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Common Fixed Points for Four Mappings in G-Metric Spaces

Anchalee Kaewcharoen

Department of Mathematics, Faculty of Science
Naresuan University, Phitsanulok, 65000, Thailand,
Centre of Excellence in Mathematics, CHE, Si Ayutthaya Rd.
Bangkok, 10400, Thailand
anchaleeka@nu.ac.th

Abstract

We prove the unique common fixed point theorems for pairs of weakly compatible mappings satisfying the certain contractive conditions in the setting of generalized metric spaces without exploiting the notion of continuity.

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1 Introduction and preliminaries

The common fixed point theory is very important and useful in Mathematics. It can be applied in various areas, for instant, variational inequalities, optimization, and approximation theory. The common fixed point theorems for mappings satisfying certain contractive conditions in metric spaces have been continually studied for decade (see [5, 7, 8, 9, 10, 15] and references contained therein). In 1976, Jungck [6] proved the existence of common fixed point theorem for commuting mappings in metric spaces where the result requires the continuity of one of two such mappings. In 1986, Jungck [7] introduced the concept of compatible mappings and proved that weakly commuting mappings are compatible mappings. In 1994, Pant [15] introduced R-weakly commuting mappings and assure the existence of common fixed points where the result requires the continuity of at least one of the mappings. In 2006, Mustafa and Sims [13] introduced a generalization of metric spaces. Namely, G-metric spaces as the following:

Definition 1.1 Let X be a nonempty set and $G : X \times X \times X \to \mathbb{R}^+$ be a function satisfying :

$(G1)$ $G(x, y, z) = 0$ if $x = y = z$,

$(G2)$ $0 < G(x, x, y)$, for all $x, y \in X$ with $x \neq y$,

$(G3)$ $G(x, x, y) \leq G(x, y, z)$, for all $x, y, z \in X$ with $z \neq y$,

$(G4)$ $G(x, y, z) = G(x, z, y) = G(y, z, x) = \cdots$ (symmetry in all three variables), and

$(G5)$ $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$, for all $x, y, z, a \in X$ (rectangle inequality).

Then the function G is called a generalized metric or more specifically a G-metric on X, and the pair (X, G) is called a G-metric space.

Since then the fixed point theory in G-metric spaces has been studied and developed by many authors (see [1, 2, 3, 4, 11, 12, 13, 14, 17, 18, 19, 20]). In 2009, Abbas and Rhoades [3], proved the unique common fixed point theorems for a pair of weakly compatible mappings which are more general than R-weakly commuting and compatible mappings in the setting of a generalized metric spaces without assuming the notion of continuity. Recently, Abbas et al. [1], proved the unique fixed point theorem for R-weakly commuting pairs of mappings assuming the continuity of at least one of the mappings on a G-metric space X.

In this paper, we prove the existence of the unique common fixed point theorems for the four mappings f, g, S, and T on a G-metric space X which satisfy the certain contractive conditions and the pairs $\{f, S\}$ and $\{g, T\}$ are weakly compatible without exploiting the notion of continuity.

We now recall some of the basic concepts and results in G-metric spaces that have been established in [13].

Definition 1.2 A G-metric space (X, G) is said to be symmetric if $G(x, y, y) = G(y, x, x)$ for all $x, y \in X$.

Proposition 1.3 *Every G-metric space (X, G), defines a metric space (X, d_G) by*

$$d_G(x, y) = G(x, y, y) + G(x, x, y), \text{ for all } x, y \in X.$$

Definition 1.4 Let (X, G) be a G-metric space. We say that a sequence $\{x_n\}$ in X is:

(i) a G-convergent sequence if, for any $\varepsilon > 0$, there exist $x \in X$ and $N \in \mathbb{N}$ such that $G(x, x_n, x_m) < \varepsilon$, for all $n, m \geq N$,

(ii) a G-Cauchy sequence if, for any $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that $G(x_n, x_m, x_l) < \varepsilon$, for all $n, m, l \geq N$.

Theorem 1.5 *Let (X, G) be a G-metric space and $\{x_n\}$ be a sequence in X. Then the followings are equivalent:*

(i) $\{x_n\}$ *is G-convergent to x,*

(ii) $G(x_n, x_n, x) \to 0$, *as* $n \to \infty$,

(iii) $G(x_n, x, x) \to 0$, *as* $n \to \infty$,

(iv) $G(x_m, x_n, x) \to 0$, *as* $m, n \to \infty$.

Theorem 1.6 *Let (X, G) be a G-metric space and $\{x_n\}$ be a sequence in X. Then the followings are equivalent:*

(i) $\{x_n\}$ *is a G-Cauchy sequence,*

(ii) for every $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that $G(x_n, x_m, x_m) < \varepsilon$, for all $n, m \geq N$,

(iii) $\{x_n\}$ *is a Cauchy sequence in* (X, d_G).

A G-metric space X is said to be complete if every G-Cauchy sequence in X is a G-convergent sequence in X.

Proposition 1.7 *Let (X, G) be a G-metric space. Then the function $G(x, y, z)$ is jointly continuous in all three of its variables.*

Definition 1.8 Let f and g be single-valued self mappings on a set X. If $w = fx = gx$ for some $x \in X$, then x is called a coincidence point of f and g, and w is called a point of coincidence of f and g.

Definition 1.9 Let f and g be single-valued self mappings on a set X. A pair $\{f, g\}$ is weakly compatible if f and g commute at their coincidence points.

The following result is a consequence of Theorem 3.1 of [16].

Theorem 1.10 *Let (X, d) be a metric space and $f, g, S, T : X \to X$ be mappings such that*

$$d(fx, gx) \leq h \max\{d(Sx, Ty), d(fx, Sx), d(gy, Ty), [d(fx, Ty) + d(gy, Sx)]/2\},$$

for all $x, y \in X$ where $h \in [0, 1)$. Suppose that $fX \subseteq TX$ and $gX \subseteq SX$. Assume that one of TX or SX is a complete subspace of X. If the pairs $\{f, S\}$ and $\{g, T\}$ are weakly compatible, then f, g, S and T have a unique common fixed point in X.

2 Common fixed point theorems for four mappings

Theorem 2.1 *Let X be a G-complete metric space. Suppose that $\{f, S\}$ and $\{g, T\}$ are weakly compatible pairs of self-mappings on X satisfying*

$$\begin{aligned} G(fx, fx, gy) \leq\ & h \max\{G(Sx, Sx, Ty), G(fx, fx, Sx), G(gy, gy, Ty), \\ & [G(fx, fx, Ty) + G(gy, gy, Sx)]/2\} \qquad (1) \end{aligned}$$

and

$$\begin{aligned} G(fx, gy, gy) \leq\ & h \max\{G(Sx, Ty, Ty), G(fx, Sx, Sx), G(gy, Ty, Ty), \\ & [G(fx, Ty, Ty) + G(gy, Sx, Sx)]/2\}, \qquad (2) \end{aligned}$$

for all $x, y \in X$ where $h \in [0, \frac{1}{2})$. Suppose that $fX \subseteq TX$ and $gX \subseteq SX$. If one of TX or SX is a G-closed subspace of X, then f, g, S, and T have a unique common fixed point.

Proof. If (X, G) is symmetric, then by using (1) and (2), we obtain that

$$\begin{aligned} d_G(fx, gy) \leq\ & \frac{h}{2} \max\{d_G(Sx, Ty), d_G(fx, Sx), d_G(gy, Ty), [d_G(fx, Ty) + d_G(gy, Sx)]/2\} \\ & + \frac{h}{2} \max\{d_G(Sx, Ty), d_G(fx, Sx), d_G(gy, Ty), [d_G(fx, Ty) + d_G(gy, Sx)]/2\} \\ =\ & h \max\{d_G(Sx, Ty), d_G(fx, Sx), d_G(gy, Ty), [d_G(fx, Ty) + d_G(gy, Sx)]/2\}, \end{aligned}$$

for all $x, y \in X$ with $h \in [0, \frac{1}{2})$. By Theorem 1.10, we have f, g, S, and T have a unique common fixed point. We now suppose that (X, G) is not symmetric. Let x_0 be an arbitrary point in X. Since $fX \subseteq TX$ and $gX \subseteq SX$, there exist x_1, x_2 in X such that $gx_0 = Sx_1$ and $fx_1 = Tx_2$. Again, there exist x_3, x_4 in X such that $gx_2 = Sx_3$ and $fx_3 = Tx_4$. By continuing this process, for each $n \in \mathbb{N} \cup \{0\}$, we can choose $x_n \in X$ such that

$$gx_{2n} = Sx_{2n+1} \text{ and } fx_{2n+1} = Tx_{2n+2}.$$

For each $n \in \mathbb{N} \cup \{0\}$, we let

$$y_{2n} = gx_{2n} = Sx_{2n+1} \text{ and } y_{2n+1} = fx_{2n+1} = Tx_{2n+2}.$$

As in the proof of Theorem 2.1 [1], we have

$$G(y_{n+1}, y_{n+1}, y_n) \leq hG(y_n, y_n, y_{n-1}), \text{ for all } n \in \mathbb{N}.$$

Therefore, for each $n \in \mathbb{N}$, we obtain that

$$G(y_{n+1}, y_{n+1}, y_n) \leq h^n G(y_1, y_1, y_0).$$

We will prove that $\{y_n\}$ is a G-Cauchy sequence in X. For $n, m \in \mathbb{N}$ with $m > n$, we have

$$\begin{aligned} G(y_n, y_m, y_m) &\leq G(y_n, y_{n+1}, y_{n+1}) + G(y_{n+1}, y_{n+2}, y_{n+2}) + \cdots + G(y_{m-1}, y_m, y_m) \\ &\leq h^n G(y_1, y_1, y_0) + h^{n+1} G(y_1, y_1, y_0) + \cdots + h^{m-1} G(y_1, y_1, y_0) \\ &\leq \frac{h^n}{1-h} G(y_1, y_1, y_0). \end{aligned}$$

By taking the limit of both sides, we obtain that $G(y_n, y_m, y_m) \to 0$ as $n, m \to \infty$. This implies that $\{y_n\}$ is a G-Cauchy sequence in X. Therefore there exists $z \in X$ such that $y_n \to z$ as $n \to \infty$. This implies that

$$\lim_{n\to\infty} y_{2n} = \lim_{n\to\infty} y_{2n+1} = z.$$

Suppose that TX is G-closed. It follows that $z = Tu$ for some $u \in X$. Using (1), we obtain that

$$\begin{aligned} G(y_{2n+1}, y_{2n+1}, gu) &= G(fx_{2n+1}, fx_{2n+1}, gu) \\ &\leq h \max\{G(Sx_{2n+1}, Sx_{2n+1}, Tu), G(fx_{2n+1}, fx_{2n+1}, Sx_{2n+1}), \\ &\qquad G(gu, gu, Tu), [G(fx_{2n+1}, fx_{2n+1}, Tu) + G(gu, gu, Sx_{2n+1})]/2\} \\ &= h \max\{G(y_{2n}, y_{2n}, Tu), G(y_{2n+1}, y_{2n+1}, y_{2n}), \\ &\qquad G(gu, gu, Tu), [G(y_{2n+1}, y_{2n+1}, Tu) + G(gu, gu, y_{2n})]/2\}. \end{aligned}$$

Thus, as $n \to \infty$, we have

$$\begin{aligned} G(z,z,gu) &\leq h\max\{0,0,G(gu,gu,z),[0+G(gu,gu,z)]/2\} \\ &= hG(gu,gu,z) \\ &\leq 2hG(z,z,gu). \end{aligned}$$

This implies that $gu = z = Tu$. Since $\{g,T\}$ is weakly compatible, we have $gz = gTu = Tgu = Tz$. We next prove that $gz = z$. Applying (1), we obtain that

$$\begin{aligned} G(y_{2n+1},y_{2n+1},gz) &= G(fx_{2n+1},fx_{2n+1},gz) \\ &\leq h\max\{G(Sx_{2n+1},Sx_{2n+1},Tz),G(fx_{2n+1},fx_{2n+1},Sx_{2n+1}), \\ &\quad G(gz,gz,Tz),[G(fx_{2n+1},fx_{2n+1},Tz)+G(gz,gz,Sx_{2n+1})]/2\} \\ &= h\max\{G(y_{2n},y_{2n},Tz),G(y_{2n+1},y_{2n+1},y_{2n}), \\ &\quad G(gz,gz,Tz),[G(y_{2n+1},y_{2n+1},Tz)+G(gz,gz,y_{2n})]/2\}. \end{aligned}$$

Taking $n \to \infty$, we have

$$\begin{aligned} G(z,z,gz) &\leq h\max\{G(z,z,Tz),0,0,[G(z,z,Tz)+G(gz,gz,z)]/2\} \\ &= h\max\{G(z,z,gz),0,0,[G(z,z,gz)+G(gz,gz,z)]/2\} \\ &= \frac{h}{2}[G(z,z,gz)+G(gz,gz,z)]. \end{aligned}$$

It follows that

$$G(z,z,gz) \leq \frac{h}{2-h}G(gz,gz,z). \tag{3}$$

By using (2), we obtain that

$$\begin{aligned} G(y_{2n+1},gz,gz) &= G(fx_{2n+1},gz,gz) \\ &\leq h\max\{G(Sx_{2n+1},Tz,Tz),G(fx_{2n+1},Sx_{2n+1},Sx_{2n+1}), \\ &\quad G(gz,Tz,Tz),[G(fx_{2n+1},Tz,Tz)+G(gz,Sx_{2n+1},Sx_{2n+1})]/2\} \\ &= h\max\{G(y_{2n},Tz,Tz),G(y_{2n+1},y_{2n},y_{2n}), \\ &\quad G(gz,Tz,Tz),[G(y_{2n+1},Tz,Tz)+G(gz,y_{2n},y_{2n})]/2\}. \end{aligned}$$

Taking $n \to \infty$, we have

$$\begin{aligned} G(z,gz,gz) &\leq h\max\{G(z,Tz,Tz),0,0,[G(z,Tz,Tz)+G(gz,z,z)]/2\} \\ &= h\max\{G(z,gz,gz),0,0,[G(z,gz,gz)+G(gz,z,z)]/2\} \\ &= \frac{h}{2}[G(z,gz,gz)+G(gz,z,z)]. \end{aligned}$$

Therefore

$$G(z, gz, gz) \leq \frac{h}{2-h} G(gz, z, z). \tag{4}$$

By (3) and (4), we obtain that

$$G(z, z, gz) \leq (\frac{h}{2-h})^2 G(z, z, gz).$$

Thus $z = gz = Tz$ which implies that z is a common fixed point of g and T. Since $gz \in SX$, there exists $v \in X$ such that $z = gz = Sv$. Using (1) to obtain that

$$\begin{aligned}
G(fv, fv, gz) &\leq h \max\{G(Sv, Sv, Tz), G(fv, fv, Sv), G(gz, gz, Tz), \\
&\quad [G(fv, fv, Tz) + G(gz, gz, Sv)]/2\} \\
&= h \max\{G(gz, gz, Tz), G(fv, fv, gz), G(gz, gz, gz), \\
&\quad [G(fv, fv, gz) + G(gz, gz, gz)]/2\} \\
&= h \max\{0, G(fv, fv, gz), 0, G(fv, fv, gz)/2\} \\
&= hG(fv, fv, gz).
\end{aligned}$$

Therefore $fv = gz = Sv$. Since $\{f, S\}$ is weakly compatible, we have $fz = fSv = Sfv = Sz$. We next prove that $fz = z$. Applying (1), we obtain that

$$\begin{aligned}
G(fz, fz, z) &= G(fz, fz, gz) \\
&\leq h \max\{G(Sz, Sz, Tz), G(fz, fz, Sz), G(gz, gz, Tz) \\
&\quad [G(fz, fz, Tz) + G(gz, gz, Sz)]/2\} \\
&= h \max\{G(fz, fz, z), [G(fz, fz, z) + G(z, z, fz)]/2\} \\
&= \frac{h}{2}[G(fz, fz, z) + G(z, z, fz)].
\end{aligned}$$

It follows that

$$G(z, fz, fz) \leq \frac{h}{2-h} G(z, z, fz). \tag{5}$$

By using (2), we obtain that

$$\begin{aligned}
G(fz, z, z) &= G(fz, gz, gz) \\
&\leq h \max\{G(Sz, Tz, Tz), G(fz, Sz, Sz), G(gz, Tz, Tz) \\
&\quad [G(fz, Tz, Tz) + G(gz, Sz, Sz)]/2\} \\
&= h \max\{G(fz, z, z), 0, 0, [G(fz, z, z) + G(z, fz, fz)]/2\} \\
&= \frac{h}{2}[G(fz, z, z) + G(z, fz, fz)].
\end{aligned}$$

Therefore

$$G(z,z,fz) \leq \frac{h}{2-h} G(z,fz,fz). \tag{6}$$

By (5) and (6), we obtain that

$$G(z,fz,fz) \leq (\frac{h}{2-h})^2 G(z,fz,fz).$$

Thus $z = fz = Sz$ which implies that z is a common fixed point of f and S. Hence we can conclude that z is a common fixed point of f, g, S, and T. Suppose that $w \in X$ is another common fixed point of f, g, S, and T. Applying (1) to obtain

$$\begin{aligned} G(z,z,w) &= G(fz,fz,gw) \\ &\leq h \max\{G(Sz,Sz,Tw), G(fz,Sz,Sz), G(gw,gw,Tw) \\ &\quad [G(fz,fz,Tw) + G(gw,gw,Sz)]/2\} \\ &= h \max\{G(z,z,w), 0, 0, [G(z,z,w) + G(w,w,z)]/2\} \\ &= \frac{h}{2}[G(z,z,w) + G(w,w,z)]. \end{aligned}$$

Therefore

$$G(z,z,w) \leq \frac{h}{2-h} G(w,w,z). \tag{7}$$

Applying (2) to obtain

$$\begin{aligned} G(z,w,w) &= G(fz,gw,gw) \\ &\leq h \max\{G(Sz,Tw,Tw), G(fz,Sz,Sz), G(gw,Tw,Tw) \\ &\quad [G(fz,Tw,Tw) + G(gw,Sz,Sz)]/2\} \\ &= h \max\{G(z,w,w), 0, 0, [G(z,w,w) + G(w,z,z)]/2\} \\ &= \frac{h}{2}[G(z,w,w) + G(w,z,z)]. \end{aligned}$$

Therefore

$$G(z,w,w) \leq \frac{h}{2-h} G(w,z,z). \tag{8}$$

By (7) and (8), we have

$$G(z,z,w) \leq (\frac{h}{2-h})^2 G(z,z,w).$$

It follows that $z = w$. In the case of being G-closed set of SX, the proof is similarly. ■

Theorem 2.2 *Let X be a G- metric space. Suppose that $\{f, S\}$ and $\{g, T\}$ are weakly compatible pairs of self-mappings on X satisfying*

$$\begin{aligned} G(fx, fx, gy) \leq\ & h \max\{G(Sx, Sx, Ty), G(fx, fx, Sx), G(gy, gy, Ty), \\ & [G(fx, fx, Ty) + G(gy, gy, Sx)]/2\} \end{aligned} \tag{9}$$

and

$$\begin{aligned} G(fx, gy, gy) \leq\ & h \max\{G(Sx, Ty, Ty), G(fx, Sx, Sx), G(gy, Ty, Ty), \\ & [G(fx, Ty, Ty) + G(gy, Sx, Sx)]/2\}, \end{aligned} \tag{10}$$

for all $x, y \in X$ where $h \in [0, \frac{1}{2})$. Suppose that $fX \subseteq TX$ and $gX \subseteq SX$. If one of TX or SX is a G-complete subspace of X, then f, g, S, and T have a unique common fixed point.

Proof. As the proof in Theorem 2.1, we can construct a Cauchy sequence $\{y_n\}$ in X satisfying

$$y_{2n} = gx_{2n} = Sx_{2n+1} \text{ and } y_{2n+1} = fx_{2n+1} = Tx_{2n+2} \text{ for } n = 0, 1, 2, \ldots.$$

Assume that TX is a G-complete subspace of X. Thus there exist $z, u \in X$ such that $y_{2n+1} \to z$ as $n \to \infty$ and $Tu = z$. This implies that $y_{2n} \to z$ as $n \to \infty$. For the rest of the proof, we can follow Theorem 2.1 to obtain a unique common fixed point of f, g, S, and T. Hence the proof is complete. ∎

If we take $g = f$ and apply Theorem 2.2, we immediately obtain the following corollary.

Corollary 2.3 *Let X be a G- metric space. Suppose that $\{f, S\}$ and $\{f, T\}$ are weakly compatible pairs of self-mappings on X satisfying*

$$\begin{aligned} G(fx, fx, fy) \leq\ & h \max\{G(Sx, Sx, Ty), G(fx, fx, Sx), G(fy, fy, Ty), \\ & [G(fx, fx, Ty) + G(fy, fy, Sx)]/2\} \end{aligned} \tag{11}$$

and

$$\begin{aligned} G(fx, fy, fy) \leq\ & h \max\{G(Sx, Ty, Ty), G(fx, Sx, Sx), G(fy, Ty, Ty), \\ & [G(fx, Ty, Ty) + G(fy, Sx, Sx)]/2\}, \end{aligned} \tag{12}$$

for all $x, y \in X$ where $h \in [0, \frac{1}{2})$. Suppose that $fX \subseteq TX \cup SX$. If one of TX or SX is a G-complete subspace of X, then f, S, and T have a unique common fixed point.

In Theorem 2.2, if we take $S = T$, then we obtain the following corollary.

Corollary 2.4 *Let X be a G- metric space. Suppose that $\{f, T\}$ and $\{g, T\}$ are weakly compatible pairs of self-mappings on X satisfying*

$$\begin{aligned} G(fx, fx, gy) \leq\ & h \max\{G(Tx, Tx, Ty), G(fx, fx, Tx), G(gy, gy, Ty), \\ & [G(fx, fx, Ty) + G(gy, gy, Tx)]/2\} \end{aligned} \tag{13}$$

and

$$\begin{aligned} G(fx, gy, gy) \leq\ & h \max\{G(Tx, Ty, Ty), G(fx, Tx, Tx), G(gy, Ty, Ty), \\ & [G(fx, Ty, Ty) + G(gy, Tx, Tx)]/2\}, \end{aligned} \tag{14}$$

for all $x, y \in X$ where $h \in [0, \frac{1}{2})$. Suppose that $fX \cup gX \subseteq TX$. If TX is a G-complete subspace of X, then $f, g,$ and T have a unique common fixed point.

Example 2.5 Let $X = [0, 2]$ and $G(x, y, z) = \max\{|x - y|, |y - z|, |x - z|\}$. Define $f, g, S, T : X \to X$ by

$$fx = gx = 1 \text{ and } Sx = Tx = 2 - x \text{ for all } x \in X.$$

We obtain that f, g, S and T satisfy (1) and (2) in Theorem 2.2. Indeed, we have $G(fx, fx, gy) = 0$ and $G(fx, gy, gy) = 0$. Therefore

$$\begin{aligned} G(fx, fx, gy) \leq\ & h \max\{G(Sx, Sx, Ty), G(fx, fx, Sx), G(gy, gy, Ty), \\ & [G(fx, fx, Ty) + G(gy, gy, Sx)]/2\} \end{aligned}$$

and

$$\begin{aligned} G(fx, gy, gy) \leq\ & h \max\{G(Sx, Ty, Ty), G(fx, Sx, Sx), G(gy, Ty, Ty), \\ & [G(fx, Ty, Ty) + G(gy, Sx, Sx)]/2\}, \end{aligned}$$

for all $x, y \in X$ where $h \in [0, \frac{1}{2})$. It is obvious that $fX \subseteq TX$ and $gX \subseteq SX$. Furthermore, we have $\{f, S\}$ and $\{g, T\}$ are weakly compatible pairs and TX is a G-complete subspace of X. Thus all assumptions in Theorem 2.2 are satisfied. This implies that f, g, S, and T have a unique common fixed point which is $x = 1$.

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