open set in X containing A^c. Since A^c is b^*-closed, $int(cl(A^c)) \subseteq F^c$ taking complement on bothsides, $F \subseteq int(cl(A^c))$. Conversely assume that F^c is contained in $cl(int(A))$ whenever F is contained in A and F is closed in X. Let G be an open set containing A^c,. Then $G^c \subseteq cl(int(A^c))$ taking complement on bothsides $int(cl(A^c)) \subseteq G$.Hence A^c is b^*-closed. Therefore A is b^*-open.

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