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Completely Bounded Fourier Multipliers Over Compact Groups

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Abstract

Vector version of Fourier multipliers over compact non necessary abelian groups are defined. A characterization of the completely bounded ones is obtained.

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1 Introduction

The theory of operator spaces as developed in [3] and [4] by Blecher and Paulsen, and in [5] and [6] by Effros and Ruan is currently in full expansion.

Recently this theory allowed Pisier to introduce the non-commutative analogue of Banach space valued L_p-spaces in the category of operator spaces [12] starting from a given von Neumann algebra equipped with a trace having some additionnal properties. He gave many concrete illustrations among which there were Fourier multipliers over compact abelian groups. Our aim in this paper is to study the vector version of the compact non necessary abelian case following the approach in [12]. Let E be an operator space. We introduce Fourier multipliers of $L_p(G, E)$, the L_p-space of $E-$valued functions on the compact group G equipped with its normalized Haar measure in Bochner's sense. The introduction of Fourier multipliers for the compact case is made possible by the work of Assiamoua and Olubummo [2] who defined the Fourier-Stieltjes of vector (Banach space valued) measures on compact groups and studied the analogue of some classical properties, for instance the Riesz-Fischer theorem and the Plancherel theorem which lead to a Fourier inversion formula. The spaces of the transformed measures are studied by Mensah and Assiamoua in [9] and [10], mainly the duality relations between them. The link to Operator spaces is made when $L_p(G, E)$ is endowed with its natural operator space structure. The morphism in the category of operator spaces are the completely bounded maps (see the definition in the next section). In this paper we characterize Fourier multipliers of $L_p(G)$ that are completely bounded.
The rest of the paper is organized as follows. In the next section we recall briefly some elements in operators spaces theory. The section 3 is devoted to Fourier multipliers over abelian compact groups as treated in [11]. In section 4 we recall some results of vector harmonic analysis over compact groups drawn from [2]. We finish by stating our main results in the last section.

2 Operator spaces

Let H and K be complex Hilbert spaces. We denote by $B(H, K)$ the complex Banach space of all bounded operators from H into K. If $K = H$, then $B(H, H)$ is simply denoted $B(H)$. The space $B(H)$ is isometric to the space M_n of all $n \times n$ matrices with complex entries when H is of finite dimension n. By definition an *operator space* is a closed subspace of $B(H)$. An abstract characterization of operator spaces (the so-called *representation theorem for operator spaces*) was given by Ruan in [13]. A simpler proof of this theorem can be found in [7]. The characterization is as follows. A complex vector space X is an operator space iff for each $n \geq 1$, there is a complete norm $\|.\|_n$ on $M_n(X)$, the space of $n \times n$ matrices with entries in X, such that the following properties are satisfied :

1. $\|x \oplus y\|_{n+m} = max\{\|x\|_n, \|y\|_m\}$,

2. $\|\alpha x \beta\|_n \leq \|\alpha\| \|x\|_n \|\beta\|$,
   for $x \in M_n(X),\ y \in M_m(X),\ \alpha,\ \beta \in M_n$.

Let l_2 be the space of all sequences (x_k) of complex numbers such that $\sum_{k=1}^{\infty} |x_k|^2 < \infty$, and l_2^n the space of finite sequences $x_1, x_2, \dots, x_n$ of complex numbers. They are Hilbert spaces with obvious inner products. Common examples of operator spaces are the $p-$Schatten spaces often denoted by S_p and defined as the space of all compact operators T on l_2 such that $tr|T|^p < \infty$ endowed with the norm

$$\|T\|_{S_p} = (tr|T|^p)^{1/p} \quad (1 \leq p < \infty). \tag{1}$$

For $p = \infty$, S_∞ is the space of all compacts operators on l_2 equipped with the operator norm.
Particularly, S_p^n denotes the space of all compacts operators on l_2^n and S_∞^n the space of all compact operators on l_2^n equipped with the norm

$$\|T\|_{S_p} = (tr|T|^p)^{1/p} \quad (1 \leq p < \infty)$$

and the operator norm respectively.
Let G be a compact group and E and operator space. One can construct a natural operator space structure on $L_p(G, E)$ by the interpolation method ([11], chapter 2). In the category of operator space, the morphisms are the completely bounded maps defined as follows.
A linear map $\varphi : X \subset B(H) \longrightarrow Y \subset B(K)$ between two operator spaces is said to be *completely bounded* (c.b.) if the linear maps
$\varphi_n : M_n(X) \to M_n(Y)$ defined by: $\varphi_n((a_{ij})_{1\leq i,j\leq n}) = (\varphi(a_{ij}))_{1\leq i,j\leq n}$
are such that $\sup_{n\geq 1} \|\varphi_n\| < \infty$. The completely bounded norm is denoted by

$$\|\varphi\|_{cb} = \sup_{n\geq 1} \|\varphi_n\|. \tag{2}$$

The space of all completely bounded maps from X into Y is denoted $cb(X, Y)$ and simply $cb(X)$ if $X = Y$.
Let μ be a measure on the set Ω and $L_p(\Omega, \mu)$ (resp. $L_p(\Omega, \mu, S_p)$) the L_p-space of complex functions (resp. L_p-space of S_p-valued functions) on Ω. The following proposition, which we may need in the sequel, was proved in [11] (page 33).

Proposition 2.1 *A linear map $u : L_p(\Omega, \mu) \to L_p(\Omega, \mu)$ is completely bounded iff the mapping $u \otimes I_{S_p}$ is bounded on $L_p(\Omega, \mu, S_p)$. Moreover, we have*

$$\|u\|_{cb(L_p(\Omega,\mu))} = \|u \otimes I_{S_p}\|_{B(L_p(\Omega,\mu,S_p))}. \tag{3}$$

3 Fourier multipliers over abelian compact groups

Let G is compact abelian group with normalized Haar measure λ and dual group Γ. Let X be a Banach space. The Fourier transform $\widehat{f}:\Gamma\to\mathbb{C}$ (resp. $\widehat{f}:\Gamma\to X$) of a function f in $L_p(G)$ (resp. in $L_p(G,X)$) for $1\le p\le\infty$ is given by

$$\forall\gamma\in\Gamma\quad \widehat{f}(\gamma)=\int_G f(t)\overline{\gamma(t)}d\lambda(t). \tag{4}$$

Then, at least, for $p\ge 2$, we have for all f in $L_p(G)$ (resp. in $L_p(G,X)$) the synthesis formula :

$$f=\sum_{\gamma\in\Gamma}\widehat{f}(\gamma)\gamma. \tag{5}$$

Let $\varphi:\Gamma\to\mathbb{C}$ be a map. One defines a *Fourier multiplier* M_φ on $L_p(G)$ (resp. on $L_p(G,X)$) by setting
$\forall f\in L_p(G)$ (resp. $f\in L_p(G,X)$) with $\widehat{f}$ finite support

$$M_\varphi f=\sum_{\gamma\in\Gamma}\varphi(\gamma)\widehat{f}(\gamma)\gamma. \tag{6}$$

Remark 3.1 *If T is the operator M_φ on $L_p(G)$, then $T\otimes I_X$ corresponds to M_φ acting on $L_p(G,X)$, but we denote T and $T\otimes I_X$ by M_φ.*

We say that φ is a bounded multiplier on $L_p(G)$ (resp. on $L_p(G,X)$) when M_φ is bounded on $L_p(G)$ (resp. on $L_p(G,X)$) and that φ is a c.b. multiplier on $L_p(G)$ (resp. on $L_p(G,X)$) if the map M_φ on $L_p(G)$ (resp. on $L_p(G,X)$) is c.b.. For more details we refer to [11] where the following proposition was proved (page 90).

Proposition 3.1 *Let G be a compact abelian group and $C\ge 0$ a constant. The following assertions are equivalent:*

1. *The multiplier φ is completely bounded on $L_p(G)$ with*
$$\|M_\varphi\|_{cb(L_p(G))}\le C.$$
2. *The multiplier φ is bounded on $L_p(G,S_p)$ with*
$$\|M_\varphi\|_{B(L_p(G,S_p))}\le C.$$
3. *For any n and any family of coefficients $(x_\gamma)_{\gamma\in\Gamma}$ with finite support, where $x_\gamma\in S_p^n$, we have*
$$\|\sum_{\gamma\in\Gamma}\varphi(\gamma)x_\gamma\gamma\|_{L_p(G,S_p^n)}\le C\|\sum_{\gamma\in\Gamma}x_\gamma\gamma\|_{L_p(G,S_p^n)}.$$

4 Fourier-Stieltjes transform of vector measures

Let G be a compact group with normalized Haar measure λ and unitary dual Σ. In any equivalence class σ belonging to Σ, we choose an element U^σ and denoted its hilbertian representation space by H_σ. Since the group G is compact, the space H_σ is of finite dimension d_σ. We fix once and for all a canonical basis $(\xi_1^\sigma, \ldots, \xi_{d_\sigma}^\sigma)$ of H_σ. We set

$$u_{ij}^\sigma(t) = \langle U_t^\sigma \xi_j^\sigma, \xi_i^\sigma \rangle \tag{7}$$

and introduce the operator $\overline{U_t^\sigma}$ of H_σ which matrix elements are given by

$$\langle \overline{U}_t^\sigma \xi_j^\sigma, \xi_i^\sigma \rangle = \overline{u_{ij}^\sigma(t)}, \tag{8}$$

the complex conjugate of $u_{ij}^\sigma(t)$.
The Fourier transform of a complex valued function f which is integrable with respect to the normalized Haar measure of G is a family of continuous endomorphisms $(\hat{f}(\sigma))_{\sigma\in\Sigma}$ of H_σ defined by

$$\langle \hat{f}(\sigma)\xi, \eta \rangle = \int_G \langle \overline{U}_t^\sigma \xi, \eta \rangle f(t)\, d\lambda(t), \quad (\xi, \eta) \in H_\sigma \times H_\sigma \tag{9}$$

For more informations on abstract harmonic analysis we refer to [8].
However, if f is a vector valued function on G (with values in the operator space E), the formula (9) is meaningless because the mapping $\eta \mapsto \int_G \langle \overline{U}_t^\sigma \xi, \eta \rangle f(t)\, dt$ from H_σ into E is no longer representable by a scalar product as far as we know.
In fact more generally, the Fourier-Stieltjes transform of a bounded vector measure $m : G \to E$ can be interpreted as a family $(\hat{m}(\sigma))_{\sigma\in\Sigma}$ of continuous sesquilinear mappings from $H_\sigma \times H_\sigma$ into E defined by :

$$\hat{m}(\sigma)(\xi, \eta) = \int_G \langle \overline{U}_t^\sigma \xi, \eta \rangle\, dm(t), \quad (\xi, \eta) \in H_\sigma \times H_\sigma \tag{10}$$

And therefore, the Fourier transform of a Haar-integrable function $f : G \to E$, when f is identified with the vector measure $f\lambda$, is a family $(\hat{f}(\sigma))_{\sigma\in\Sigma}$ of continuous sesquilinear mappings from $H_\sigma \times H_\sigma$ into E defined by

$$\hat{f}(\sigma)(\xi, \eta) = \int_G \langle \overline{U}_t^\sigma \xi, \eta \rangle f(t)\, d\lambda(t), \quad (\xi, \eta) \in H_\sigma \times H_\sigma. \tag{11}$$

From a result in [2] (the analogue of Riesz-Fischer theorem), we derived that at least for $p \geq 2$ we have for all f in $L_p(G, E)$,

$$f = \sum_\sigma d_\sigma \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \widehat{f}(\sigma)(\xi_j^\sigma, \xi_i^\sigma) u_{ij}^\sigma. \tag{12}$$

In the sequel, we set $a_{ij}^{\sigma} = \widehat{f}(\sigma)(\xi_j^{\sigma}, \xi_i^{\sigma})$.

5 Main Results

In this section, G is a compact group, E is an operator space. Let $\varphi : \Sigma \to \mathbb{C}$ be a map. We define a *Fourier multiplier* M_{φ} on $L_p(G)$ by: $\forall f \in L_p(G)$ with $\widehat{f}$ finite support,

$$M_{\varphi} f = \sum_{\sigma \in \Sigma} d_{\sigma} \sum_{i=1}^{d_{\sigma}} \sum_{j=1}^{d_{\sigma}} \varphi(\sigma) \langle \widehat{f}(\sigma) \xi_j^{\sigma}, \xi_i^{\sigma} \rangle u_{ij}^{\sigma}. \tag{13}$$

Similarly $\forall f \in L_p(G, E)$ with $\widehat{f}$ finite support, we define a *Fourier multiplier* on $L_p(G, E)$ by:

$$M_{\varphi} f = \sum_{\sigma \in \Sigma} d_{\sigma} \sum_{i=1}^{d_{\sigma}} \sum_{j=1}^{d_{\sigma}} \varphi(\sigma) \widehat{f}(\sigma)(\xi_j^{\sigma}, \xi_i^{\sigma}) u_{ij}^{\sigma}. \tag{14}$$

We say that φ defines a bounded multiplier on $L_p(G)$ (resp. on $L_p(G, E)$) if the map M_{φ} is bounded on $L_p(G)$ (resp. on $L_p(G, E)$). We say that φ defines a c.b. multiplier on $L_p(G)$ (resp. on $L_p(G, E)$) if the map M_{φ} is c.b. on $L_p(G)$ (resp. on $L_p(G, E)$).
We can now prove the following theorem which can be considered as a Wendel type characterization of multipliers [1].

Theorem 5.1 *These two assertions are equivalent:*

1. *M_{φ} is a Fourier multiplier on $L_p(G, E)$.*

2. *$\forall f \in L_p(G, E)$, $\widehat{M_{\varphi} f} = \varphi \widehat{f}$.*

Proof. Let M_{φ} be a Fourier multiplier on $L_p(G, E)$ and $\sigma' \in \Sigma$, then for all (ξ, η) in $H_{\sigma'} \times H_{\sigma'}$ with $\xi = \sum_{s=1}^{d_{\sigma'}} \alpha_s \xi_s^{\sigma'}$ and $\eta = \sum_{r=1}^{d_{\sigma'}} \beta_r \xi_r^{\sigma'}$ in the canonical basis

$(\xi_1^{\sigma'}, \ldots, \xi_{d_{\sigma'}}^{\sigma'})$ of $H_{\sigma'}$. We have

$$\begin{aligned}
\widehat{M_\varphi f}(\sigma')(\xi,\eta) &= \widehat{M_\varphi f}(\sigma')(\sum_{s=1}^{d_{\sigma'}} \alpha_s \xi_s^{\sigma'}, \sum_{r=1}^{d_{\sigma'}} \beta_r \xi_r^{\sigma'}) \\
&= \sum_{r=1}^{d_{\sigma'}} \sum_{s=1}^{d_{\sigma'}} \alpha_s \overline{\beta_r} \widehat{M_\varphi f}(\sigma')(\xi_s^{\sigma'}, \xi_r^{\sigma'}) \\
&= \sum_{r=1}^{d_{\sigma'}} \sum_{s=1}^{d_{\sigma'}} \alpha_s \overline{\beta_r} \int_G \langle \overline{U}_t^{\sigma'} \xi_s^{\sigma'}, \xi_r^{\sigma'} \rangle (M_\varphi f)(t) d\lambda(t) \\
&= \sum_{r=1}^{d_{\sigma'}} \sum_{s=1}^{d_{\sigma'}} \alpha_s \overline{\beta_r} \int_G \overline{u}_{rs}^{\sigma'}(t) (M_\varphi f)(t) d\lambda(t) \\
&= \sum_{r=1}^{d_{\sigma'}} \sum_{s=1}^{d_{\sigma'}} \alpha_s \overline{\beta_r} \int_G \overline{u}_{rs}^{\sigma'}(t) \sum_{\sigma\in\Sigma} d_\sigma \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \varphi(\sigma) a_{ij}^\sigma u_{ij}^\sigma(t) d\lambda(t) \\
&= \sum_{r=1}^{d_{\sigma'}} \sum_{s=1}^{d_{\sigma'}} \alpha_s \overline{\beta_r} \sum_{\sigma\in\Sigma} d_\sigma \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \varphi(\sigma) a_{ij}^\sigma \int_G \overline{u}_{rs}^{\sigma'}(t) u_{ij}^\sigma(t) d\lambda(t).
\end{aligned}$$

The Schur's orthogonality relations

$$\int_G \overline{u}_{rs}^{\sigma'}(t) u_{ij}^\sigma(t) d\lambda(t) = 0 \text{ if } \sigma \neq \sigma', \ \int_G \overline{u}_{rs}^{\sigma}(t) u_{ij}^\sigma(t) d\lambda(t) = \frac{1}{d_\sigma} \delta_{ir}\delta_{js}, \tag{15}$$

where δ is the Kronecker delta symbol, lead to :

$$\begin{aligned}
\widehat{M_\varphi f}(\sigma')(\xi,\eta) &= \sum_{r=1}^{d_{\sigma'}} \sum_{s=1}^{d_{\sigma'}} \alpha_s \overline{\beta_r} \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} d_{\sigma'} \varphi(\sigma') a_{ij}^{\sigma'} (\frac{1}{d_{\sigma'}} \delta_{ir}\delta_{js}) \\
&= \sum_{r=1}^{d_{\sigma'}} \sum_{s=1}^{d_{\sigma'}} \alpha_s \overline{\beta_r} \varphi(\sigma') a_{rs}^{\sigma'} \\
&= \sum_{r=1}^{d_{\sigma'}} \sum_{s=1}^{d_{\sigma'}} \alpha_s \overline{\beta_r} \varphi(\sigma') \widehat{f}(\sigma')(\xi_s^{\sigma'}, \xi_r^{\sigma'}) \\
&= \varphi(\sigma') \widehat{f}(\sigma')(\sum_{s=1}^{d_{\sigma'}} \alpha_s \xi_s^{\sigma'}, \sum_{r=1}^{d_{\sigma'}} \beta_r \xi_r^{\sigma'}) \\
&= \varphi(\sigma') \widehat{f}(\sigma')(\xi,\eta).
\end{aligned}$$

Thus $\widehat{M_\varphi f} = \varphi \widehat{f}$.

In the converse, if $\widehat{M_\varphi f} = \varphi \widehat{f}$, then we have

$$\begin{aligned}
M_\varphi f &= \sum_{\sigma\in\Sigma} d_\sigma \sum_{i=1}^{d_\sigma} \sum_{j=1}^{d_\sigma} \widehat{M_\varphi f}(\sigma)(\xi_j^\sigma, \xi_i^\sigma) u_{ij}^\sigma \\
&= \sum_{\sigma\in\Sigma} d_\sigma \sum_{i,j=1}^{d_\sigma} \varphi(\sigma) \widehat{f}(\sigma)(\xi_j^\sigma, \xi_i^\sigma) u_{ij}^\sigma.
\end{aligned}$$

So M_φ is a Fourier multiplier on $L_p(G,E)$. $\square$

The following theorem is the compact group analogue of Proposition 3.1

Theorem 5.2 *Let G be a compact group with unitary dual Σ, $\varphi:\Sigma\to\mathbb{C}$ a function and $C\geq 0$ a constant. The following assertions are equivalent:*

i) The multiplier φ is completely bounded on $L_p(G)$ with

$$\|M_\varphi\|_{cb(L_p(G))}\leq C.$$

ii) The multiplier φ is bounded on $L_p(G,S_p)$ with

$$\|M_\varphi\|_{B(L_p(G,S_p))}\leq C.$$

iii) For all n and all family of coefficients with finite support $(a^\sigma_{ij})_{\sigma\in\Sigma}$, (i.e. $\{\sigma:\exists(i,j),a^\sigma_{ij}\neq 0\}$ is finite) with $a^\sigma_{ij}\in S^n_p$ $(1\leq i,j\leq d_\sigma)$, we have

$$\|\sum_{\sigma\in\Sigma}d_\sigma\sum_{i=1}^{d_\sigma}\sum_{j=1}^{d_\sigma}\varphi(\sigma)a^\sigma_{ij}u^\sigma_{ij}\|_{L_p(G,S^n_p)}\leq C\|\sum_{\sigma\in\Sigma}d_\sigma\sum_{i=1}^{d_\sigma}\sum_{j=1}^{d_\sigma}a^\sigma_{ij}u^\sigma_{ij}\|_{L_p(G,S^n_p)}.$$

Proof. Let us prove firstly that $i)$ and $ii)$ are equivalent.
$(i)\Leftrightarrow M_\varphi:L_p(G)\to L_p(G)$ is c.b on $L_p(G)$ with $\|M_\varphi\|_{cb(L_p(G),L_p(G))}\leq C$
$\Leftrightarrow M_\varphi:L_p(G,S_p)\to L_p(G,S_p)$ is bounded on $L_p(G,S_p)$ with

$$\|M_\varphi\|_{L_p(G,S_p)}=\|M_\varphi\|_{cb(L_p(G))}\leq C$$

according to Proposition 2.1.
So $(i)\Leftrightarrow M_\varphi$ is bounded on $L_p(G,S_p)$ with $\|M_\varphi\|_{B(L_p(G,S_p))}\leq C$,
hence $(i)\Leftrightarrow(ii)$.
Now, let us we prove that (ii) and (iii) are equivalent.
Assume (ii). φ being bounded on $L_p(G,S_p)$ with $\|M_\varphi\|_{B(L_p(G,S_p))}\leq C$, then $\forall n\geq 1$, φ is bounded on $L_p(G,S^n_p)$ with $\|M_\varphi\|_{B(L_p(G,S^n_p))}\leq C$, because $S_p=\cup_{n\geq 1}S^n_p$. Moreover , for $f\in L_p(G,S^n_p)$ whose corresponding family of matrix elements of Fourier coefficients $(a^\sigma_{ij})_{\sigma\in\Sigma,1\leq i,j\leq d_\sigma}$ of finite support, we have

$$f=\sum_{\sigma\in\Sigma}d_\sigma\sum_{i=1}^{d_\sigma}\sum_{j=1}^{d_\sigma}a^\sigma_{ij}u^\sigma_{ij}.$$

So

$$\|M_\varphi f\|_{L_p(G,S^n_p)}=\|\sum_{\sigma\in\Sigma}d_\sigma\sum_{i=1}^{d_\sigma}\sum_{j=1}^{d_\sigma}\varphi(\sigma)a^\sigma_{ij}u^\sigma_{ij}\|_{L_p(G,S^n_p)}.$$

Since M_φ is bounded we have also

$$\|M_\varphi f\|_{L_p(G,S_p^n)} \le \|M_\varphi\|_{B(L_p(G,S_p^n))}\|f\|_{L_p(G,S_p^n)}.$$

Thus $\|\sum_{\sigma\in\Sigma} d_\sigma \sum_{i=1}^{d_\sigma}\sum_{j=1}^{d_\sigma}\varphi(\sigma)a_{ij}^\sigma u_{ij}^\sigma\|_{L_p(G,S_p^n)} \le C\|\sum_{\sigma\in\Sigma} d_\sigma \sum_{i=1}^{d_\sigma}\sum_{j=1}^{d_\sigma} a_{ij}^\sigma u_{ij}^\sigma\|_{L_p(G,S_p^n)}$,
so $(ii) \Rightarrow (iii)$.
Reciprocally, let $(\widehat{f}(\sigma))_{\sigma\in\Sigma}$ be the family of continuous sesquilinear mappings from $H_\sigma \times H_\sigma$ into S_p^n with finite support, corresponding to the Fourier transform of a function f in $L_p(G,S_p^n)$, and we have according to (iii): $\forall f \in L_p(G,S_p^n)$

$$\|\sum_{\sigma\in\Sigma} d_\sigma \sum_{i=1}^{d_\sigma}\sum_{j=1}^{d_\sigma}\varphi(\sigma)\widehat{f}(\sigma)(\xi_j^\sigma,\xi_i^\sigma)u_{ij}^\sigma\|_{L_p(G,S_p^n)} \le C\|\sum_{\sigma\in\Sigma} d_\sigma \sum_{i=1}^{d_\sigma}\sum_{j=1}^{d_\sigma}\widehat{f}(\sigma)(\xi_j^\sigma,\xi_i^\sigma)u_{ij}^\sigma\|_{L_p(G,S_p^n)},$$

hence $\|M_\varphi f\|_{L_p(G,S_p^n)} \le C\|f\|_{L_p(G,S_p^n)}$,
thus M_φ is bounded on $L_p(G,S_p^n)$ and $\|M_\varphi\|_{B(L_p(G,S_p^n)} \le C$. Thus $(ii) \Rightarrow (iii)$. □

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