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Some Properties of $\tau_1\tau_2$-s^*g Locally Closed Sets

K. Kannan and K. Chandrasekhara Rao

Department of Mathematics
Srinivasa Ramanujan Centre, SASTRA University, Kumbakonam, India
anbukkannan@rediffmail.com, k.chandrasekhara@rediffmail.com

Abstract

In this paper, we continue the study of $\tau_1\tau_2$-s^*g locally closed sets and $\tau_1\tau_2$-s^*g submaximal spaces in bitopology. In particular, it is proved that $\tau_1\tau_2$-s^*g locally closed sets are closed under finite intersections. Also some implications of $\tau_1\tau_2$-s^*g locally closed sets are given and we establish that some implications are not reversible, which are justified with suitable examples. Further some distinct notions of pairwise s^*glc-continuity are introduced and we discuss some of their consequences like the composition of two pairwise s^*glc-continuous functions and the restriction maps of pairwise s^*glc-continuity.

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1 Introduction

In recent years generalization of closed sets plays an important role in developing separation axioms in both unital and bitopological spaces. The difference of two closed subsets of an n-dimensional Euclidean space was considered by Kuratowski and Sierpinski [19] in 1921 and the implicit in their work is the notion of a locally closed subset of a topological space (X, τ). Following Bourbaki [1], Ganster and Reilly [12] introduced locally closed sets in topological spaces and studied three different notions of generalized continuity, namely, lc-continuity, lc-irresolteness and sub lc-continuity.

According to them, a subset of (X, τ) is locally closed in X if it is the intersection of an open subset of X and a closed subset of X. Also they gave the decomposition that a function between two topological spaces is continuous if and only if it sub - lc continuous and nearly continuous. Stone [27] has used the term FG for a locally closed subset.

In 1996, H. Maki, P. Sundaram and K. Balachandran [22] introduced the concept of generalized locally closed sets and obtained different notions of generalized continuities. Ganster, Arockiarani and Balachandran [11] introduced regular generalized locally closed sets and $rglc$-continuous functions and discussed some of their properties in the same year.

In the next year K. Balachandran , Y. Gnanambal and P. Sundaram introduced sg-locally closed sets, semi locally closed , locally semi closed and investigated some of their topological properties. Jin Han Park and Jin Keun Park [17] continued the study of sg-locally closed sets and $sglc$-continuity on topological spaces and gave the following graphical representation.

locally closed → sg-locally closed*
↓ ↘
locally semi closed → sg-locally closed** → sg-locally closed
↘ ↗
semi locally closed

K. Chandrasekhara Rao and K. Joseph [2] introduced the concepts of semi star generalized open sets { named omega-open sets in [28] and $g^\wedge$-open sets in [29]}and semi star generalized closed sets { named omega-closed sets in [28] and $g^\wedge$-closed sets in [29]} in topological spaces. K. Chandrasekhara Rao and K. Kannan [5, 6] introduced the concepts of semi star generalized locally closed sets, s^*g-submaximal spaces with the help of s^*g-closed sets and studied their basic properties in topological spaces.

Mean while J.C. Kelly [18] introduced the study of bitopological spaces. M. Jelic [16] introduced locally closed sets and lc-continuity in bitopological settings. K. Chandrasekhara Rao and K. Kannan [3, 4] introduced the concepts of semi star generalized closed sets in bitopological spaces.

A subfamily mX of the power set $P(X)$ of a nonempty set X is said to be a minimal structure on X if mX contains X and empty set [24, 25] and it is extended to bitopological settings also[26]. Each member of mX is said to be m-open and the complement of an m-open set is said to be m-closed. Obviously the set of s^*g-open sets is a minimal structure on X and the set of $\tau_i\tau_j$-s^*g open sets is also a minimal structure on (X, τ_1, τ_2).

K. Chandrasekhara Rao and K. Kannan [7] introduced the concepts of semi star generalized locally closed sets and s^*g-submaximal spaces with the help of s^*g-closed sets and studied their basic properties in bitopological spaces.

In this sequel it needs to be defined some distinct notions of s^*glc-continuity and some salient properties by using s^*g-locally closed sets in bitopological settings. In the next section some prerequisites and abbreviations are established.

2 Preliminaries

Let (X, τ_1, τ_2) or simply X denote a bitopological space. For any subset $A \subseteq X$, τ_i-$int(A)$ and τ_i-$cl(A)$ denote the interior and closure of a set A with respect to the topology τ_i, respectively. A^C denotes the complement of A in X unless explicitly stated.

A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-semi star generalized closed ($\tau_1\tau_2$-s^*g closed) if τ_2-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-semi open in X. A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-semi star generalized open ($\tau_1\tau_2$-s^*g open) if $X - A$ is $\tau_1\tau_2$-s^*g closed in X.

A subset A of a bitopological space (X, τ_1, τ_2) is said to be a

(a) $\tau_1\tau_2$-locally semi closed set if $A = G \cap F$ where G is τ_1-open and F is τ_2-semi closed in X.

(b) $\tau_1\tau_2$-semi locally closed set if $A = G \cap F$ where G is τ_1-semi open and F is τ_2-semi closed in X.

(c) $\tau_1\tau_2$-g locally closed set if $A = G \cap F$ where G is τ_1-g open and F is τ_2-g closed in X.

(d) $\tau_1\tau_2$-sg locally closed set if $A = G \cap F$ where G is τ_1-sg open and F is τ_2-sg closed in X.

(e) $\tau_1\tau_2$-sg locally closed* set if $A = G \cap F$ where G is τ_1-sg open and F is τ_2-closed in X.

(f) $\tau_1\tau_2$-sg locally closed** set if $A = G \cap F$ where G is τ_1-open and F is τ_2-sg closed in X.

(g) $\tau_1\tau_2$-gs locally closed set if $A = G \cap F$ where G is τ_2-gs open and F is τ_2-gs closed in X.

A map $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise lc-continuous if $f^{-1}(U)$ is $\tau_i\tau_j$-locally closed for each σ_j-open set U in Y, i, j = 1,2, $i \neq j$.

3 Semi Star Generalized Locally Closed Sets

Definition 3.1 [4] A subset A of a bitopological space (X, τ_1, τ_2) is said to be

(a) $\tau_1\tau_2$-s^*g locally closed if $A = G \cap F$ where G is a τ_1-s^*g open set and F is a τ_2-s^*g closed set in X,

(b) $\tau_1\tau_2$-s^*g locally closed* if $A = G \cap F$ where G is a τ_1-s^*g open set and F is τ_2-closed in X,

(c) $\tau_1\tau_2$-s^*g locally closed** if $A = G \cap F$ where G is τ_1-open and F is τ_2-s^*g closed in X.

Remark 3.2 The class of all $\tau_1\tau_2$-s^*g locally closed sets, $\tau_1\tau_2$-s^*g locally closed* sets, $\tau_1\tau_2$-s^*g locally closed** sets in (X, τ_1, τ_2) are denoted by $\tau_1\tau_2$-$S^*GLC(X, \tau_1, \tau_2)$, $\tau_1\tau_2$-$S^*GLC^*(X, \tau_1, \tau_2)$ and $\tau_1\tau_2$-$S^*GLC^{**}(X, \tau_1, \tau_2)$, respectively.

Remark 3.3 Since every $\tau_1\tau_2$-s^*g locally closed set is the intersection of a τ_1-s^*g open set and τ_2-s^*g closed set, we can conclude the following.

Theorem 3.4 A subset A of (X, τ_1, τ_2) is $\tau_1\tau_2$-s^*g locally closed if and only if A^C is the union of a τ_2-s^*g open set and τ_1-s^*g closed set.

Remark 3.5 Every τ_1-open set {resp. τ_2-closed set } is $\tau_1\tau_2$-s^*g open {resp. $\tau_1\tau_2$-s^*g closed }. Accordingly, we conclude the following.

Theorem 3.6 (a) Every τ_1-open set is $\tau_1\tau_2$-s^*g locally closed and every τ_2-closed set is $\tau_1\tau_2$-s^*g locally closed.

(b) Every $\tau_1\tau_2$-locally closed set is $\tau_1\tau_2$-s^*g locally closed, $\tau_1\tau_2$-s^*g locally closed* and $\tau_1\tau_2$-s^*g locally closed**.

Remark 3.7 But the converses of the assertions of above theorem are not true in general as can be seen in the following examples.

Example 3.8 (a) Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}\}$, $\tau_2 = \{\phi, X, \{a\}, \{b, c\}\}$. Then $\{b\}$ is a $\tau_1\tau_2$-s^*g locally closed set, but not τ_1-open in (X, τ_1, τ_2) and $\{c\}$ is a $\tau_1\tau_2$-s^*g locally closed set, but not τ_2-closed in (X, τ_1, τ_2).

(b) Let $X = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Then $\{a, c, d\}$ is a $\tau_1\tau_2$-s^*g locally closed set, but not $\tau_1\tau_2$-locally closed in (X, τ_1, τ_2) and $\{b, c, d\}$ is a $\tau_1\tau_2$-s^*g locally closed** set, but not $\tau_1\tau_2$-locally closed in (X, τ_1, τ_2).

(c) Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}, \{b, c\}\}$, $\tau_2 = \{\phi, X, \{a\}\}$. Then $\{c\}$ is a $\tau_1\tau_2$-s^*g locally closed set*, but not τ_1-open in (X, τ_1, τ_2) and $\{c\}$ is a $\tau_1\tau_2$-s^*g locally closed set, but not $\tau_1\tau_2$-locally closed in (X, τ_1, τ_2).

Remark 3.9 Since every $\tau_1\tau_2$-s^*g closed set is $\tau_1\tau_2$-g closed, $\tau_1\tau_2$-sg closed and $\tau_1\tau_2$-gs closed, we conclude the following.

Theorem 3.10 (a) Every $\tau_1\tau_2$-s^*g locally closed is $\tau_1\tau_2$-g locally closed.

(b) Every $\tau_1\tau_2$-s^*g locally closed is $\tau_1\tau_2$-sg locally closed.

(c) Every $\tau_1\tau_2$-s^*g locally closed is $\tau_1\tau_2$-gs locally closed.

Remark 3.11 But none of the assertions of the above theorem are reversible in general as can be seen in the following example.

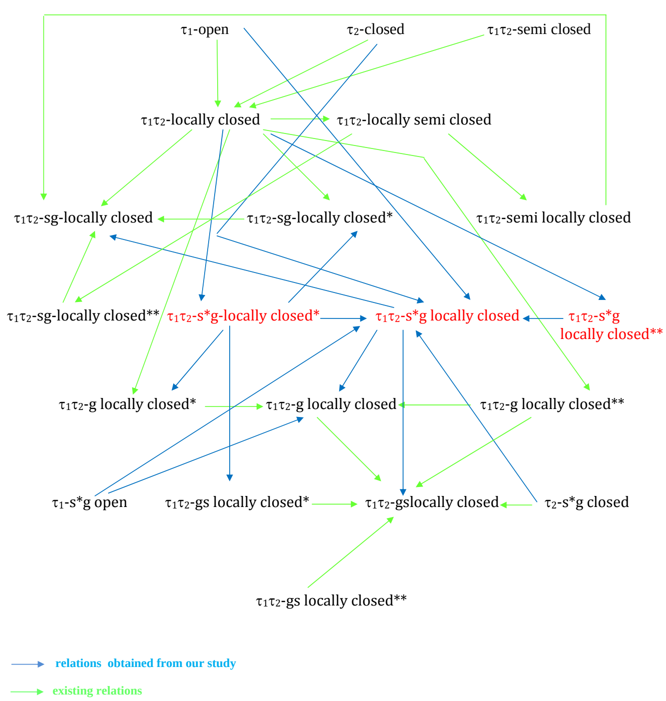
Example 3.12 In Example 3.8 (b),

(a) $\{a, c\}$ is a $\tau_1\tau_2$-g locally closed set, but not $\tau_1\tau_2$-s^*g locally closed in (X, τ_1, τ_2).

(b) $\{a, b, d\}$ is a $\tau_1\tau_2$-sg locally closed set, but not $\tau_1\tau_2$-s^*g locally closed in (X, τ_1, τ_2).

(c) $\{b, d\}$ is a $\tau_1\tau_2$-gs locally closed set, but not $\tau_1\tau_2$-s^*g locally closed in (X, τ_1, τ_2).

From the above results we conclude the following.



Since the finite intersection of τ_1-open sets is τ_1-open and the intersection of two τ_2-s^*g closed sets is τ_2-s^*g closed, we immediately get

Theorem 3.13 In any bitopological space (X, τ_1, τ_2), intersection of two $\tau_1\tau_2$-s^*g locally closed** sets is $\tau_1\tau_2$-s^*g locally closed**.

In this sequel our next result exhibits the intersection of a $\tau_1\tau_2$-s^*g locally closed set and a τ_2-closed set in a bitopological space.

Theorem 3.14 If $A \in \tau_1\tau_2$-$S^*GLC(X, \tau_1, \tau_2)$ and B is τ_2-closed in X, then $A \cap B \in \tau_1\tau_2$-$S^*GLC(X, \tau_1, \tau_2)$.

Proof It is obvious since every τ_2-closed set is τ_2-s^*g closed and the intersection of two τ_2-s^*g closed sets is τ_2-s^*g closed.

Our next result is an immediate consequence of the above theorem.

Theorem 3.15 If $A \in \tau_1\tau_2$-$S^*GLC(X, \tau_1, \tau_2)$ and B is τ_2-s^*g closed in X, then $A \cap B \in \tau_1\tau_2$-$S^*GLC(X, \tau_1, \tau_2)$.

Remark 3.16 The complement of a $\tau_1\tau_2$-s^*g locally closed set in (X, τ_1, τ_2) is not $\tau_1\tau_2$-s^*g locally closed in general and hence the finite union of $\tau_1\tau_2$-s^*g locally closed sets need not be $\tau_1\tau_2$-s^*g locally closed in (X, τ_1, τ_2). The next examples show the claim.

Example 3.17 In Example 3.8 (b), $\{d\}$ is a $\tau_1\tau_2$-s^*g locally closed set,but its complement $\{a, b, c\}$ is not $\tau_1\tau_2$-s^*g locally closed in (X, τ_1, τ_2).

Example 3.18 In Example 3.8 (b), $A = \{d\}$, $B = \{a, b\}$ are $\tau_1\tau_2$-s^*g locally closed sets, but $A \cup B = \{a, b, d\}$ is not $\tau_1\tau_2$-s^*g locally closed in (X, τ_1, τ_2).

Theorem 3.19 In a bitopological space (X, τ_1, τ_2), the following are equivalent.

(a) A is $\tau_1\tau_2$-s^*g locally closed if and only if A^C is $\tau_1\tau_2$-s^*g locally closed.

(b) $\tau_1\tau_2$-s^*g locally closed sets are closed under finite union.

Proof $(a) \Rightarrow (b)$: Suppose that A is $\tau_1\tau_2$-s^*g locally closed if and only if A^C is $\tau_1\tau_2$-s^*g locally closed. Let A, B be $\tau_1\tau_2$-s^*g locally closed. Then by our assumption, A^C, B^C are $\tau_1\tau_2$-s^*g locally closed. Consequently, $(A \cup B)^C = A^C \cap B^C$ is $\tau_1\tau_2$-s^*g locally closed. Therefore,$A \cup B$ is $\tau_1\tau_2$-s^*g locally closed.

$(b) \Rightarrow (a)$: Suppose that $\tau_1\tau_2$-s^*g locally closed sets are closed under finite union. Let A be $\tau_1\tau_2$-s^*g locally closed. Then $A = G \cap F$ where G is τ_1-s^*g open and F is τ_2-s^*g closed in X. Since G^C is τ_1-s^*g closed and F^C is τ_2-s^*g open in X and every τ_2-s^*g open is $\tau_1\tau_2$-s^*g locally closed and τ_1-s^*g closed set is $\tau_1\tau_2$-s^*g locally closed, we have A^C is $\tau_1\tau_2$-s^*g locally closed by our assumption. Similarly, we can prove if A^C is $\tau_1\tau_2$-s^*g locally closed then A is $\tau_1\tau_2$-s^*g locally closed.

4 s^*glc-Continuity

In this section, we define some distinct notions of pairwise s^*glc-continuity and study some of their consequences.

Definition 4.1 A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $\tau_1\tau_2$-s^*glc continuous {resp. $\tau_1\tau_2$-s^*glc^* continuous, $\tau_1\tau_2$-s^*glc^{**} continuous } if $f^{-1}(U)$ is $\tau_1\tau_2$-s^*g locally closed {resp. $\tau_1\tau_2$-s^*g locally closed*, $\tau_1\tau_2$-s^*g locally closed**} for each σ_1-open set U in Y.

Definition 4.2 A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $\tau_2\tau_1$-s^*glc continuous {resp. $\tau_2\tau_1$-s^*glc^* continuous, $\tau_2\tau_1$-s^*glc^{**} continuous } if $f^{-1}(U)$ is $\tau_2\tau_1$-s^*g locally closed {resp. $\tau_2\tau_1$-s^*g locally closed*, $\tau_2\tau_1$-s^*g locally closed**} for each σ_2-open set U in Y.

Definition 4.3 A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise s^*glc-continuous {resp. pairwise s^*glc^*-continuous, pairwise s^*glc^{**}-continuous} if f is both $\tau_1\tau_2$-s^*glc continuous {resp. $\tau_1\tau_2$-s^*glc^* continuous, $\tau_1\tau_2$-s^*glc^{**} continuous} and $\tau_2\tau_1$-s^*glc continuous {resp. $\tau_2\tau_1$-s^*glc^*-continuous, $\tau_2\tau_1$-s^*glc^{**} continuous}.

Example 4.4 Let $X = Y = \{a, b, c, d\}$,$\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$, $\sigma_1 = \{\phi, Y, \{a\}\}$, $\sigma_2 = \{\phi, Y, \{a, b\}, \{a, b, c\}\}$. Let f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a function defined by $f(\phi) = \phi$, $f(X) = Y$, $f(a) = \{b\}$, $f(b) = \{a\}$, $f(c) = \{d\}$, $f(d) = \{c\}$, $f(\{a, b\}) = \{a, b\}$, $f(\{a, c\}) = \{b, d\}$, $f(\{a, d\}) = \{b, c\}$, $f(\{b, c\}) = \{a, d\}$, $f(\{b, d\}) = \{a, c\}$, $f(\{c, d\}) = \{c, d\}$, $f(\{a, b, c\}) = \{a, b, d\}$,$f(\{a, b, d\}) = \{a, b, c\}$, $f(\{a, c, d\}) = \{b, c, d\}$, $f(\{b, c, d\}) = \{a, c, d\}$. Then f is both pairwise s^*glc-continuous and pairwise s^*glc^{**}-continuous.

Example 4.5 Let $X = Y = \{a, b, c, d\}$,$\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$, $\sigma_1 = \{\phi, Y, \{a\}\}$, $\sigma_2 = \{\phi, Y, \{a, b\}, \{a, b, c\}\}$. Let f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a function defined by $f(\phi) = \phi$, $f(X) = Y$, $f(a) = \{a\}$, $f(b) = \{b\}$, $f(c) = \{d\}$, $f(d) = \{c\}$, $f(\{a, b\}) = \{a, b\}$, $f(\{a, c\}) = \{b, d\}$, $f(\{a, d\}) = \{b, c\}$, $f(\{b, c\}) = \{a, d\}$, $f(\{b, d\}) = \{a, c\}$, $f(\{c, d\}) = \{c, d\}$, $f(\{a, b, c\}) = \{a, b, d\}$, $f(\{a, b, d\}) = \{a, b, c\}$, $f(\{a, c, d\}) = \{b, c, d\}$, $f(\{b, c, d\}) = \{a, c, d\}$. Then f is both pairwise s^*glc^*-continuous

Definition 4.6 A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is strongly $\tau_1\tau_2$-s^*glc continuous {resp. strongly $\tau_1\tau_2$-s^*glc^* continuous, strongly $\tau_1\tau_2$-s^*glc^{**} continuous } if $f^{-1}(U)$ is τ_1-open for each $\sigma_1\sigma_2$-s^*g locally closed set {resp. $\sigma_1\sigma_2$-s^*g locally closed*, $\sigma_1\sigma_2$-s^*g locally closed** } U in Y.

Definition 4.7 A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is strongly $\tau_2\tau_1$-s^*glc continuous {resp. strongly $\tau_2\tau_1$-s^*glc^* continuous, strongly $\tau_2\tau_1$-s^*glc^{**} continuous } if $f^{-1}(U)$ is τ_2-open for each $\sigma_2\sigma_1$-s^*g locally closed set {resp. $\sigma_2\sigma_1$-s^*g locally closed*, $\sigma_2\sigma_1$-s^*g locally closed** } U in Y.

Definition 4.8 A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is strongly pairwise s^*glc-continuous {resp. strongly pairwise s^*glc^*-continuous, strongly pairwise s^*glc^{**} continuous } if f is both strongly $\tau_1\tau_2$-s^*glc continuous {resp. strongly $\tau_1\tau_2$-s^*glc^* continuous, strongly $\tau_1\tau_2$-s^*glc^{**} continuous } and strongly $\tau_2\tau_1$-s^*glc continuous {resp. strongly $\tau_2\tau_1$-s^*glc^*-continuous, strongly $\tau_2\tau_1$-s^*glc^{**} continuous }.

Definition 4.9 A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $\tau_1\tau_2$-s^*glc irresolute {resp. $\tau_1\tau_2$-s^*glc^* irresolute, $\tau_1\tau_2$-s^*glc^{**} irresolute } if $f^{-1}(U)$ is $\tau_1\tau_2$-s^*g locally closed {resp. $\tau_1\tau_2$-s^*g locally closed*, $\tau_1\tau_2$-s^*g locally closed**} for each $\sigma_1\sigma_2$-s^*g locally closed set {resp. $\sigma_1\sigma_2$-s^*g locally closed*, $\sigma_1\sigma_2$-s^*g locally closed**} U in Y.

Definition 4.10 A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $\tau_2\tau_1$-s^*glc irresolute {resp. $\tau_2\tau_1$-s^*glc^* irresolute, $\tau_2\tau_1$-s^*glc^{**} irresolute} if $f^{-1}(U)$ is $\tau_2\tau_1$-s^*g locally closed {resp. $\tau_2\tau_1$-s^*g locally closed*, $\tau_2\tau_1$-s^*g locally closed**} for each $\sigma_2\sigma_1$-s^*g locally closed set {resp. $\sigma_2\sigma_1$-s^*g locally closed*, $\sigma_2\sigma_1$-s^*g locally closed**} U in Y.

Definition 4.11 A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise s^*glc-irresolute {resp. pairwise s^*glc^*-irresolute, pairwise s^*glc^{**} irresolute} if f is both $\tau_1\tau_2$-s^*glc irresolute {resp. $\tau_1\tau_2$-s^*glc^* irresolute, $\tau_1\tau_2$-s^*glc^{**} irresolute} and $\tau_2\tau_1$-s^*glc irresolute {resp. $\tau_2\tau_1$-s^*glc^*-irresolute, $\tau_2\tau_1$-s^*glc^{**} irresolute}.

Example 4.12 Let $X = Y = \{a, b, c, d\}$,$\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$, $\sigma_1 = \{\phi, Y, \{a\}\}$, $\sigma_2 = \{\phi, Y, \{a, b\}, \{a, b, c\}\}$. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a function defined by $f(\phi) = \phi$, $f(X) = Y$, $f(a) = \{a\}$, $f(b) = \{b\}$, $f(c) = \{d\}$, $f(d) = \{c\}$, $f(\{a, b\}) = \{a, b\}$, $f(\{a, c\}) = \{a, d\}$, $f(\{a, d\}) = \{a, c\}$, $f(\{b, c\}) = \{b, d\}$, $f(\{b, d\}) = \{b, c\}$, $f(\{c, d\}) = \{c, d\}$, $f(\{a, b, c\}) = \{a, b, d\}$,$f(\{a, b, d\}) = \{a, b, c\}$, $f(\{a, c, d\}) = \{a, c, d\}$, $f(\{b, c, d\}) = \{b, c, d\}$. Then f is pairwise s^*glc-irresolute.

Theorem 4.13 (a) Every pairwise lc-continuous function is pairwise s^*glc-continuous.

(b) Every pairwise s^*glc^*-continuous function is pairwise s^*glc-continuous.

(c) Every pairwise s^*glc^{**}-continuous function is pairwise s^*glc-continuous.

(d) Every pairwise s^*glc-irresolute function is pairwise s^*glc-continuous.

Proof (a) Suppose that $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise lc-continuous. Then $f^{-1}(U)$ is $\tau_i\tau_j$-locally closed for each σ_i-open set U in Y, $i \neq j$, $i, j = 1, 2$. Then, $f^{-1}(U)$ is $\tau_i\tau_j$-s^*g locally closed in X for each σ_i-open set U in Y, $i \neq j$, $i, j = 1, 2$. Consequently $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise s^*glc-continuous.
The proofs of (b)-(d) are similar.

Remark 4.14 But the converses of the assertions of above theorem are not true in general as can be seen in the following example.

Example 4.15 (a) In Example 4.4, f is pairwise s^*glc-continuous, but neither pairwise lc-continuous nor pairwise s^*glc^*-continuous. Also f is not pairwise s^*glc-irresolute.

(b) Let $X = Y = \{a, b, c, d\}$,$\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$, $\sigma_1 = \{\phi, Y, \{a\}\}$, $\sigma_2 = \{\phi, Y, \{a, b\}, \{a, b, c\}\}$. Let f: (X, τ_1, τ_2) $\to$ (Y, σ_1, σ_2) be a function defined by $f(\phi) = \phi$, $f(X) = Y$, $f(a) = \{a, b\}$, $f(b) = \{b\}$, $f(c) = \{c\}$, $f(d) = \{d\}$, $f(a, b) = \{a\}$, $f(a, c) = \{a, c\}$, $f(a, d) = \{a, d\}$, $f(b, c) = \{b, c\}$, $f(b, d) = \{b, d\}$, $f(c, d) = \{c, d\}$, $f(a, b, c) = \{a, b, c\}$, $f(a, b, d) = \{a, b, d\}$, $f(a, c, d) = \{a, c, d\}$, $f(b, c, d) = \{b, c, d\}$. Then f is pairwise s^*glc-continuous, but not pairwise s^*glc^{**}-continuous.

Since every $\tau_i\tau_j$-s^*g closed set is $\tau_i\tau_j$-g closed, $\tau_i\tau_j$-sg closed and $\tau_i\tau_j$-gs closed , we have every pairwise s^*glc-continuous function is pairwise glc-continuous, pairwise $sglc$-continuous and pairwise $gslc$-continuous. However the none of these implications can be reversed. The following example supports the claim.

Example 4.16 Let $X = Y = \{a, b, c, d\}$,$\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$, $\sigma_1 = \{\phi, Y, \{a\}\}$, $\sigma_2 = \{\phi, Y, \{a, b\}, \{a, b, c\}\}$. Let f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a function defined by $f(\phi) = \phi$, $f(X) = Y$, $f(a) = \{b, c\}$, $f(b) = \{b\}$, $f(c) = \{c\}$, $f(d) = \{d\}$, $f(a, b) = \{a, b\}$, $f(a, c) = \{a, c\}$, $f(a, d) = \{a, d\}$, $f(b, c) = \{a\}$, $f(b, d) = \{b, d\}$, $f(c, d) = \{c, d\}$, $f(a, b, c) = \{a, b, c\}$, $f(a, b, d) = \{a, b, d\}$, $f(a, c, d) = \{a, c, d\}$, $f(b, c, d) = \{b, c, d\}$. Then f is pairwise glc-continuous, pairwise $sglc$-continuous and pairwise gs-continuous but not pairwise s^*glc-continuous.

Concerning composition of functions , the composition of two pairwise s^*glc-irresolute functions is both pairwise s^*glc-irresolute and pairwise s^*glc-continuous. Also if a map f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise s^*glc-irresolute and $g : (Y, \sigma_1, \sigma_2) \to (Z, \mu_1, \mu_2)$ is pairwise s^*glc-continuous then the composition map gof: $(X, \tau_1, \tau_2) \to (Z, \mu_1, \mu_2)$ pairwise s^*glc-continuous.

If a map f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise s^*glc-continuous and g: $(Y, \sigma_1, \sigma_2) \to (Z, \mu_1, \mu_2)$ is pairwise strongly s^*glc-continuous then the composition map gof: $(X, \tau_1, \tau_2) \to (Z, \mu_1, \mu_2)$ pairwise s^*glc-irresolute.

But the composition of two pairwise s^*glc-continuous functions need not be pairwise s^*glc-continuous. The following example supports the claim.

Example 4.17 Let $X = Y = Z = \{a, b, c, d\}$,$\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$, $\sigma_1 = \{\phi, Y, \{a\}\}$, $\sigma_2 = \{\phi, Y, \{a, b\}, \{a, b, c\}\}$, $\mu_1 = \{\phi, Z, \{a\}\}$, $\mu_2 = \{\phi, Z, \{b, c\}\}$. Let f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a function defined by $f(\phi) = \phi$, $f(X) = Y$, $f(a) = \{b\}$, $f(b) = \{a\}$, $f(c) = \{d\}$, $f(d) = \{c\}$, $f(\{a, b\}) = \{a, b\}$, $f(\{a, c\}) = \{b, d\}$, $f(\{a, d\}) = \{b, c\}$, $f(\{b, c\}) = \{a, d\}$, $f(\{b, d\}) = \{a, c\}$, $f(\{c, d\}) = \{c, d\}$, $f(\{a, b, c\}) = \{a, b, d\}$,$f(\{a, b, d\}) = \{a, b, c\}$, $f(\{a, c, d\}) = \{b, c, d\}$, $f(\{b, c, d\}) = \{a, c, d\}$. Then f is pairwise s^*glc-continuous.

Let g: $(Y, \sigma_1, \sigma_2) \to (Z, \mu_1, \mu_2)$ be a function defined by $g(\phi) = \phi$, $g(Y) = Z$, $g(a) = \{a\}$, $g(b) = \{c\}$, $g(c) = \{b\}$, $g(d) = \{a, b\}$, $g(\{a, b\}) = \{d\}$, $g(\{a, c\}) = \{a, d\}$, $g(\{a, d\}) = \{a, c\}$, $g(\{b, c\}) = \{b, c, d\}$, $g(\{b, d\}) = \{c, d\}$, $g(\{c, d\}) = \{b, d\}$, $g(\{a, b, c\}) = \{a, b, d\}$, $g(\{a, b, d\}) = \{a, b, c\}$, $g(\{a, c, d\}) = \{a, c, d\}$, $g(\{b, c, d\}) = \{b, c\}$. Then g is pairwise s^*glc-continuous. But gof is not pairwise s^*glc-continuous.

Our next result exhibits the restriction maps on pairwise s^*glc - continuities.

Theorem 4.18 (a) Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise s^*glc^{**}-continuous and Z is a subset of X. Then the restriction map $f/_Z : (Z, \tau_{1z}, \tau_{2z}) \to (Y, \sigma_1, \sigma_2)$ is pairwise s^*glc^{**}-continuous.

(b) Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise s^*glc^*-continuous and Z is a subset of X. Then the restriction map $f/_Z : (Z, \tau_{1z}, \tau_{2z}) \to (Y, \sigma_1, \sigma_2)$ is pairwise s^*glc^*-continuous.

Proof Suppose that $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is pairwise s^*glc^{**}-continuous. Then $f^{-1}(U)$ is $\tau_i\tau_j$-s^*g locally closed** for each σ_i-open set U in Y. Consequently $f^{-1}(U)$ is the intersection of an τ_i-open set G and τ_j-s^*g closed set F in (X, τ_1, τ_2). Now, $(f/_Z)^{-1}(U) = (G \cap Z) \cap (F \cap Z)$. Since G is τ_i-open in X, $G \cap Z$ is τ_i-open in Z and since F is τ_j-s^*g closed in X, $F \cap Z$ is $\tau_j - s^*g$ closed in Z. Therefore,$f/_Z : (z, \tau_{1z}, \tau_{2z}) \to (Y, \sigma_1, \sigma_2)$ is pairwise s^*glc^{**}-continuous.
The proof of (b) is similar.

Recall that a bitopological space (X, τ_1, τ_2) is a $\tau_i\tau_j$-submaximal space {resp. $\tau_i\tau_j$-s^*g submaximal space} [5] if every τ_i-dense subset of X is τ_j-open {resp. τ_j-s^*g open} in X. Also a bitopological space (X, τ_1, τ_2) is a $\tau_1\tau_2$-s^*g submaximal space if and only if $\tau_2\tau_1$-$s^*glc^*(X, \tau_1, \tau_2) = P(X)$.

Definition 4.19 A bitopological space (X, τ_1, τ_2) is a pairwise submaximal space {resp. pairwise s^*g-submaximal space} if X is both $\tau_1\tau_2$-submaximal {resp. $\tau_1\tau_2$-s^*g submaximal} and $\tau_2\tau_1$-submaximal {resp. $\tau_2\tau_1$-s^*g submaximal}

Theorem 4.20 A bitopological space (X, τ_1, τ_2) is a pairwise s^*g-submaximal space if and only if $\tau_1\tau_2$-$s^*glc^*(X, \tau_1, \tau_2) = \tau_2\tau_1$-$s^*glc^*(X, \tau_1, \tau_2) = P(X)$.

Proof It is obvious since a bitopological space (X, τ_1, τ_2) is a $\tau_i\tau_j$-s^*g submaximal space if and only if $\tau_j\tau_i$-$S^*GLC^*(X, \tau) = P(X)$, $i \neq j$, $i, j = 1$, 2.

Theorem 4.21 A bitopological space (X, τ_1, τ_2) is $\tau_i\tau_j$-s^*g submaximal if every function having (X, τ_1, τ_2) as its domain is $\tau_j\tau_i$-s^*glc^* continuous, $i \neq j$, $i, j = 1$, 2.

Proof Necessity: Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a function and $V \in \sigma_j$. Then $f^{-1}(V) \in P(X) = \tau_j\tau_i - S^*GLC^*(X, \tau_1, \tau_2)$ since (X, τ_1, τ_2) is $\tau_i\tau_j$-s^*g submaximal, $i \neq j$, $i, j = 1$, 2. Therefore,every function having (X, τ_1, τ_2) as its domain is $\tau_j\tau_i$-s^*glc^*-continuous.

Sufficiency: The proof is similar.

Corollary 4.22 A bitopological space (X, τ_1, τ_2) is pairwise s^*g-submaximal if every function having (X, τ_1, τ_2) as its domain is pairwise s^*glc^*-continuous.

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