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An Application of Interval Valued Fuzzy Matrices in Medical Diagnosis

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Abstract

In this paper, we extend Sanchez's approach for medical diagnosis using the representation of an interval valued fuzzy matrix as an interval matrix of two fuzzy matrices. We introduce arithmetic mean of an interval valued fuzzy matrix as the arithmetic mean of its lower and upper limit matrices and propose a method to study Sanchez's approach of medical diagnosis through the arithmetic mean of an interval valued fuzzy matrix, which is a simpler technique than that of using intuitionistic fuzzy sets available in the literature.

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1 Introduction

The concept of Interval valued fuzzy matrix (IVFM) is one of the recent topics developed for dealing with the uncertainties present in most of our real life situations. The parameterization tool of Interval valued fuzzy matrix enhances the flexibility of its applications. Most of our real life problems in medical sciences, engineering, management environment and social sciences often involve data which are not necessarily crisp, precise and deterministic in character due to various uncertainties associated with these problems. Such uncertainties are usually being handled with the help of the topics like probability, fuzzy sets, intuitionistic fuzzy sets, interval mathematics and rough sets etc. The concept of IVFM as a generalization of fuzzy matrix was introduced and developed by Shyamal and Pal [8], by extending the max. min operations on Fuzzy algebra

$\mathcal{F}$ =[0,1], for elements a, b $\in \mathcal{F}$, a + b = max { a, b} and a . b = min {a, b}. Let $\mathcal{F}_{mn}$be the set of all m n Fuzzy Matrices over the Fuzzy algebra with support [0, 1], that is matrices whose entries are intervals and all the intervals are subintervals of the interval [0, 1]. De et.al.[2] have studied Sanchez's [5, 6] method of medical diagnosis using intuitionistic fuzzy set. Saikia et.al.[7] have extended the method in[2] using intuitionistic fuzzy soft set theory. In [1], Chetia and Das have studied Sanchez's approach of medical diagnosis through IVFSS obtaining an improvement of the same presented in De et.al. [2 and 7]. In our earlier work [3], we have represented an IVFM $A = (a_{ij}) = ([a_{ijL}, a_{ijU}])$ where each a_{ij} is a subinterval of the interval [0, 1], as the Interval matrix $A = [A_L, A_U]$ whose ij^{th} entry is the interval $[a_{ijL}, a_{ijU}]$, where the lower limit $A_L = (a_{ijL})$ and the upper limit $A_U = (a_{ijU})$ are fuzzy matrices such that $A_L \leq A_U$. By using this representation we have discussed the consistency of Interval valued fuzzy relational equations in [4].

In this paper, by using the representation of interval valued fuzzy matrix, we provide the techniques to study Sanchez's approach of medical diagnosis of Interval valued fuzzy matrices. We have compared our technique with the One found in [1], for the same hypothetical case study presented in [1] and have exhibited that our technique is much simpler in the computation of matrices involved. In section 2, we present the basic definition and notations of an IVFM. In section 3, by using the representation of an interval valued fuzzy matrix as an interval matrix of two fuzzy matrices, we propose a method based on Sanchez's approach to study medical diagnosis. In our method the matrix operations involved are max.min, which is uniform and simpler than that are found in [1, 2 and 7] where intuitionistic fuzzy matrix operations (ie, max-min, min-max) are involved in the computation of medical knowledge. In section 4, we have introduced the arithmetic mean (am) matrix of an IVFM A as the average of the lower and upper limit matrices A_L and A_U and directly apply Sanchez's method of medical diagnosis for the am(A),which is a fuzzy matrix.

2 Preliminaries

In this section, some basic definitions and notations are given. Let IVFM denote the set of all interval valued fuzzy matrices, that is, fuzzy matrices whose entries are all subintervals of the interval [0, 1].

Definition 2.1.

For a pair of fuzzy matrices E= (e_{ij}) and F= (f_{ij}) in $\mathcal{F}_{mn}$ such that E $\leq$ F, let us define the interval matrix denoted as [E,F], whose ij^{th} entry is the interval with lower limit e_{ij} and upper limit f_{ij} ,that is,[e_{ij} , f_{ij}].

In particular for E = F, IVFM [E, E] reduces to the fuzzy matrix E $\in \mathcal{F}_{mn}$. For $A = (a_{ij}) = ([a_{ijL}, a_{ijU}]) \in$(IVFM)$_{m\times n}$, let us define $A_L = (a_{ijL})$ and $A_U = (a_{ijU})$ clearly A_L and A_U belong to $\mathcal{F}_{mn}$ such that $A_L \leq A_U$. Therefore A can be written as

$$A = [A_L, A_U] \tag{2.1}$$

where A_L and A_U are called the lower and upper limits of A respectively.

Here we shall follow the basic operation on IVFM as given in [7].

For $A = (a_{ij}) = ([a_{ijL}, a_{ijU}])$ and $B = (b_{ij}) = ([b_{ijL}, b_{ijU}])$ of order $m \times n$ their sum denoted as $A + B$ is defined as $A + B = (a_{ij} + b_{ij}) = [(a_{ijL} + b_{ijL}), (a_{ijU} + b_{ijU})]$, then addition is

$$A + B = max\{a_{ij}, b_{ij}\} = [max\{a_{ijL}, b_{ijL}\}, max\{a_{ijU}, b_{ijU}\}] \tag{2.2}$$

For $A = (a_{ij})_{m\times n}$ and $B = (b_{ij})_{n\times p}$ their product denoted as AB is defined as

$$\begin{aligned} AB = (C_{ij}) &= [\sum_{n}^{k=1} a_{ik}b_{kj}] \quad \text{i =1,2,...m and j=1,2,...p} \\ &= [\sum_{n}^{k=1}(a_{ikL}.b_{kjL}), \sum_{n}^{k=1}(a_{ikU}b_{kjU})] \\ &= [\text{max min}(a_{ikL}.b_{kjL}), \text{max min}(a_{ikU}b_{kjU})] \end{aligned} \tag{2.3}$$

$$If A = [A_L, A_U] \ \text{and} B = [B_L, B_U] \text{ then } \quad A + B = [A_L + B_L, A_U + B_U]$$
$$AB = [A_L B_L, A_U B_U]. \tag{2.4}$$

$$A \geq B \ \text{ if and only if } a_{ijL} \geq b_{ijL} \text{and } a_{ijU} \geq b_{ijU} \ \text{ if and only } if A + B = A \tag{2.5}$$

In particular if $a_{ijL} = a_{ijU}$ and $b_{ijL} = b_{ijU}$ then (2.2) reduces to the standard max. min composition of Fuzzy Matrices.

3 Application of Interval valued fuzzy matrices in medical diagnosis

Suppose S is a set of symptoms of certain diseases, D is a set of diseases and P is a set of patients, construct an Interval valued fuzzy matrix (F, D) over

S, where F is a mapping F: D $\rightarrow \tilde{F}$(S), $\tilde{F}$(S) is a set of all interval valued fuzzy sets of S. A relation matrix say, R_1 is constructed from the interval valued fuzzy matrix (F, D) and called symptom - disease matrix. Similarly its compliment $(F, \text{D})^c$ gives another relation matrix, say R_2,called non symptom diseases matrix. Analogous to Sanchez's notion of medical knowledge, we refer to each of the matrices R_1 and R_2 as medical knowledge of an interval valued fuzzy matrix. Again we construct another interval valued fuzzy matrix (F_1, S) over P, where F_1 is a mapping given by F_1: S $\rightarrow \tilde{F}$(P). This Interval valued fuzzy matrix gives another relation matrix Q called patient-symptom matrix. Then we obtain two new relation matrices T_1 = Q.R_1 and T_2 = Q.R_2 called symptom patient matrix and non symptom patient matrix respectively.
Now,

$$T_1 = Q.R_1 \tag{3.1}$$

$$T_2 = Q.R_2 \tag{3.2}$$

Let $T_1 = [T_{1L}, T_{1U}]$, $Q = [Q_L, Q_U]$, $R_1 = [R_{1L}, R_{1U}]$, $T_2 = [T_{2L}, T_{2U}]$, and $R_2 = [R_{2L}, R_{2U}]$, be the representation of the form (2.1) for the IVFM T_1, Q, R_1, T_2 and R_2. Then by using the IVFM operation (2.4) in (3.1) and (3.2) we get,

$$T_{1L} = Q_L.R_{1L} \quad \text{and} \quad T_{1U} = Q_U.R_{1U} \tag{3.3}$$

$$T_{2L} = Q_L.R_{2L} \quad \text{and} \quad T_{2U} = Q_U.R_{2U} \tag{3.4}$$

Let us define the non-disease matrices T_{3L}, T_{3U}, T_{4L} and T_{4U} Corresponding to T_{1L},T_{1U},T_{2L} and T_{2U} respectively as

$$T_{3L} = Q_L.(J - R_{1L}) \quad \text{and} \quad T_{3U} = Q_U.(J - R_{3U}) \tag{3.5}$$

$$T_{4L} = Q_L.(J - R_{2L}) \quad \text{and} \quad T_{4U} = Q_U.(J - R_{2U}) \tag{3.6}$$

Where J is the matrix with all entries '1'. Now,

$$S_{T1L} = \max_{i,j}[T_{1L}(p_i, d_j), T_{4L}(p_i, d_j)] \quad \text{and} \quad S_{T1U} = \max_{i,j}[T_{1U}(p_i, d_j), T_{4U}(p_i, d_j)] \tag{3.7}$$

$\forall$ i = 1,2,3 and j = 1,2.

$$S_{T2L} = \max_{i,j}[T_{2L}(p_i, d_j), T_{3L}(p_i, d_j)] \text{ and } S_{T2U} = \max_{i,j}[T_{2U}(p_i, d_j), T_{3U}(p_i, d_j)] \tag{3.8}$$

$\forall$ i = 1,2,3 and j = 1,2.

We calculate the diagnosis score S_{T1} and S_{T2} for and against the diseases respectively

$$S_{T1} = \max_{i,j} [S_{T1U}(p_i, d_j), S_{T2L}(p_i, d_j)] \, \forall i = 1, 2, 3 \text{ and } j = 1, 2 \tag{3.9}$$

$$S_{T2} = \max_{i,j} [S_{T1L}(p_i, d_j), S_{T2U}(p_i, d_j)] \, \forall i = 1, 2, 3 \text{ and } j = 1, 2 \tag{3.10}$$

$$\text{Now if } \max_{j} [S_{T1}(p_i, d_j) - S_{T2}(p_i, d_j)] \tag{3.11}$$

Occurs for exactly (p_i, d_k) only, then we conclude that the acceptable diagnostic hypothesis for patient p_i is the disease d_k. In case there is a tie, the process has to be repeated for patient p_i by reassessing the symptoms.

Algorithm 3.1

(1) Input the interval valued fuzzy matrices (F, D) and $(F, D)^C$ over the set S of symptoms S, where D is the set of diseases. Also write the medical knowledge matrix R_1 and R_2 representing the relation matrices of the IVFM (F, D) and $(F, D)^c$ respectively.

(2) $R_2 = 1 - R_1 = [1 - R_{IU}, 1 - R_{IL}]$

(3) Input the IVFM (F_1, S) over the set P of patients and write its relation matrix Q.

(4) Compute the relation matrices
(i) $T_{1L} = Q.R_{IL}$ and $T_{1U} = Q_U.R_{IU}$
(ii) $T_{2L} = Q_L.R_{2L}$ and $T_{2U} = Q_U.R_{2U}$ then we get,
$T_{3L} = Q_L.(J - R_{1L})$ and $T_{3L} = Q_U.(J - R_{1U})$
$T_{4L} = Q_L.(J - R_{2L})$ and $T_{4L} = Q_U.(J - R_{2U})$

(5) Compute $S_{T1L}, S_{T1U}, S_{T2L}$ and S_{T2U}.

(6) Compute the diagnosis scores S_{T1} and S_{T2}

(7) Find $S_K = \max_j\{S_{T1}(p_i, d_j) - S_{T2}(p_i, d_j)\}$ then we conclude that the patient p_i is suffering from the disease d_k.

(8) If S_k has more than one value then go to step one and repeat the process by reassessing the symptoms for the patient

Illustration 3.2

Suppose there are three patient's p_1, p_2 and p_3 in a hospital with symptoms temperature, headache, cough and stomach problem. Let the possible diseases relating to the above symptoms be viral fever and malaria. We consider the set S $= \{e_1, e_2, e_3, e_4\}$ as universal set, where e_1 , e_2 , e_3 and e_4 represent the symptoms temperature , headache, cough and stomach problem respectively and the set D $= \{d_1, d_2\}$ where d_1 and d_2 represent the parameters viral fever and malaria respectively. Suppose that F $(d_1) = [<e_1,[.7,1]>, <e_2,[.1,.4]>, <e_3,[.5,.6]>, <e_4,[.2,.4]>]$, F $(d_2) = [<e_1,[.6,.9]>, <e_2,[.4,.6]>, <e_3,[.3,.6]>, <e_4,[.8,1]>]$. The Interval valued fuzzy matrix (F, D) is a parameterized family [F(d_1), F(d_2)] of all interval valued fuzzy matrix over the set S and are determined from expert medical documentation. Thus the fuzzy matrix (F, D) gives an approximate description of the interval valued fuzzy matrix medical knowledge of the two diseases and their symptoms. This interval valued fuzzy matrix (F, D) and its complement $(F, D)^C$ are represented by two relation matrices R_1 and R_2 called symptom - disease matrix and non symptom disease matrix respectively given by

$$R_1 = \begin{matrix} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \end{matrix} \begin{matrix} \begin{matrix} d_1 & \quad d_2 \end{matrix} \\ \begin{pmatrix} [0.7,1] & [0.6,0.9] \\ [0.1,0.4] & [0.4,0.6] \\ [0.5,0.6] & [0.3,0.6] \\ [0.2,0.4] & [0.8,0.1] \end{pmatrix} \end{matrix} \quad \text{and} \quad R_2 = \begin{matrix} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \end{matrix} \begin{matrix} \begin{matrix} d_1 & \quad d_2 \end{matrix} \\ \begin{pmatrix} [0.0,0.3] & [0.1,0.4] \\ [0.6,0.9] & [0.4,0.6] \\ [0.4,0.5] & [0.4,0.7] \\ [0.6,0.8] & [0.0,0.2]] \end{pmatrix} \end{matrix}$$

By our representation (2.1) we have,

$$R_{1L} = \begin{matrix} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \end{matrix} \begin{matrix} \begin{matrix} d_1 & d_2 \end{matrix} \\ \begin{pmatrix} .7 & .6 \\ .1 & .4 \\ .5 & .3 \\ .2 & .8 \end{pmatrix} \end{matrix} \quad \text{and} \quad R_{1U} = \begin{matrix} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \end{matrix} \begin{matrix} \begin{matrix} d_1 & d_2 \end{matrix} \\ \begin{pmatrix} 1 & .9 \\ .4 & .6 \\ .6 & .6 \\ .4 & 1 \end{pmatrix} \end{matrix}$$

$$R_{2L} = \begin{matrix} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \end{matrix} \begin{matrix} \begin{matrix} d_1 & d_2 \end{matrix} \\ \begin{pmatrix} .0 & .1 \\ .6 & .4 \\ .4 & .4 \\ .6 & .0 \end{pmatrix} \end{matrix} \quad \text{and} \quad R_{2U} = \begin{matrix} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \end{matrix} \begin{matrix} \begin{matrix} d_1 & d_2 \end{matrix} \\ \begin{pmatrix} .3 & .4 \\ .9 & .6 \\ .5 & .7 \\ .8 & .2 \end{pmatrix} \end{matrix}$$

Again we take $P = \{p_1, p_2, p_3\}$ as the universal set where p_1, p_2 and p_3 represent patients respectively and $S = \{e_1, e_2, e_3, e_4\}$ as the set of parameters suppose that,

$$F_1(e_1) = [< p_1, [.6, .9] >, < p_2, [.3, .5] >, < p_3, [.6, .8] >]$$

$F_2(e_2) = [< p_1, [.3, .5] >, < p_2, [.3, .7] >, < p_3, [.2, .6] >]$

$F_3(e_3) = [< p_1, [.8, 1] >, < p_2, [.2, .4] >, < p_3, [.5, .7] >]$ and

$F_4(e_4) = [< p_1, [.6, .9] >, < p_2, [.3, .5] >, < p_3, [.2, .5] >]$.

The Interval valued fuzzy matrix (F_1, S) is another parameterized family of all interval valued fuzzy matrices and gives a collections of approximate description of the patient-symptoms in the hospital. This interval valued fuzzy matrix (F_1, S) represents a relation matrix Q called patient-symptom matrix given by

$$Q = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \end{array} \\ \begin{pmatrix} [.6, .9] & [.3, .5] & [.8, 1] & [.6, .9] \\ [.3, .5] & [.3, .7] & [.2, .4] & [.3, .5] \\ [.6, .8] & [.2, .6] & [.5, .7] & [.2, .5] \end{pmatrix} \end{array}$$

By our representation (2.1) we have, $Q = [Q_L, Q_U]$

$$Q_L = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \end{array} \\ \begin{pmatrix} .6 & .3 & .8 & .6 \\ .3 & .3 & .2 & .3 \\ .6 & .2 & .5 & .2 \end{pmatrix} \end{array} \quad \text{and} \quad Q_U = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \end{array} \\ \begin{pmatrix} .9 & .5 & 1 & .9 \\ .5 & .7 & .4 & .5 \\ .8 & .6 & .7 & .5 \end{pmatrix} \end{array}$$

Then combining the relation matrices R_{1L}, R_{1U} and R_{2L}, R_{2U} separately with Q_L and Q_U we get the matrices $T_1 = [T_{1L}, T_{1U}]$ and $T_2 = [T_{2L}, T_{2U}]$. From equations (3.3), (3.4), (3.5) and (3.6) we have,

$$T_{1L} = Q_L.R_{1L} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{c} \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{pmatrix} .6 & .6 \\ .3 & .3 \\ .6 & .6 \end{pmatrix} \end{array} \quad \text{and} \quad T_{1U} = Q_U.R_{1U} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{c} \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{pmatrix} .9 & .9 \\ .5 & .6 \\ .8 & .8 \end{pmatrix} \end{array}$$

$$T_{2L} = Q_L.R_{2L} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{c} \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{pmatrix} .6 & .4 \\ .3 & .3 \\ .4 & .4 \end{pmatrix} \end{array} \quad \text{and} \quad T_{2U} = Q_U R_{2U} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{c} \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{pmatrix} .8 & .7 \\ .7 & .6 \\ .6 & .7 \end{pmatrix} \end{array}$$

$$T_{3L} = Q_L.(J - R_{1L}) = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{c} \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{pmatrix} .6 & .7 \\ .3 & .3 \\ .5 & .5 \end{pmatrix} \end{array} \quad \text{and} \quad T_{3U} = Q_U(J - R_{1U}) = \quad d_1 \quad d_2$$

$$\begin{matrix} p_1 \\ p_2 \\ p_3 \end{matrix} \begin{pmatrix} .6 & .4 \\ .6 & .4 \\ .6 & .4 \end{pmatrix}$$

$$T_{4L} = Q_L.(J - R_{2L}) = \begin{matrix} \\ p_1 \\ p_2 \\ p_3 \end{matrix} \begin{matrix} d_1 & d_2 \\ \begin{pmatrix} .6 & .6 \\ .3 & .3 \\ .6 & .6 \end{pmatrix} \end{matrix} \quad \text{and} \quad T_{4U} = Q_U(J - R_{2U}) = \begin{matrix} & d_1 & d_2 \\ p_1 \\ p_2 \\ p_3 \end{matrix} \begin{pmatrix} .7 & .8 \\ .5 & .5 \\ .7 & .6 \end{pmatrix}$$

Now, from equations (3.7) and (3.8) we have,

$$S_{T1L} = \begin{matrix} \\ p_1 \\ p_2 \\ p_3 \end{matrix} \begin{matrix} d_1 & d_2 \\ \begin{pmatrix} .6 & .6 \\ .3 & .3 \\ .6 & .6 \end{pmatrix} \end{matrix} \quad \text{and} \quad S_{T1U} = \begin{matrix} \\ p_1 \\ p_2 \\ p_3 \end{matrix} \begin{matrix} d_1 & d_2 \\ \begin{pmatrix} .9 & .9 \\ .5 & .6 \\ .8 & .8 \end{pmatrix} \end{matrix}$$

$$S_{T2L} = \begin{matrix} \\ p_1 \\ p_2 \\ p_3 \end{matrix} \begin{matrix} d_1 & d_2 \\ \begin{pmatrix} .6 & .7 \\ .3 & .3 \\ .5 & .5 \end{pmatrix} \end{matrix} \quad \text{and} \quad S_{T1U} = \begin{matrix} \\ p_1 \\ p_2 \\ p_3 \end{matrix} \begin{matrix} d_1 & d_2 \\ \begin{pmatrix} .8 & .7 \\ .7 & .6 \\ .6 & .7 \end{pmatrix} \end{matrix}$$

We calculate the diagnosis score for and against the diseases S_{T1} and S_{T2} from equations (3.9) and (3.10) we have,

$$S_{T1} = \begin{matrix} \\ p_1 \\ p_2 \\ p_3 \end{matrix} \begin{matrix} d_1 & d_2 \\ \begin{pmatrix} .9 & .9 \\ .5 & .6 \\ .8 & .8 \end{pmatrix} \end{matrix} \quad \text{and} \quad S_{T1U} = \begin{matrix} \\ p_1 \\ p_2 \\ p_3 \end{matrix} \begin{matrix} d_1 & d_2 \\ \begin{pmatrix} .8 & .7 \\ .7 & .6 \\ .6 & .7 \end{pmatrix} \end{matrix}$$

Now, for equation (3.11) we have the difference for and against the diseases by IVFM Method

S_{T1} - S_{T2}	d_1	d_2
p_1	.1	.2
p_2	-.2	.0
p_3	.2	.1

We conclude the patient p_3 is suffering from the disease d1 and patient's p_1 and p_2 both suffering from disease d_2.

The same conclusion is available in [1], by which in matrix computations involved are not uniform.

4 Arithmetic mean method

In this section, we apply Sanchez's method of medical diagnosis for the arithmetic mean of interval valued fuzzy matrix. In this method is an attempt to improve the above section.

Definition 4.1

Let $A = [A_L, A_U]$.
Arithmetic mean of an IVFM A = arithmetic mean of A_L and A_U denoted as am(A) is defined as am$(A) = \frac{A_L + A_U}{2} = \left[\frac{a_{ijL} + a_{ijU}}{2}\right]$ is the fuzzy matrix.

The relation matrices R_1, R_2 and Q are constructed as in step 1, 2, 3 of the algorithm (3.1). By using the Definition (4.1) of the arithmetic mean of an IVFM,let as compute the am(R_1), am(R_2) and am(Q) for the matrices $R_1 = [R_{1L}, R_{2U}], R_2 = [R_{2L}, R_{2U}]$ and $Q = [Q_L, Q_U]$.
By using definition 4.1,

$$am(R_1) = \frac{R_{1L} + R_{1U}}{2} \tag{4.1}$$

$$am(R_2) = \frac{R_{2L} + R_{2U}}{2} \tag{4.2}$$

And

$$am(Q) = \frac{Q_L + Q_U}{2} \tag{4.3}$$

Then combining the relation matrices $am(R_1)$ and $am(R_2)$ separately with $am(Q)$ under the max.min composition of fuzzy matrices we get

$$T_1 = am(Q).am(R_1) \tag{4.4}$$

$$T_2 = am(Q).am(R_2) \tag{4.5}$$

$$T_3 = am(Q).(J - am(R_1)) \tag{4.6}$$

$$T_4 = am(Q).(J - am(R_2)) \tag{4.7}$$

Where J is the matrix with all entries '1'.
By using Sanchez's technique [5, 6], we calculate the diagnosis score S_{T1} and S_{T2} for and against the disease respectively.

$$S_{T1} = \max_{i,j} [T_1(p_i, d_j), T_4(p_i, d_j)] \, \forall i = 1, 2, 3 \text{ and } j = 1, 2 \tag{4.8}$$

$$S_{T2} = \max_{i,j} [T_2(p_i, d_j), T_3(p_i, d_j)] \,\forall i = 1, 2, 3 \text{ and } j = 1, 2 \tag{4.9}$$

$$\text{Now if } \max_{j} [S_{T1}(p_i, d_j) - S_{T2}(p_i, d_j)] \tag{4.10}$$

Occurs for exactly (p_i, d_i) only, then we conclude that the acceptable diagnostic hypothesis for patient p_i is the disease d_k. In case there is a tie, the process has to be repeated for patient p_i by reassessing the symptoms. Let us illustrate the am method by considering the case study in Illustration 3.2.

Example 4.2

We shall calculate the Average symptom disease matrix $am(R_1)$, average non symptom disease matrix $am(R_2)$ and average patient symptom matrix $am(Q)$ by using (4.1), (4.2) and (4.3) for the matrices R_1, R_2 and Q respectively.

$$am(R_1) = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \end{array} \begin{array}{c} \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{pmatrix} 0.85 & 0.75 \\ 0.25 & 0.5 \\ 0.55 & 0.45 \\ 0.3 & 0.45 \end{pmatrix} \end{array} \tag{4.11}$$

$$am(R_2) = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \end{array} \begin{array}{c} \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{pmatrix} 0.15 & 0.25 \\ 0.75 & 0.5 \\ 0.45 & 0.55 \\ 0.7 & 0.1 \end{pmatrix} \end{array} \tag{4.12}$$

and

$$am(Q) = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{c} \begin{array}{cccc} d_1 & d_2 & d_3 & d_4 \end{array} \\ \begin{pmatrix} 0.75 & 0.4 & 0.9 & 0.75 \\ 0.4 & 0.5 & 0.3 & 0.4 \\ 0.7 & 0.4 & 0.6 & 0.3 \end{pmatrix} \end{array} \tag{4.13}$$

Then combining the relation matrices $am(R_1)$ and $am(R_2)$ separately with $am(Q)$ we have

$$T_1 = (amQ).(amR_1) = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{c} \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{pmatrix} 0.75 & 0.75 \\ 0.4 & 0.5 \\ 0.7 & 0.7 \end{pmatrix} \end{array} \tag{4.14}$$

$$T_2 = (amQ).(J - amR_1) = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{c} \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{pmatrix} 0.7 & 0.55 \\ 0.5 & 0.5 \\ 0.7 & 0.55 \end{pmatrix} \end{array} \quad (4.15)$$

$$T_3 = (amQ).(J - amR_1) = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{c} \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{pmatrix} 0.7 & 0.55 \\ 0.5 & 0.5 \\ 0.45 & 0.55 \end{pmatrix} \end{array} \quad (4.16)$$

$$T_4 = (amQ).(J - amR_2) = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{c} \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{pmatrix} 0.75 & 0.75 \\ 0.4 & 0.5 \\ 0.7 & 0.7 \end{pmatrix} \end{array} \quad (4.17)$$

Then by equations (4.8) and (4.9) we have,

$$S_{T1} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{c} \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{pmatrix} 0.75 & 0.75 \\ 0.4 & 0.5 \\ 0.7 & 0.7 \end{pmatrix} \end{array} \quad \text{and} \quad S_{T2} = \begin{array}{c} e_1 \\ e_2 \\ e_3 \\ e_4 \end{array} \begin{array}{c} \begin{array}{cc} d_1 & d_2 \end{array} \\ \begin{pmatrix} 0.7 & 0.55 \\ 0.5 & 0.5 \\ 0.45 & 0.55 \end{pmatrix} \end{array}$$

Now we calculate from equation (4.10) we have, the difference for and against the diseases by am Method

S_{T1} - S_{T2}	d_1	d_2
p_1	0.05	0.15
p_2	-0.1	0
p_3	0.25	0.15

Now, we conclude that the patient p_3 is suffering from the disease d_1 and patient p_1 and p_2 both suffering from the disease d_2.

5. Conclusion

We have applied Sanchez's approach to study medical diagnosis by using the representation of an interval valued fuzzy matrix as an interval matrix of two fuzzy matrices. In our method the matrix operations involved are max.min, which is uniform and much simpler than that are found in [1, 2 and 7].

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