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A Ninth-Order Iterative Method Free from Second Derivative for Solving Nonlinear Equations

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Abstract

In this paper, we study and analyze an iterative method for solving nonlinear equations with ninth order of convergence. The new proposed method is obtained by composing an iterative method obtained in Noor et al. [9] with Newton's method and approximating the first-appeared derivative in the last step by a combination of already evaluated function values. The convergence analysis of our method is also considered in this paper. Several numerical examples are presented to illustrate the efficiency and performance of our new proposed method.

Mathematics Subject Classification: 41A25, 65H05, 65K05

Keywords: Newton's method, Iterative methods, Nonlinear equations, Efficiency index, Order of convergence

1 Introduction

In this paper, we propose a three-step iterative method with order of convergence nine for finding a simple root α of a nonlinear equation $f(x) = 0$.

Now, consider the nonlinear equation

$$f(x) = 0, \tag{1}$$

where $f : I \subseteq R \to R$ is a real and sufficiently smooth function in I, with I a real open interval. We assume that f has a simple zero at α in I, i.e., $f(\alpha) = 0$ and $f'(\alpha) \neq 0$.

It is well known that Newton's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{2}$$

is a good choice for approximating the simple zero α of f with order of convergence 2.

In recent years, several two-step, three-step and fourth-step iterative methods for solving the nonlinear equation (1) have been proposed, see [1-22]. These methods have been constructed using different techniques. Recently, Noor et al. [9] proposed several iterative methods for solving nonlinear equation (1), one of these methods is a two-step iterative method (Algorithm 2.4) in [9] free from second derivative with order of convergence six, which they called it the new two-step modified Halley's method. Per one iteration, this method requires two evaluations of the function and two evaluations of its first derivative.

In this paper, we will propose a new three-step iterative method based on composing Algorithm (2.4) in [9] with the classical Newton's method. According to the next theorem, which can be found in [18], the order of convergence of the composed method will be 12 and per one iteration this method requires three evaluations of the function and three evaluations of its first derivative with its efficiency index $12^{1/6} \approx 1.5131$,where the efficiency index of a method is defined to be $\rho^{1/\theta}$ with ρ is the order of convergence and θ is the total number of functional evaluations per one iteration.

Theorem 1.1 (18) *: Let $\phi_1(x)$ and $\phi_2(x)$ be two iterative methods with order of convergence p and q, respectively, then the order of convergence of the iterative method $\phi(x) = \phi_2(\phi_1(x))$ is pq.*

To improve the efficiency index, we approximate the first-appeared derivative in the last step by a combination of already evaluated function values. In this case, the order of convergence will be decreased to nine as we will prove in section 3, while its efficiency index will increase to become $9^{1/5} \approx 1.552$ instead of $12^{1/6} \approx 1.5131$, which will be equal to the efficiency index of (Algorithms (2.8) and (2.9)) of Noor et al. [9] and the same order of convergence.

In Section 2, the construction of our new proposed three-step iterative method will be presented. The analysis convergence of the method will be discussed in Section 3. Several numerical examples are given in Section 4 to illustrate the efficiency and the accuracy of the new proposed iterative method. Finally, some conclusions are pointed in Section 5.

2 Description of the method

To explain the procedure of constructing our new proposed three-step iterative method for solving equation (1) , consider (Algorithm (2.4)) proposed by Noor et al. in [9]:

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)},$$

$$z_n = y_n - \frac{2f(y_n)f'(y_n)}{2(f'(y_n))^2 - f(y_n)P_f(x_n, y_n)}, \tag{3}$$

where

$$P_f(x_n, y_n) = \frac{2}{(y_n - x_n)}\{2f'(y_n) + f'(x_n) - 3\frac{f(y_n) - f(x_n)}{y_n - x_n}\} \approx f''(y_n). \tag{4}$$

The new proposed three-step iterative method in this paper is obtained by composing method (3) which has order of convergence six with Newton's method

$$x_{n+1} = z_n - \frac{f(z_n)}{f'(z_n)},$$

to obtain

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$z_n = y_n - \frac{2f(y_n)f'(y_n)}{2(f'(y_n))^2 - f(y_n)P_f(x_n, y_n)},$$

$$x_{n+1} = z_n - \frac{f(z_n)}{f'(z_n)} \tag{5}$$

The order of convergence of method (5) is then 12. Per one iteration this method requires three evaluations of the function and three evaluations of its first derivative, with its efficiency index $12^{1/6} \approx 1.5131$. To improve the efficiency index, we approximate the first-appeared derivative in the last step $f'(z_n)$ by a combination of already evaluated function values using divided differences. This procedure was used by A. Cordero et al. [3].

To explain the idea, consider the Taylor polynomial of degree 2 for the function $f(z_n)$:

$$f(z_n) = f(y_n) + f'(y_n)(z_n - y_n) + \frac{f''(y_n)}{2}(z_n - y_n)^2. \tag{6}$$

This implies that

$$f'(y_n) \approx \frac{f(z_n) - f(y_n)}{z_n - y_n} - \frac{f''(y_n)}{2}(z_n - y_n) = f[z_n, y_n] - \frac{f''(y_n)}{2}(z_n - y_n). \tag{7}$$

But $f''(y_n)$ can be expressed as:

$$f''(y_n) \approx \frac{2(f[z_n, x_n] - f'(x_n))}{(z_n - x_n)}. \tag{8}$$

Also, from (6), we have

$$f'(z_n) \approx f'(y_n) + f''(y_n)(z_n - y_n). \tag{9}$$

Substitute the estimation of $f'(y_n)$ and $f''(y_n)$ into the last expression, to get

$$f'(z_n) \approx f[z_n, y_n] + f[z_n, x_n, x_n](z_n - y_n), \tag{10}$$

where $f[z_n, x_n, x_n] = \frac{f[z_n,x_n]-f'(x_n)}{(z_n-x_n)}$. Now by substituting (10) into (5), we obtain the following new proposed three-step iterative method for solving equation (1):

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$z_n = y_n - \frac{2f(y_n)f'(y_n)}{2(f'(y_n))^2 - f(y_n)P_f(x_n, y_n)}$$

$$x_{n+1} = z_n - \frac{f(z_n)}{f[z_n, y_n] + f[z_n, x_n, x_n](z_n - y_n)}. \tag{11}$$

In the next section, we will show that our new proposed method (11) has ninth order of convergence. Per one iteration, method (11) requires three evaluations of the function and two evaluations of its first derivative, so its efficiency index attains $9^{1/5} \approx 1.552$ and this is the main motivation of our paper.

3 Convergence Analysis

The convergence analysis of the new proposed three-step iterative method (11) for solving equation (1) will be established in this section.

Theorem 3.1 *: Let $\alpha \in I$ be a simple zero of a sufficiently differentiable function $f : I \subseteq R \to R$ for an open interval I. If x_0 is sufficiently close to α, then method (11) has ninth order of convergence.*

Proof: Let α be a simple zero of (1) and $x_n = \alpha + e_n$. By Taylor expansion, we have

$$f(x_n) = f'(\alpha)[e_n + c_2e_n^2 + c_3e_n^3 + c_4e_n^4 + c_5e_n^5 + c_6e_n^6 + c_7e_n^7 + c_8e_n^8 + \boldsymbol{O}(e_n^9)], \tag{12}$$

$$\begin{aligned} f'(x_n) &= f'(\alpha)[1 + 2c_2e_n + 3c_3e_n^2 + 4c_4e_n^3 + 5c_5e_n^4 + 6c_6e_n^5 + 7c_7e_n^6 + 8c_8e_n^7 \\ &\quad + \boldsymbol{O}(e_n^8)], \end{aligned} \tag{13}$$

where $c_k = \frac{f^{(k)}(\alpha)}{k!f'(\alpha)}, k = 2, 3, \ldots$.
Substituting (12) and (13) into y_n in (11), to obtain

$$\begin{aligned} y_n &= x_n - \frac{f(x_n)}{f'(x_n)} = \alpha + c_2e_n^2 + 2(c_3 - c_2^2)e_n^3 + (3c_4 - 7c_2c_3 + 4c_2^3)e_n^4 + \ldots \\ &\quad +(-444c_5c_3c_2^2 + 8c_9 - 22c_2c_8 - 54c_3^4 - 128c_2^8 + 164c_5c_2c_4 - 20c_5^2 \\ &\quad +96c_5c_3^2 + 224c_5c_2^4 - 38c_6c_4 - 112c_6c_2^3 - 32c_7c_3 + 52c_7c_2^2 + 104c_3c_4^2 \\ &\quad -240c_4^2c_2^2 + 144c_6c_2c_3 - 558c_4c_2c_3^2 + 1120c_4c_3c_2^3 - 416c_4c_2^5 + 648c_3^3c_2^2 \\ &\quad -1200c_3^2c_2^4 + 704c_3c_2^6)e_n^9 + \boldsymbol{O}(e_n^{10}). \end{aligned} \tag{14}$$

By expanding $f(y_n)$ about α, we obtain

$$\begin{aligned} f(y_n) &= f'(\alpha)[c_2e_n^2 + 2(c_3 - c_2^2)e_n^3 + (3c_4 - 7c_2c_3 + 5c_2^3)e_n^4 + \ldots \\ &\quad +(-584c_5c_3c_2^2 + 8c_9 - 22c_2c_8 - 46c_3^4 - 320c_2^8 + 188c_5c_2c_4 - 20c_5^2 \\ &\quad +96c_5c_3^2 + 380c_5c_2^4 - 38c_6c_4 - 164c_6c_2^3 - 32c_7c_3 + 64c_7c_2^2 + 104c_3c_4^2 \\ &\quad -324c_4^2c_2^2 + 164c_6c_2c_3 - 626c_4c_2c_3^2 + 1686c_4c_3c_2^3 - 808c_4c_2^5 + 774c_3^3c_2^2 \\ &\quad -1904c_3^2c_2^4 + 1424c_3c_2^6)e_n^9 + \boldsymbol{O}(e_n^{10})]. \end{aligned} \tag{15}$$

Expanding $f'(y_n)$ about α, to obtain

$$\begin{aligned} f'(y_n) &= f'(\alpha)[1 + 2c_2^2e_n^2 + (4c_2c_3 - 4c_2^3)e_n^3 + (6c_2c_4 - 11c_3c_2^2 + 8c_2^4)e_n^4 + \ldots \\ &\quad +(-28c_7c_2c_3 - 76c_6c_2c_4 + 132c_6c_3c_2^2 + 72c_5c_3c_4 - 264c_5c_2c_3^2 \\ &\quad -380c_5c_3c_2^3 + 376c_5c_4c_2^2 + 100c_3c_2c_4^2 + 1952c_4c_3c_2^4 + 16c_2c_9 \\ &\quad +104c_7c_2^3 - 44c_8c_2^2 + 60c_6c_3^2 - 224c_6c_2^4 - 40c_2c_5^2 + 408c_5c_2^5 \\ &\quad -280c_4c_3^3 - 744c_4^2c_2^3 - 1152c_4c_2^6 + 832c_3c_2^7 + 648c_2c_3^4 - 1464c_3^3c_2^3 \\ &\quad -256c_2^9 + 252c_4c_3^2c_2^2)e_n^9 + \boldsymbol{O}(e_n^{10})]. \end{aligned} \tag{16}$$

Substituting (12)-(16) into $P_f(x_n, y_n)$ in (4), to obtain

$$\begin{aligned} P_f(x_n, y_n) &= \frac{2}{(y_n - x_n)}[2f'(y_n) + f'(x_n) - 3\frac{f(y_n) - f(x_n)}{(y_n - x_n)}] \\ &= f'(\alpha)[2c_2 + (6c_2c_3 - 2c_4)e_n^2 + \ldots + (4c_2c_4 - 12c_3c_2^2 - 4c_5 \\ &\quad +12c_3^2)e_n^3 + (164c_5c_3c_2^2 + 56c_5c_2c_4 - 72c_6c_2c_3 + 284c_4c_2c_3^2 \\ &\quad +200c_4c_3c_2^3 - 4c_2c_8 - 112c_5c_3^2 - 4c_5c_2^4 + 20c_6c_4 - 12c_6c_2^3 \\ &\quad +32c_7c_3 + 8c_7c_2^2 - 20c_3c_4^2 - 208c_4^2c_2^2 - 256c_4c_2^5 - 756c_3^3c_2^2 \\ &\quad +768c_3^2c_2^4 - 192c_3c_2^6 - 12c_9 + 108c_3^4 + 8c_5^2)e_n^7 + \boldsymbol{O}(e_n^8)]. \quad (17) \end{aligned}$$

Substituting (14)-(17) into z_n in (11), to get

$$\begin{aligned} z_n &= y_n - \frac{2f(y_n)f'(y_n)}{2f'^2(y_n) - f(y_n)P_f(x_n, y_n)} \\ &= \alpha + (-c_3c_2^3 + c_4c_2^2 + c_2^5)e_n^6 + (4c_3c_2c_4 - 6c_3^2c_2^2 + 12c_3c_2^4 - 6c_4c_2^3 + 2c_5c_2^2 \\ &\quad -6c_2^6)e_n^7 + \ldots + (-62c_5c_3c_2^2 - 8c_3^4 - 56c_2^8 + 20c_5c_2c_4 + 8c_5c_3^2 + 34c_5c_2^4 \\ &\quad -12c_6c_2^3 + 4c_7c_2^2 + 12c_3c_4^2 - 50c_4^2c_2^2 + 12c_6c_2c_3 - 100c_4c_2c_3^2 + 248c_4c_3c_2^3 \\ &\quad -114c_4c_2^5 + 134c_3^3c_2^2 - 294c_3^2c_2^4 + 224c_3c_2^6)e_n^9 + \boldsymbol{O}(e_n^{10}). \quad (18) \end{aligned}$$

Now, expand $f(z_n)$ about α to get

$$\begin{aligned} f(z_n) &= f'(\alpha)[(-c_3c_2^3 + c_4c_2^2c_2^5)e_n^6 + (4c_3c_2c_4 - 6c_3^2c_2^2 + 12c_3c_2^4 - 6c_4c_2^3 + 2c_5c_2^2 - 6c_2^6)e_n^7 \\ &\quad + \ldots + (-62c_5c_3c_2^2 - 8c_3^4 - 56c_2^8 + 20c_5c_2c_4 + 8c_5c_3^2 + 34c_5c_2^4 - 12c_6c_2^3 + 4c_7c_2^2 \\ &\quad +12c_3c_4^2 - 50c_4^2c_2^2 + 12c_6c_2c_3 - 100c_4c_2c_3^2 + 248c_4c_3c_2^3 - 114c_4c_2^5 + 134c_3^3c_2^2 \\ &\quad -294c_3^2c_2^4 + 224c_3c_2^6)e_n^9 + \boldsymbol{O}(e_n^{10})]. \quad (19) \end{aligned}$$

Using (12)-(15) and (18)-(19), then the estimation of $f'(z_n)$ described in (10) can be written in terms of e_n as

$$\begin{aligned} f'(z_n) &\approx f[z_n, y_n] + f[z_n, x_n, x_n](z_n - x_n) = f'(\alpha)[1 - 2c_2c_3e_n^3 + (-4c_3^2 + 5c_3c_2^2 - 3c_2c_4)e_n^4 \\ &+ \ldots + (-132c_4c_3c_2^2 + 62c_5c_2c_3 - 12c_2^7 - 6c_2c_7 - 24c_4c_5 + 30c_2c_4^2 + 64c_4c_3^2 + 6c_4c_2^4 \\ &- 12c_5c_2^3 - 20c_6c_3 + 10c_6c_2^2 - 106c_2c_3^3 + 174c_3^2c_2^3 - 38c_3c_2^5)e_n^7 + \boldsymbol{O}(e_n^8)]. \quad (20) \end{aligned}$$

Substituting (18)-(20) into x_{n+1} in (11), to obtain

$$x_{n+1} = z_n - \frac{f(z_n)}{f[z_n, y_n] + f[z_n, x_n, x_n](z_n - x_n)} \quad (21)$$

$$= \alpha + (-2c_4c_3c_2^3 + 2c_3^2c_2^4 - 2c_3c_2^6)e_n^9 + \boldsymbol{O}(e_n^{10}). \quad (22)$$

Thus, we have

$$e_{n+1} = x_{n+1} - \alpha = (-2c_4c_3c_2^3 + 2c_3^2c_2^4 - 2c_3c_2^6)e_n^9 + \boldsymbol{O}(e_n^{10}). \quad (23)$$

Which shows that the order of convergence of our new proposed method defined in (11) is nine. This completes the proof.

4 Numerical examples

In this section, the obtained theoretical results are confirmed by numerical experiments and compared with Algorithms (2,8) and (2.9) presented recently by Noor et al. [9] whose order of convergence of these methods is nine, where Algorithm (2.8) is given as comes next:

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$z_n = y_n - \frac{2f(y_n)f'(y_n)}{2(f'(y_n))^2 - f(y_n)P_f(x_n, y_n)}$$

$$x_{n+1} = z_n - \frac{f'(x_n) + f'(y_n)}{3f'(y_n) - f'(x_n)} \frac{f(z_n)}{f'(x_n)}. \tag{24}$$

and Algorithm (2.9) is given as comes next:

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$z_n = y_n - \frac{2f(y_n)f'(y_n)}{2(f'(y_n))^2 - f(y_n)P_f(x_n, y_n)}$$

$$x_{n+1} = z_n - \frac{2(f'(x_n))^2}{(f'(x_n))^2 - 4f'(x_n)f'(y_n) + (f'(y_n))^2} \frac{f(z_n)}{f'(x_n)}. \tag{25}$$

The test functions and their roots, found up to the 28th decimal places, are as follows:

Example	the approximate zero α
$f_1(x) = x^3 + 4x^2 - 10,$	1.3652300134140968457608068289,
$f_2(x) = \sin^2 x - x^2 + 1,$	1.4044916482153412260350868178,
$f_3(x) = \cos x - x,$	.7390851332151606416553120877,
$f_4(x) = (x - 1)^3 - 1,$	2.0000000000000000000000000000,
$f_5(x) = x^3 - 10,$	2.1544346900318837217592935665,
$f_6(x) = e^{-x} + \cos x,$	1.7461395304080124176507030889,
$f_7(x) = \sin x - x/2,$	1.8954942670339809471440357381.

All computations were done using MATLAB 7.6 with 200 digit floating arithmetic (VPA=200). The following criteria

$$|x_n - x_{n-1}| < \varepsilon \quad \textit{and} \quad |f(x_n)| < \varepsilon,$$

are used for stopping computer programmes. Displayed in Table 1 are the number of iterations (IT), such that the stopping criteria satisfied, where ε is taken to be 10^{-15}, the value of $|f(x_n)|$ after the required iterations. Moreover, displayed is the distance of two consecutive approximations $\delta = |(x_n - x_{n-1})|$ and the computational order of convergence (COC) which can be approximated using the formula

$$COC \approx \frac{\ln |(x_n - x_{n-1})/(x_{n-1} - x_{n-2})|}{\ln |(x_{n-1} - x_{n-2})/(x_{n-2} - x_{n-3})|}.$$

Table 1:

	Method (24)	Method (25)	Our method (11)
$f_1(x), x_0 = 1$			
IT	3	3	3
$\lvert f(x_n)\rvert$	0.60000e-198	0.60000e-198	0.60000e-198
δ	0.18133e-054	0.45245e-049	0.38021e-059
COC	9.10	9.15	9.10
$f_2(x), x_0 = 1.3$			
IT	3	3	3
$\lvert f(x_n)\rvert$	0.10000e-198	0.10000e-198	0.10000e-198
δ	0.33790e-085	0.32919e-081	0.33243e-092
COC	9.03	9.04	9.04
$f_3(x), x_0 = 1.7$			
IT	3	3	3
$\lvert f(x_n)\rvert$	0	0	0
δ	0.69976e-72	0.46057e-71	0.60043e-71
COC	8.65	8.64	8.70
$f_4(x), x_0 = 2.5$			
IT	3	3	3
$\lvert f(x_n)\rvert$	0	0	0
δ	0.39755e-35	0.73243e-33	0.22197e-37
COC	8.68	8.60	8.72
$f_5(x), x_0 = 2$			
IT	3	3	3
$\lvert f(x_n)\rvert$	0.20000e-198	0.20000e-198	0.20000e-198
δ	0.60404e-091	0.18525e-086	0.60129e-094
COC	9.02	9.03	9.02
$f_6(x), x_0 = 2$			
IT	3	3	3
$\lvert f(x_n)\rvert$	0,10000e-199	0.10000e-199	0.10000e-199
δ	0.20203e-088	0.20922e-087	0.16976e-089
COC	9.08	9.09	9.08
$f_7(x), x_0 = 2$			
IT	3	3	3
$\lvert f(x_n)\rvert$	0.40000e-199	0.40000e-199	0.40000e-199
δ	0.41708e-097	0.84446e-095	0.19730e-104
COC	8.97	8.97	8.98

5 Conclusion

In this paper, we proposed a new three-step iterative method free from second derivative for solving nonlinear equations and we have proved that the order of convergence of the suggested method is nine with its efficiency index $9^{1/5} \approx 1.552$. Several numerical examples are presented and compared with Algorithms (2.8) and (2.9) proposed recently by Noor et. al [9] to illustrate the efficiency and accuracy of our proposed method. From Table (1), we observe that our iterative method is comparable with all the methods cited in that table.

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