

On Generalized α b- Continuous Maps, On Generalized α b - open maps and On Generalized α b - closed maps

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Abstract

In this paper we study new class is called of generalized α b – Continuous mappings (denoted by $g\alpha$ b-Continuous) and study some of their properties. Also we study On Generalized- α b open maps and On Generalized- α b closed maps.

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1. Introduction

Balachandran et al in[16] introduced the concept of generalized continuous maps and gc-irresolute maps of a topological space. In this chapter we introduce and study the concept of a new class of maps called generalized α b-Continuous maps which includes

the includes generalized b – continuous maps, Moreover we introduce and study the concept of generalized α b - closed maps, generalized α b - open maps and ¹Ahmad Al-Omari and ²Mohd. Salmi Md. Noorani are introduced on Generalized b -closed sets, and they introduced ap - b -continuous, ap - b -closed and contra- b -continuous maps.

2. Preliminaries

In this section let us recall some definitions and results which are used in this section

Definition 2.1: A subset A of a topological space (X, τ) is called α - open [16] if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$

Definition 2.2: A subset A of a topological space (X, τ) is called semi- open [1] if $A \subseteq \text{cl}(\text{int}(A))$

Definition 2.3: A subset A of a topological space (X, τ) is called pre-open [6] if $A \subseteq \text{int}(\text{cl}(A))$

Definition 2.4: A subset A of a topological space (X, τ) is called semi-pre open [1] if $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$

Definition 2.5: A subset A of a topological space (X, τ) is called b -open [4] if $A \subseteq \text{cl}(\text{int}(A)) \cup \text{int}(\text{cl}(A))$.

Definition 2.6: A is said to be generalized closed set (g -closed) [12] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open

Definition 2.7: A is said to be α -generalized closed set (α g -closed) [16] if $\alpha \text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

Definition 2.8: A is said to be generalized pre-closed set (gp -closed) [12] if $A^* \subseteq U$ whenever $A \subseteq U$ and U is open.

Definition 2.9: A is said to be generalized semi-preclosed(gsp-closed) set [6] if $\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

Definition 2.10: A is said to be generalized semi-closed set(gs-closed) set [3] if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

Definition 2.11: A is said to be semi generalized closed set (sg-closed) [3] if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open.

Definition 2.12: A is said to be generalized b-closed set(gb-closed) [18] if $\text{bcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

Definition 2.13: A subset A of a topological space X is called generalized α b-closed ($g\alpha$ b-closed) [19] if $\text{bcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open.

Remark 2.14: The complement of the above open sets are known as their respective closed sets and vice-versa

3. On Generalized α b-Continuous Functions

Definition 3.1: Let X and Y be topological space. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be Generalized- α b-Continuous if the inverse image of every open set in Y is $g\alpha$ b-open in X.

Theorem 3.2: If a map $f : (X, \tau) \rightarrow (Y, \sigma)$ from a topological space X into a topological space Y is continuous then it is $g\alpha$ b-Continuous but not conversely.

Proof: Let V be an open set in Y since f is continuous then $f^{-1}(V)$ is open in X. As open set is $g\alpha$ b-open in X. Therefore f is $g\alpha$ b-Continuous.

Remark 3.3: The converse of the above theorem need not be true as seen from the following example.

Example 3.4: Let $X=Y=\{a,b,c\}$ with $\tau=\{X, \Phi, \{b\}, \{b,c\}\}$ and $\sigma = \{Y, \Phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = a, f(b) = c, f(c) = a$, then f

is $g\alpha$ b-Continuous but not continuously as the inverse image of an open set $\{a,b\}$ in Y is $\{a,c\}$ is not $g\alpha$ b-open in X .

Theorem 3.3: If a map $f : (X,\tau) \rightarrow (Y,\sigma)$ is $g\alpha$ b-Continuous if and only if the inverse image of every closed set in Y is $g\alpha$ b-Closed set in X .

Proof: Let E be closed in Y . Then E^c is open in Y , since f is $g\alpha$ b-Continuous $f^{-1}(E)$ is $g\alpha$ b-Closed in X . Conversely assume that the inverse image of every closed set in Y is $g\alpha$ b-Closed in X . Let V be an open set in Y then V^c is closed in Y , therefore $f^{-1}(V^c) = X - f^{-1}(V)$ is $g\alpha$ b-Closed in X , and so $f^{-1}(V)$ is $g\alpha$ b-open in X . Therefore $g\alpha$ b-Continuous.

Theorem 3.4: Let X and Y be topological space. If a map $f : (X,\tau) \rightarrow (Y,\sigma)$ is sg-continuous then it is $g\alpha$ b-Continuous but not conversely.

Proof: Assume that a map $f : (X,\tau) \rightarrow (Y,\sigma)$ is sg-continuous. Let V be an open set in Y , since f is sg-continuous then $f^{-1}(V)$ is sg-open and hence every sg-open is $g\alpha$ b-open in X . Therefore f is $g\alpha$ b-Continuous.

Remark 3.5: The converse of the theorem need not be true.

Example 3.6: Let $X=Y=\{a,b,c\}$ with $\tau=\{X,\Phi,\{a\},\{a,b\}\}$ and $\sigma = \{Y, \Phi, \{b\},\{a,b\},\{b,c\}\}$ and f be the identity map therefore $f(b) = b$, $f(c) = c$. Thus f is $g\alpha$ b-Continuous but not sg-continuous as the inverse image of the open set $\{b,c\}$ in Y is $\{b,c\}$ in X is not sg-open.

Theorem 3.5: Let X and Y be topological space. If a map $f : (X,\tau) \rightarrow (Y,\sigma)$ is $g\alpha$ b-Continuous then it is gs-continuous but not conversely.

Proof: Assume that a map $f : (X,\tau) \rightarrow (Y,\sigma)$ is $g\alpha$ b-Continuous let V be an open set in Y , since f is $g\alpha$ b-Continuous $f^{-1}(V)$ is $g\alpha$ b-open and hence every $g\alpha$ b-open is gs-open in X therefore f is gs-continuous.

Remark 3.6: The converse of the theorem is need not be true.

Example 3.7: Let $X=Y=\{a,b,c\}$ with $\tau=\{X,\Phi,\{c\}\}$ and $\sigma = \{Y, \Phi,\{a\},\{a,b\}\}$ and $f : (X,\tau) \rightarrow (Y,\sigma)$ be continuous and then $f(a)=a=f(b)$, the f is gs-continuous but not $g\alpha$ b-Continuous as the inverse image of the open set $\{a\}$ in Y is $\{a,b\}$ is not $g\alpha$ b-open in X .

Theorem 3.8: Let X and Y be topological space. If a map $f : (X, \tau) \rightarrow (Y, \sigma)$ is $g\alpha$ b-Continuous then it is α g-Continuous but not conversely.

Proof: Assume that a map $f : (X, \tau) \rightarrow (Y, \sigma)$ is $g\alpha$ b-Continuous. Let E be an open set in Y since f is $g\alpha$ b-Continuous $f^{-1}(E)$ is $g\alpha$ b-open is α g-open in X therefore f is α g-continuous.

Remark 3.9: The converse of the theorem need not be true.

Example 3.10: Let $X=Y=\{a,b,c\}$ with $\tau=\{X, \Phi, \{a\}\}$ and $\sigma = \{Y, \Phi, \{c\}, \{b,c\}\}$ and $f : (X, \tau) \rightarrow (Y, \sigma)$ be continuous and then $f(c)=c=f(b)$, and $f(a)=a$ then the function f is α g-continuous but not $g\alpha$ b-Continuous and the pre-image of c is $\{b,c\}$ open in Y is not $g\alpha$ b-open in X .

Theorem 3.11: Let X and Y be topological space If a map $f : (X, \tau) \rightarrow (Y, \sigma)$ is α g-continuous then it is $g\alpha$ b-Continuous.

Proof: Assume that a $f : (X, \tau) \rightarrow (Y, \sigma)$ is α g-continuous. Let E be an α -open set in Y since f is α g-continuous and then $f^{-1}(E)$ is α g-open then every α g-open is $g\alpha$ b-open in X therefore f is $g\alpha$ b-continuous.

Remark 3.12: The converse of the theorem is need not be true.

Example 3.13: Let $X=Y=\{a,b,c\}$ with $\tau=\{X, \Phi, \{b\}, \{b,c\}\}$ and $\sigma = \{Y, \Phi, \{a\}, \{c\}, \{a,c\}\}$ and $f : (X, \tau) \rightarrow (Y, \sigma)$ be continuous and then $f(a)=c$ and $f(c)=a$, and $f(b)=b$ then the inverse image open set $\{a,c\}$ in Y is $\{a,c\}$ is not $g\alpha$ b-open in X .

Remark 3.14: From the above examples α g-continuous $\begin{matrix} \xrightarrow{\hspace{1cm}} \\ \downarrow \\ \xleftarrow{\hspace{1cm}} \end{matrix}$ $g\alpha$ b-continuous.

Theorem 3.15: Let X and Y be topological space. If a map $f : (X, \tau) \rightarrow (Y, \sigma)$ is swg-continuous then it is $g\alpha$ b-Continuous but not conversely.

Proof: Assume that a $f : (X, \tau) \rightarrow (Y, \sigma)$ is swg-continuous. Let V be an open set in Y since f is swg-continuous and then $f^{-1}(V)$ is swg-open and hence $g\alpha$ b-open in X . Therefore f is $g\alpha$ b-continuous.

Remark 3.16: The converse of the theorem need not be true.

Example 3.17: Let $X=Y=\{a,b,c\}$ with $\tau=\{X,\Phi,\{b\},\{a,b\}\}$ and $\sigma = \{Y, \Phi, \{a\},\{c\},\{a,c\}\}$ and f be the identity map. Then f is $g\alpha$ b -continuous but not swg -continuous then $f(b)=b$ and $f(a)=b$, and $f(a)=b$ then the inverse image open set $\{a,b\}$ in Y is $\{a,b\}$ is not swg -open in X .

Theorem 3.18: Let X and Y be topological space. If a map $f : (X,\tau)\rightarrow(Y,\sigma)$ is $g\alpha$ b -continuous then it is gb -continuous but not conversely.

Proof: Assume that a $f : (X,\tau)\rightarrow(Y,\sigma)$ is $g\alpha$ b -Continuous. Let V be an open set in Y since f is $g\alpha$ b -Continuous and then $f^{-1}(V)$ is $g\alpha$ b -open set is gb -open set in X . Therefore f is gb -continuous.

Remark 3.19: The converse of the theorem need not be true.

Example 3.20: Let $X=Y=\{a,b,c\}$ with $\tau=\{X,\Phi,\{a\},\{a,c\}\}$ and $\sigma = \{Y, \Phi, \{a\},\{a,c\}\}$ and f be the identity map. Then f is gb -continuous but not $g\alpha$ b -continuous then $f(a)=b$ and $f(c)=c$ then the inverse image of open set $\{a,c\}$ in Y is $\{b,c\}$ is not $g\alpha$ b -open in X .

Theorem 3.21: Let X and Y be topological space. If a map $f : (X,\tau)\rightarrow(Y,\sigma)$ is $g\alpha$ b -Continuous then it is gpr -continuous but not conversely.

Proof: Assume that a $f : (X,\tau)\rightarrow(Y,\sigma)$ is $g\alpha$ b -Continuous..Let E be an open set in Y since f is $g\alpha$ b -Continuous and then $f^{-1}(E)$ is $g\alpha$ b -open set in X . Therefore every $g\alpha$ b -open is gpr -open set in X . Therefore f is gpr -continuous.

Remark 3.22: The converse of the theorem need not be true.

Example 3.23: Let $X=Y=\{a,b,c\}$ with $\tau=\{X,\Phi,\{b\},\{b,c\}\}$ and $\sigma = \{Y, \Phi, \{a\},\{a,b\}\}$ and f be the identity map. Then f is gpr -continuous but not $g\alpha$ b -continuous then $f(a)=a=f(c)$ and $f(b)=b$ then the inverse image of open set $\{a\}$ in Y is $\{a,c\}$ is not $g\alpha$ b -open in X .

Theorem 3.24: Let X and Y be topological space. If a map $f : (X,\tau)\rightarrow(Y,\sigma)$ is $g\alpha$ b -Continuous then it is gp -continuous but not conversely.

Proof: Assume that a $f : (X,\tau)\rightarrow(Y,\sigma)$ is $g\alpha$ b -Continuous. Let V be an open set in Y since f is $g\alpha$ b -Continuous and then $f^{-1}(V)$ is $g\alpha$ b -open in X . Therefore every $g\alpha$ b -open is gp -open in X . Therefore f is gp -continuous.

Remark 3.25:The converse of the theorem need not be true.

Example 3.26: Let $X=Y=\{a,b,c\}$ with $\tau=\{X,\Phi,\{a\},\{a,c\}\}$ and $\sigma = \{Y, \Phi, \{b\},\{c\},\{b,c\}\}$ and f be the identity map. Then f is $g\alpha$ -continuous but not $g\alpha$ b-continuous then $f(b)=b$ and $f(c)=c$ then the inverse image of open set $\{b,c\}$ in Y is $\{b,c\}$ is not $g\alpha$ b-open in X .

Theorem 3.27: Let X and Y be topological space. If a map $f : (X,\tau)\rightarrow(Y,\sigma)$ is $g\alpha$ b-Continuous then it is g -continuous but not conversely.

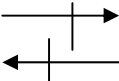
Proof: Assume that $f : (X,\tau)\rightarrow(Y,\sigma)$ is $g\alpha$ b-Continuous. Let E be an open set in Y since f is g -Continuous and then $f^{-1}(E)$ is g -open in X . Therefore every g -open set is $g\alpha$ b- open sets in X . Therefore f is $g\alpha$ b -continuous.

Remark 3.28:The following examples shows that $g\alpha$ b –continuous and g -continuous are need not be same.

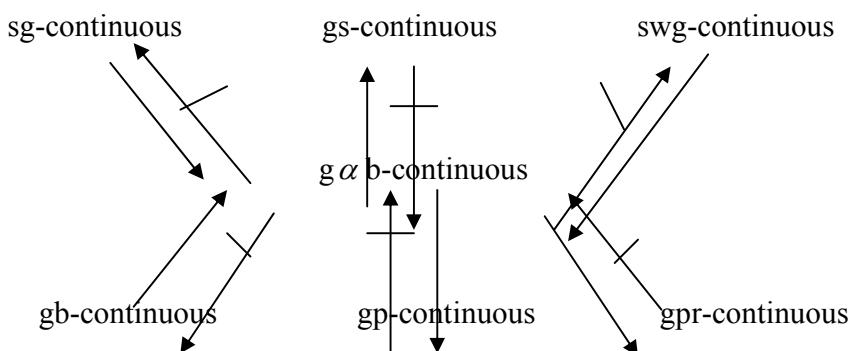
Example 3.29:Let $X=Y=\{a,b,c\}$ with $\tau =\{X,\Phi,\{a\}\}$ and $\sigma = \{Y, \Phi, \{b\},\{c\},\{a,c\},\{b,c\}\}$ and f be the identity map. Then f is g -continuous but not $g\alpha$ b-continuous then $f(a)=a$ and $f(c)=c$ then the inverse image of open set $\{a,c\}$ in Y is $\{b,c\}$ is not $g\alpha$ b-open in X .

Example 3.30:Let $X=Y=\{a,b,c\}$ with $\tau=\{X,\Phi,\{a\},\{c\},\{a,c\}\}$ and $\sigma =\{Y,\Phi, \{a\},\{b\},\{a,b\}\}$ and f be the identity map. Then f is $g\alpha$ b-continuous but not g -continuous then $f(a)=a$ and $f(b)=a$ and $f(c)=c$ then the inverse image of open set $\{a,b\}$ in Y is $\{a,b\}$ is not g -open in X .

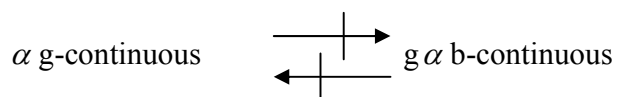
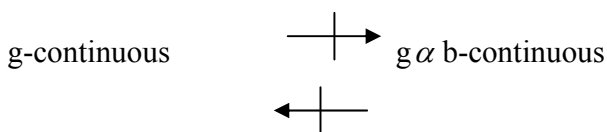
Remark 3.31:The following examples shows that

$g\alpha$ b –continuous  g -continuous are need not be same.

1. From the above results we form the following diagram:



2.



4. Generalized- αb open maps and Generalized- αb closed maps.

In this section we introduce generalized- αb open ($g\alpha b$ -open) maps and generalized- αb closed ($g\alpha b$ -closed) maps.

Definition 4.1: Let X and Y be two topological space. A map $f:(X,\tau)\rightarrow(Y,\sigma)$ is called generalized- αb - open ($g\alpha b$ -open) map if the image of every open set in X is $g\alpha b$ -open in Y .

Theorem 4.2: Every open map is gab -open but not conversely.

Proof: Let $f:(X,\tau)\rightarrow(Y,\sigma)$ is an open map and V be an open set in X then $f(V)$ is open in Y and hence gab -open in Y . Thus f is gab - open the converse of the above theorem need not be true as seen from the following example.

Example 4.3: Consider $X=Y =\{a,b,c\}$, $\tau =\{X,\Phi, \{b\},\{a,b\},\{b,c\}\}$ and $\sigma =\{Y, \Phi, \{b\}\}$. Let a map $f:(X,\tau)\rightarrow(Y,\sigma)$ be defined by $f(a)=a=f(c),f(b)=b$ then this function is gab -open but not open as the image of the open set $\{a,b\}$ in X is $\{a,b\}$ not open in Y .

Definition 4.4 : Let X and Y be topological space. A map $f:(X,\tau)\rightarrow(Y,\sigma)$ is called gab -closed map if the image of every closed set in X is gab -closed set in Y .

Theorem 4.5: Every closed map is gab -closed but not conversely.

Proof: Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be closed map and V be a closed set in X . Then $f(V)$ is closed and hence gab -closed in Y . Thus f is gab -closed. The converse of the above theorem need not be true as seen from the following example.

Example 4.6: Consider $X=Y=\{a,b,c\}$, $\tau =\{X,\Phi, \{b\}\}$ and $\sigma =\{Y, \Phi, \{b\},\{a,b\}\}$ and a map $f:(X,\tau)\rightarrow(Y,\sigma)$ be defined by $f(a)=a,f(b)=b,f(c)=c$. This function f is gab -closed but not closed as $f(\{a,b\})=\{a,b\}$ is not closed in Y .

Theorem 4.7: A map $f:(X,\tau)\rightarrow(Y,\sigma)$ is gab -closed if and only if for each subset S of Y and for each α -open set U containing $f^{-1}(S)$ there is a gab -open set V of Y such that $S\subseteq V$ and $f^{-1}(V)\subseteq U$.

Proof: Suppose f is gab -closed. Let S be a subset of Y and U be an α -open set of X such that $f^{-1}(S)\subseteq U, V=Y -f(X-U)$ is gab -open set containing S such that $f^{-1}(V)\subseteq U$. Converse, suppose that F is a closed set of X . Then $f^{-1}(Y-f(F))\subseteq X-F$ and $X-F$ is open. By hypothesis there is a gab -open set V of Y such that $X-f(F)\subseteq V$ and $f^{-1}(V)\subseteq X-F$. Therefore $F\subseteq X- f^{-1}(V)$, hence $Y-V\subseteq f(F)\subseteq f(X- f^{-1}(V))\subseteq Y-V$ which implies $f(F)=Y-V$, since $Y-V$ is gab -closed, $f(F)$ is gab -closed and hence f is gab -closed map.

Theorem 4.8: If a map $f:(X,\tau)\rightarrow(Y,\sigma)$ is continuous and gab -closed A is gab -closed set of X then $f(A)$ is gab -closed in Y .

Proof: Let $f(A) \subseteq U$ where U is α -open set in Y , since f is continuous $f^{-1}(U)$ is an open set containing A hence $\text{bcl}(A) \subseteq f^{-1}(U)$ as A is $g\alpha b$ -closed since f is $g\alpha b$ -closed $f(\text{bcl}(A)) \subseteq U$ is $g\alpha b$ -closed U is an α -open set which implies $\text{bcl}(f(\text{bcl}(A))) \subseteq U$ and hence $\text{bcl}(f(A)) \subseteq U$ so $f(A)$ is $g\alpha b$ -closed set in Y .

Corollary 4.9: If map $f:(X,\tau) \rightarrow (Y,\sigma)$ is continuous and closed and A is $g\alpha b$ -closed then $f(A)$ is $g\alpha b$ -closed in Y .

Corollary 4.10: If a map $f:(X,\tau) \rightarrow (Y,\sigma)$ is $g\alpha b$ -closed and A is closed set of X the $f_A:A \rightarrow Y$ is $g\alpha b$ -closed.

Corollary 4.11: If a map $f:(X,\tau) \rightarrow (Y,\sigma)$ is $g\alpha b$ -closed and continuous and A is $g\alpha b$ -closed set of X then $f_A:A \rightarrow Y$ is continuous and $g\alpha b$ -closed.

Proof: Let F be a closed set of A then F is $g\alpha b$ -closed set of X by the theorem $f(A)$ is $g\alpha b$ -closed hence $f_A(F) = f(F)$ is $g\alpha b$ -closed set of Y . Here f_A is $g\alpha b$ -closed and also continuous.

Definition 4.12: A space X is said to be α -normal if for every two disjoint closed subsets A and B of X there exist two disjoint α -open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Theorem 4.13: If a map $f:(X,\tau) \rightarrow (Y,\sigma)$ is continuous $g\alpha b$ -closed map from a α -normal space X into a space then is α -normal.

Proof: Let A, B be disjoint closed sets of Y , then f is continuous A and B are closed sets implies $f^{-1}(A), f^{-1}(B)$ are disjoint closed sets of X , since X is α -normal there are two disjoint open sets U, V in X such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$ since f is $g\alpha b$ -closed by the theorem 4.2, there are $g\alpha b$ -open sets G, H in Y such that $A \subseteq G, B \subseteq H$ and $f^{-1}(G) \subseteq U$ and $f^{-1}(H) \subseteq V$ since U, V are disjoint α -open sets $\text{bint}(G)$ and $\text{bint}(H)$ are disjoint open sets since G is α -open, A is closed and $A \subseteq G, A \subseteq \text{bint}(G)$ and H is α -open set is closed and $B \subseteq H$ then $B \subseteq \text{bint}(H)$ hence Y is α -normal.

Definition 4.14: A space X is said to be α -regular if for each closed set F of X and each $x \in X - F$ there exist disjoint α -open sets U and V such that $x \in U$ and $F \subseteq V$.

Theorem 4.15: If a map $f:(X,\tau) \rightarrow (Y,\sigma)$ is α -open, continuous, $g\alpha b$ -closed and surjection where X is α -regular then Y is α -regular.

Proof: Let U be an α -open set containing a part x of X such that $f(x)=p$ since X is α -regular and f is continuous there is an α -open set V such that $x \in V \subseteq f^{-1}(U)$ hence $p \in f(V) \subseteq f(\text{bcl}(V)) \subseteq U$ since f is gab -closed $f(\text{bcl}(V))$ is gab -closed then $f(\text{bcl}(V)) \subseteq U, U$ is and α -open set it follows that $\text{cl}(\text{int}(f(\text{bcl}(V)))) \subseteq U$ and hence $p \in f(V) \subseteq \text{bcl}(f(V)) \subseteq U$ and $f(U)$ is α -open, since f is α -open hence y is α -regular.

Remark: The following example show that the α -regular closed maps and gab -closed maps are independent.

Example 4.16: Let $X=Y=\{a,b,c\}$ and a map $f:(X,\tau) \rightarrow (Y,\sigma)$ be identity map with $\tau = \{X, \Phi, \{a\}, \{b\}, \{a,b\}\}$ and $\sigma = \{y, \Phi, \{a\}, \{a,b\}\}$. then f is gab -closed but not α -regular closed as the image of the α -regular closed set $\{a,c\}$ in X is $\{a,c\}$ in Y is not gab -closed.

Definiton 4.17: (approximately- αb -continuous)

A map $f:(X,\tau) \rightarrow (Y,\sigma)$ is said to be approximately gab -continuous if $\text{bcl}(F) \subseteq f^{-1}(U)$ whenever U is α -open subset of subset of Y and F is gab -closed subset of X such that $F \subseteq f^{-1}(U)$.

Definition 4.18:(approximately- αb -closed)

A map $f:(X,\tau) \rightarrow (Y,\sigma)$ is said to be approximately αb -closed if $f(F) \subseteq \text{bint}(V)$, whenever V is gab -open subset of Y . F is an α -closed subset of X and $f(F) \subseteq V$

Definition 4.19: (approximately- αb -continuous map)

A map $f:(X,\tau) \rightarrow (Y,\sigma)$ is said to be approximately b -open if $\text{bcl}(F) \subseteq f(V)$ whenever V is an α -open subset of X . F is gab -closed subset of Y and $F \subseteq f(V)$.

Theorem 4.20: If a map $f:(X,\tau) \rightarrow (Y,\sigma)$ is ap - αb -continuous and b -closed then the image of each gab -closed set in X is gab -closed in Y .

Proof: Let F be a gab -closed subset of X . Let $f(F) \subseteq V$ where V is α -open set of Y . Then $F \subseteq f^{-1}(V)$ since f is ap - αb -continuous we have $\text{bcl}(F) \subseteq f^{-1}(V)$ then $f(\text{bcl}(F)) \subseteq V$ therefore we have $\text{bcl}f(F) \subseteq \text{bcl}(f(\text{bcl}(F)))=f(\text{bcl}(F)) \subseteq V$ hence $f(F)$ is gab -closed in Y .

Theorem 4.21: If $f:(X,\tau) \rightarrow (Y,\sigma)$ is ap - αb -closed and b -closed function then $f(A)$ is gab -closed in Y for every gab -closed set A of X .

Proof: Let A be $g\alpha b$ -closed in X . Let $f(A) \subseteq V$ where V be any α -open set in Y since f is continuous $f^{-1}(V)$ is α -open in X and $A \subseteq f^{-1}(V)$. Then we have $bcl(A) \subseteq f^{-1}(V)$ and so $f(bcl(A)) \subseteq V$ since f is closed, $f(bcl(A))$ is $g\alpha b$ -closed in Y and hence $bcl(f(A)) \subseteq bcl(f(bcl(A))) = f(bcl(A)) \subseteq V$. This shows that $bcl(f(A)) \subseteq V$ therefore $f(A)$ is $g\alpha b$ -closed in Y .

References

- [1]. D. Andrijevic, Semi-preopen sets , Mat. Vesnik 38 (1986), 24–32.
- [2]. D. Andrijevic, On b-open sets, Mat. Vesnik 48(1996), no.1-2,59-64.
- [3]. P.Bhattacharya and B.K. Lahiri , Semi –generalized closed sets on topology , Indian. J.Maths 29 (3) (1987) 375 -382.
- [4]. M. Caldas and S. Jafari, On some applications of b-open sets in topological spaces,. Kochi J. Math.. 2(2007), 11-19.
- [5]. J. Cao, M. Ganster and I. Reilly, Submaximality, extremal disconnectedness and gen-eralized closed sets, Houston J. Math., 24 (4) (1998), 681–688.
- [6]. J.Dontchev, On generalizing semi-preopen sets, Mem. Fac. Sci. Kochi Univ. Ser.A. Math., 16 (1995), 35-48.
- [7]. J. Dontchev, Contra-continuous functions and strongly S-closed space, Internat. J. Math. Math. Sci., 19 (2) (1996), 303--310.
- [8]. J. Dontchev, M. Ganster and T. Noiri, On p-closed spaces, Internat. J. Math. Math. Sci. 24 (2000), 203– 212.
- [9].M. Ganster and M. Steiner, On some questions about b-open sets, Questions Answers Gen. Topology 25 (2007), no. 1, 45–52.
- [10]. M. Ganster and M. Steiner, On $b\tau$ -closed sets, Appl. Gen. Topol. 8 (2007), no.2,243-247.

- [11]. E. Ekici and M. Caldas, Slightly γ -continuous functions, Bol. Parna, Mat. (3)22(2004).
- [12]. N. Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo (2) 19 (1970), 89-96.
- [13]. N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly 70(1963), 36-41.
- [14]. Nasef, A.A., On b-locally closed sets and related topics, Chaos Solitons. Fractals, 12, No.10, (2001), 1909-1915 .
- [15]. O. Njastad, On some classes of nearly open sets pacific. J. Math. 15(1965), 961-970.
- [16]. H. Maki, R. Devi and K. Balachandran, Associated topologies of generalized α -closed sets and α -generalized closed sets Mem. Fac. Sci. Kochi. Univ. Ser. A. Math. 15(1994), 51-63.
- [17]. M.E. Abd El-Monsef, A.A. El-Atik and M.M. El-Sharkasy some topologies induced by b-open sets, kyungpook Math. J. 45(2005) no.4, 539-547.
- [18]. Ahmad Al-Omari and Mohd. Salmi Md. Noorani On Generalized b-closed sets. Bull. Malays. Math. Sci. Soc(2) 32(1) (2009), 19-30.
- [19]. L. Vinayagamoorthi, N. Nagaveni On Generalized - α b closed sets, and Generalized α b -Continuous Maps ICMD-2010 yet to Publish.

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