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On Riemann Boundary Value Problem in Hardy Classes with Variable Summability Exponent

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Abstract

In this paper, the Riemann boundary value problem with piecewise-continuous coefficient is considered. Under certain conditions on the coefficients, the general solution of the conjugation problem in the Hardy classes with variable summability exponent is constructed.

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1 Introduction

Study of basis properties of perturbed system of exponents $\left\{e^{i\lambda_n t}\right\}_{n\in Z}$ ($\{\lambda_n\} \subset C$ is a complex sequence, Z is the set of all integers) in various spaces of functions dates back to Paley-Wiener's known paper [12] and is of great interest up to now. There are different approaches and methods for establishing basis properties of this system, and this direction is called "nonharmonic analysis". A.M.Sedletskii's monograph [14] has been devoted to it.

One of the generalizations of specific cases of this system is the double system of exponents

$$\left\{A(t)e^{int}; B(t)e^{-int}\right\}_{n\in N}, \tag{1}$$

with complex-valued coefficients $A(t)$ and$B(t)$, where N is the set of all positive integers. The methods of theory of boundary value problems for analytic functions are widely used in studying basis properties of the systems of type (1) in the spaces L_p, $1 \le p \le +\infty$, ($L_\infty \equiv C[a;b]$) and this is illustrated well in [1,2,4,6,11,13].

Recently, in the light of applications in the problems of mechanics, there arose a great interest in studying various problems in generalized classes of Lebesgue $L_{p(\cdot)}$ and Sobolev $W^m_{p(\cdot)}$ (see. [7;10]).

In this paper, we study the Riemann boundary value problem in the Hardy classes $H^{\pm}_{p(\cdot)}$ with variable summability exponent $p(\cdot)$. These classes have been earlier introduced in [3;8]. It should be noted that such a problem was considered in [9].

2 Needful concepts and facts. Main assumptions

We state some ideas from the theory of $L_{p(\cdot)}$ spaces. Let $p : [-\pi, \pi] \to [1, +\infty)$ be some Lebesgue measurable function. Denote by L_0 the class of all functions measurable on $[-\pi, \pi]$(with respect to Lebesgue measure). Denote

$$I_p(f) \overset{def}{\equiv} \int_{-\pi}^{\pi} |f(t)|^{p(t)}\, dt.$$

Let $L \equiv \{f \in L_0 : I_p(f) < +\infty\}$ and $p^{\pm} = \sup\limits_{[-\pi,\pi]} vrai\, p(t)^{\pm 1}$. With the condition $1 \leq p^- \leq p^+ < +\infty$ fulfilled, L turns into a linear space with respect to ordinary linear operations of addition of functions and multiplication of the function by a number. With a norm

$$\|f\|_{p(\cdot)} \overset{def}{\equiv} \inf\left\{\lambda > 0 : I_p\left(\frac{f}{\lambda}\right) \leq 1\right\},$$

L is a Banach space, denote it by $L_{p(\cdot)}$. Assume

$WL_\pi \overset{def}{\equiv} \{p(t) : p(\pi) = p(-\pi), \exists C > 0, \forall t_1, t_2 \in [-\pi, \pi] : |t_1 - t_2| \leq \frac{1}{2} \Rightarrow$ $\Rightarrow |p(t_1) - p(t_2)| \leq \frac{C}{-ln|t_1 - t_2|}\}$. This is a weakly Lipschitz class of functions periodic on $[-\pi, \pi]$. Throughout this paper $q(t)$ will denote the function $\frac{1}{p(t)} + \frac{1}{q(t)} \equiv 1$ conjugated to $p(t)$.

The following Holder's inequality holds:

$$\int_{-\pi}^{\pi} |f(t)\, g(t)|\, dt \leq c\left(p^-; p^+\right) \|f\|_{p(\cdot)} \|g\|_{q(\cdot)}$$

where $c(p^-; p^+) = 1 + \frac{1}{p^-} - \frac{1}{p^+}$.

It follows directly from the definition:

Property A. *If $|f(t)| \leq |g(t)|$ a.e. on $(-\pi, \pi)$, then $\|f\|_{p(\cdot)} \leq \|g\|_{p(\cdot)}$.*

We'll often use this property. The following statement is easily proved:

Statement 1. *Let $p \in WL_\pi$, $p(t) > 0, \forall t \in [-\pi, \pi]$ and $\{\alpha_i\}_1^m \subset R$(R is a real axis). The function $\omega(t) = \prod_{i=1}^m |t - t_i|^{\alpha_i}$ belongs to the space $L_{p(\cdot)}$ if $\alpha_i > -\frac{1}{p(t_i)}$, $\forall i = \overline{1, m}$; where $\{t_i\}_1^m \subset [-\pi, \pi]$, $t_i \neq t_j$, for $i \neq j$.*

Let‘s introduce the following Hardy classes.

Let $\omega = \{z : |z| < 1\}$ be a unit circle on a complex plane and $\partial\omega$ be a unit circumference.

The Hardy class $H^+_{p(\cdot)}$ is: $H^+_{p(\cdot)} \equiv \left\{ f : \; f \text{ analytic in } \omega \text{ and } \|f\|_{H^+_{p(\cdot)}} < +\infty \right\}$, where $\|f\|_{H^+_{p(\cdot)}} \equiv \sup\limits_{0<r<1} \|f(re^{it})\|_{p(\cdot)}$. $H^+_{p(\cdot)}$ is a Banach space if $1 \le p^- \le p^+ < +\infty$.

Define the Hardy class ${}_mH^-_{p(\cdot)}$ of functions analytic outside the unit circle and of order less than or equal to m at infinity. Let $f(z)$ be a function analytic on $C\backslash\bar{\omega}$ ($\bar{\omega} = \omega \cup \partial\omega$) of finite order $m_0 \le m$ at infinity, i.e. $f(z) = f_1(z) + f_2(z)$, where $f_1(z)$ is a polynomial of power m_0, $f_2(z)$ is a right part of the expansion of $f(z)$ in Lorentz series in the vicinity of the point at infinity. If the function $\varphi(z) = \overline{f_2\left(\frac{1}{\bar{z}}\right)}$ ($(\bar{\cdot})$ is a complex conjugation) belongs to the class $H^+_{p(\cdot)}$, we‘ll say that the function $f(z)$ belongs to the class ${}_mH^-_{p(\cdot)}$.

We make the following main assumptions concerning function $\alpha(t)$:

(a) $\alpha(t)$ is piecewise Holder on $[-\pi, \pi]$, $\{s_k\}_1^r : -\pi = s_0 < s_1 < ... < < s_r < s_{r+1} = \pi$ are its discontinuity points on $(-\pi, \pi)$. Let $\{h_k\}_1^r : \; h_k = = \alpha(s_k + 0) - \alpha(s_k - 0)$, $k = \overline{1, r}$ be the jumps of the function $\alpha(t)$ at the points s_k, and let $h_0 = \frac{\alpha(-\pi) - \alpha(\pi)}{\pi}$;

(b) $\left\{ \frac{h_k}{\pi} - \frac{1}{p(s_k)} : k = \overline{0, r} \right\} \bigcap Z = \emptyset$.

Determine $\{n_k\}_1^r \subset Z$ from the following relations:

$$-\frac{1}{q(s_k)} < \frac{h_k}{\pi} + n_{k-1} - n_k < \frac{1}{p(s_k)}, n_0 = 0\,,\; k = \overline{1, r}.$$

Assume $\omega_\pi = h_0 + n_r$.

3 Riemann homogeneous problem in generalized Hardy classes

Consider the following Riemann problem in the class $H^+_{p(\cdot)} \times {}_mH^-_{p(\cdot)}$:

$$\left.\begin{array}{l} F^+(\tau) - G(\tau)F^-(\tau) = 0, \; \tau \in \partial\omega, \\ F^+ \in H^+_{p(\cdot)}, \; F^- \in {}_mH^-_{p(\cdot)}, \end{array}\right\} \tag{2}$$

where $G(\tau)$ is the coefficient of the problem. By the solution of problem (2) in the class $H^+_{p(\cdot)} \times {}_mH^-_{p(\cdot)}$ we mean a pair of analytic functions $(F^+; F^-) \in H^+_{p(\cdot)} \times {}_mH^-_{p(\cdot)}$ whose non-tangential boundary values on a unit circle $\partial\omega$ satisfy relation (2) almost everywhere. Consider the case $G(\tau) \equiv e^{2i\alpha(\arg\tau)}$, $\tau \in \partial\omega$. Assume

$$X^\pm(z) \equiv \exp\left\{ \pm\frac{i}{2\pi} \int_{-\pi}^{\pi} \alpha(t) \frac{e^{it} + z}{e^{it} - z} dt \right\},$$

and let

$$Z(z) \equiv \begin{cases} X^{+}(z), & |z| < 1, \\ \left[X^{-}(z)\right]^{-1}, & |z| > 1. \end{cases}$$

Using the Sokhotski-Plemelj formula [5], we get

$$G(\tau) = Z^{+}(\tau)\left[Z^{-}(\tau)\right]^{-1}, \ \tau \in \partial\omega,$$

where $Z^{\pm}(\tau)$ are non-tangential boundary values on $\partial\omega$ of the function $Z(z)$ from within (the sign "+") and from the outside (the sign "-") of a unit circle ω. $Z(z)$ is called a canonic solution of homogeneous problem (2). Taking into account the expression for $G(\tau)$ in (2), we get

$$\frac{F^{+}(\tau)}{Z^{+}(\tau)} = \frac{F^{-}(\tau)}{Z^{-}(\tau)}, \ \tau \in \partial\omega. \tag{3}$$

Let $\Phi(z) = \frac{F(z)}{Z(z)}$, where

$$F(z) = \begin{cases} F^{+}(z), & |z| < 1, \\ F^{-}(z), & |z| > 1. \end{cases}$$

It is easy to see that the function $Z(z)$ has no zeros and no poles for $z \notin \partial\omega$. Therefore, the functions $\Phi(z)$ and $F(z)$have the same order at infinity.

Let $\alpha(t) = \alpha_0(t) + \alpha_1(t)$ be Jordan decomposition of the function $\alpha(t)$, where $\alpha_0(t)$ is a continuous part, and $\alpha_1(t)$ is a jump function on $[-\pi, \pi]$. Assume $h_0^{(0)} = 2\left[\alpha_0(\pi) - \alpha_0(-\pi)\right]$.

Accept $u_0(t) \equiv \left\{\sin\left|\frac{t-\pi}{2}\right|\right\}^{-\frac{h_0^{(0)}}{2\pi}} \exp\left\{-\frac{1}{2\pi}\int_{-\pi}^{\pi}\alpha_0(\tau) ctg\frac{t-\tau}{2} d\tau\right\}$,
$u(t) \equiv \prod_{k=1}^{r}\left\{\sin\left|\frac{t-s_k}{2}\right|\right\}^{-\frac{h_k}{\pi}}$.

By the results of the monograph [5] the function $|Z^{-}(\tau)|$, $\tau \in \partial\omega$, can be written as

$$\left|Z^{-}\left(e^{it}\right)\right| = u_0(t)\, u(t)\left\{\sin\left|\frac{t-\pi}{2}\right|\right\}^{-\frac{h_0}{\pi}}, \tag{4}$$

where $h_0 = \alpha(-\pi) - \alpha(\pi)$. By definition of the solution it is clear that $F^{-}(\tau) \in L_{p(\cdot)}$. Thus, if $|Z^{-}(\tau)|^{-1} \in L_{q(\cdot)}$, then applying the generalized Holder inequality, we get from (3) $\Phi^{\pm}(\tau) \in L_1$. Since $\Phi^{+}(\tau) = \Phi^{-}(\tau)$ a.e. $\tau \in \partial\omega$, then it follows from the uniqueness theorem [5] that $\Phi(z)$ is an entire function on the plane and as a result, from $\Phi^{-} \in {}_mH_1^{-}$ (since the functions $\Phi(z)$and $F(z)$ have the same order at infinity) it follows that $\Phi(z)$ is a polynomial of at most m^{th} degree. Consequently, $\Phi(z) = F(z)P_{m_0}(z)$, where $P_{m_0}(z)$ is a polynomial of degree $m_0 \leq m$.

Suppose that the following inequalities are fulfilled:

$$\frac{h_0}{\pi} > -\frac{1}{q\left(s_k\right)}; \frac{h_k}{\pi} > -\frac{1}{q(s_k)}, k = \overline{1, r}. \tag{5}$$

Show that when inequalities (5) are fulfilled, the function $\left|Z^{-}(\tau)\right|^{-1}$ belongs to $L_{q(\cdot)}$.

By $f\left(t\right) \sim g\left(t\right)$, $t \to t_0$, we denote the asymptotic equivalence of the functions f and g near the point $t = t_0$, i.e.

$$\exists m > 0 : \ m \leq \left|\frac{f(t)}{g(t)}\right| \leq m^{-1} \ , \text{as } t \to t_0.$$

Thus, it is evident that
$\sin\left|\frac{t-\pi}{2}\right| \sim t^2 - \pi^2$, as $t \to \pm\pi$ holds.

As a result,

$$\left\{\sin\left|\frac{t-\pi}{2}\right|\right\}^{\frac{h_0}{\pi}} \sim |t-\pi|^{\frac{h_0}{\pi}} \left|t+\pi\right|^{\frac{h_0}{\pi}} , \ t \to \pm\pi.$$

Then, from the expression (3) we get

$$\left|Z^{-}\left(e^{it}\right)\right|^{-1} \sim u_0^{-1}\left(t\right) \prod_{k=1}^{r} |t-s_k|^{\frac{h_k}{\pi}} \left|t^2-\pi\right|^{\frac{h_0}{\pi}} , t \in \left(-\pi, \pi\right). \tag{6}$$

From statement 1 it follows that when inequalities (5) are fulfilled, the function $\left|Z^{-}(\tau)\right|^{-1}$ belongs to $L_{q(\cdot)}(\tau)$. As a result we have: $\Phi(z) = F(z)P_{m_0}(z)$, $m_0 \leq m$.

Now, let $\frac{h_0}{\pi} < \frac{1}{p(\pi)}$; $\frac{h_k}{\pi} < \frac{1}{p(s_k)}$, $k = \overline{1, r}$ be fulfilled.

Then it follows from Statement 1 and relation (6) that $Z^{-}(\tau) \in L_{p(\cdot)}(\partial\omega)$, and so $\Phi^{-} \in L_{p(\cdot)}$. From the Smirnov theorem [8] we get that the function $\Phi(z) \equiv (\Phi^{+}(z); \Phi^{-}(z))$ belongs to the class $H^{+}_{p(\cdot)} \times {}_{m}H^{-}_{p(\cdot)}$. Thus, the following theorem is proved.

Theorem 3.1. *Let* $p \in WL_{\pi}$, $p^{-} > 1$, $G(e^{it}) = e^{2i\alpha(t)}$, *where* $\alpha(t)$ *satisfies the conditions (a),(b), and let the following inequalities be fulfilled:*

$$-\frac{1}{q\left(\pi\right)} < \frac{h_0}{\pi} < \frac{1}{p\left(\pi\right)}; -\frac{1}{q\left(s_k\right)} < \frac{h_k}{\pi} < \frac{1}{p\left(s_k\right)}, k = \overline{1, r}.$$

Then, the general solution of Riemann homogeneous problem (2) *has the following form:*

$$F\left(z\right) = Z\left(z\right) P_{m_0}\left(z\right) \ , \ m_0 \leq m,$$

where $Z(z)$ *is a canonical solution.*

Hence, the following corollary is true:

Corollary 3.2. *Let all the conditions of Theorem 3.1 be fulfilled. Then for* $F^{-}(\infty) = 0$, *i.e. in the class* $H^{+}_{p(\cdot)} \times {}_{-1}H^{-}_{p(\cdot)}$, *homogeneous problem* (2) *has only a trivial solution.*

4 Nonhomogeneous conjugation problem

Consider the nonhomogeneous problem

$$\left. \begin{array}{l} F^+(\tau) - G(\tau)F^-(\tau) = f(\tau),\ \tau \in \partial\omega, \\ F^+ \in H^+_{p(\cdot)},\ F^- \in {}_mH^-_{p(\cdot)}, \end{array} \right\} \tag{7}$$

where $f \in L_{p(\cdot)}$ and $G(\tau) = e^{2i\alpha(\arg\tau)}$. Obviously, the general solution of problem (7) is of the form $F(z) = F_0(z) + F_1(z)$, where F_0 is a general solution to appropriate homogeneous problem and F_1 is one of the particular solutions of the problem (7). As it was already shown, $F_0(z)$ is of the form $F_0(z) = Z(z)P_m(z)$, where $Z(z)$ is a canonical solution of homogeneous problem (2), $P_m(z)$ is a polynomial of degree $m_0 \le m$. Consider the Cauchy type integral

$$F_1(z) = \frac{Z(z)}{2\pi i} \int_{\partial\omega} \frac{f(\tau)}{Z^+(\tau)} \frac{d\tau}{\tau - z}, \tag{8}$$

where $Z^+(\tau)$ are the non-tangential boundary values of the functions $Z(z)$ on $\partial\omega$ from within a unit circle ω. It follows directly from the Sokhotski-Plemelj formula that the function $F_1(z)$ satisfies relation (7) a.e. on $\partial\omega$. Show that $F(z)$ belongs to the class $H^+_{p(\cdot)} \times {}_mH^-_{p(\cdot)}$. So, it follows from the relation

$$Z^+(\tau) = G(\tau)Z^-(\tau),\ t \in \partial\omega$$

that

$$\left|Z^+(\tau)\right| = \left|Z^-(\tau)\right|,\ t \in \partial\omega.$$

Taking into account expression (6), we have

$$\left|Z^+\left(e^{it}\right)\right|^{-1} \sim u_0^{-1}(t)\left|t^2 - \pi^2\right|^{\frac{h_0}{\pi}} \prod_{k=1}^{r} |t - s_k|^{\frac{h_k}{\pi}},\ t \in (-\pi, \pi). \tag{9}$$

From the Sokhotski-Plemelj formula we get

$$F_1^{\pm}(\xi) = \frac{1}{2}f(\xi) \pm \frac{Z^{\pm}(\tau)}{2\pi i} \int_{\partial\omega} \frac{f(\tau)}{Z^+(\tau)} \frac{d\tau}{\tau - \xi},\ \xi \in \partial\omega.$$

Assume

$$S[f] = \frac{Z^+(\tau)}{2\pi i} \int_{\partial\omega} \frac{f(\tau)}{Z^+(\tau)} \frac{d\tau}{\tau - \xi},\ \xi \in \partial\omega.$$

We have

$$\left|Z^+\left(e^{it}\right)\right| \sim u_0(t)\left|t^2 - \pi^2\right|^{\frac{h_0}{\pi}} \prod_{k=1}^{r} |t - s_k|^{-\frac{h_k}{\pi}},\ t \in (-\pi, \pi).$$

Let the following inequalities be fulfilled

$$\frac{h_k}{\pi} > -\frac{1}{q(s_k)},\ k = \overline{0, r}. \tag{10}$$

From the expression for $|Z^+(e^{it})|^{-1}$ it follows that when inequalities (10) are fulfilled, the function $|Z^+(\tau)|^{-1}$ belongs to $L_{q(\cdot)}(\partial\omega)$. Since $f \in L_{q(\cdot)}(\partial\omega)$, then from generalized Holder's inequality it follows that the product $f(\tau)\,|Z^+(\tau)|^{-1}$ belongs to $L_1(\partial\omega)$. Then, as is known (see [5]), the Cauchy type integral

$$\Phi(z) = \int_{\partial\omega} \frac{f(\tau)}{Z^+(\tau)} \frac{d\tau}{\tau - z},$$

belongs to the Hardy class H_1^+. As a result, from $Z(z) \in H^+_{p(\cdot)}$ we get that $F_1(z) = \frac{1}{2\pi i} Z(z)\, \Phi(z)$ belongs to the Hardy class H_δ^+ for some $\delta > 0$. Show that $F_1^+ \in L_{p(\cdot)}(\partial\omega)$. Suppose that the following inequalities are fulfilled:

$$-\frac{1}{q(s_k)} < \frac{h_k}{\pi} < \frac{1}{p(s_k)},\ k = \overline{0, r}. \tag{11}$$

Then, by the results of the paper [9], the singular operator

$$S[f] = \int_{\partial\omega} \frac{f(\tau)\, d\tau}{\tau - \xi},\ \ \xi \in \partial\omega,$$

acts boundedly in the weight space $L_{p(\cdot),\rho}$, where $\rho(t) = |t^2 - \pi^2|^{\frac{h_0}{\pi}} \prod_{k=1}^{r} |t - s_k|^{\frac{h_k}{\pi}}$. Consequently, it follows from expression (9) that $S[f]$ belongs to $L_{p(\cdot)}$, and as a result, $F_1^+(\tau) \in L_{p(\cdot)}$. From the results of [8] we get that the analytic function $F_1(z)$ belongs to $H^+_{p(\cdot)}$.

It can be similarly proved that with inequalities (11) fulfilled the function $F_1^-(\tau)$ belongs to $L_{p(\cdot)}(\partial\omega)$. It is obvious that $F_1(\infty) = 0$. Consequently, $F_1 \in {}_mH^-_{p(\cdot)}$ for $m \geq 0$. Thus, $F_1(z)$ is a particular solution of problem (7). So we proved the following

Theorem 4.1. *Let $p \in WL_\pi$, $G(e^{it}) = e^{2i\alpha(t)}$, $\alpha(t)$ satisfy the conditions (a),(b) and inequalities (11) be fulfilled. Then, the general solution of Riemann nonhomogeneous problem (7) in the class $H^+_{p(\cdot)} \times {}_{-m}H^-_{p(\cdot)}$ $(m \geq 0)$ is of the form $F(z) = Z(z)P_{m_0}(z) + F_1(z)$, where*

$$F_1(z) = \frac{Z(z)}{2\pi i} \int_{\partial\omega} \frac{f(\tau)}{Z^+(\tau)} \frac{d\tau}{\tau - z}, \tag{12}$$

$Z(z)$ is a canonical solution of corresponding homogeneous problem and $P_{m_0}(z)$ is a polynomial of degree $m_0 \leq m$.

Taking into account Corollary 3.2, we get from Theorem 4.1:

Corollary 4.2. *Let all the conditions of Theorem 2 be fulfilled. Then under condition $F(\infty) = 0$ the problem (7) has a unique solution $F_1(z)$ determined by expression* (12).

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