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On Indefinite Almost Paracontact Metric Manifold

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Abstract

The object of the present paper is to study various properties of fundamental 2-form of Indefinite almost paracontact metric manifold. We have also studied an affine connection satisfying various conditions.

Keywords: Indefinite almost paracontact metric manifold, ϵ-para Sasakian manifold, 2-Killing vector fields, quasi-projectively flat manifold

1 Introduction

The notion of almost paracontact manifold was introduced by Sato [4] in 1976. The structure is an analogue of the almost contact structure [3,12] and is closely related to almost product structure [in contrast to almost contact structure, which is related to almost complex structure]. An almost contact manifold is always odd dimensional but an almost paracontact manifold could be even dimensional as well. In 1969, Takahashi [13] introduced almost contact manifold equipped with associated pseudo-Riemannian metrics. In particular, he studied Sasakian manifolds equipped with an associated pseudo-Riemannian metric. These indefinite almost contact metric manifold and indefinite Sasakian manifolds are also known as ϵ- almost contact metric manifolds and ϵ- Sasakian manifolds, respectively [1,6,7]. Also, in 1989, Matsumoto [8] replaced the structure vector field ξ by $-\xi$ in an almost paracontact manifold and associated a Lorentzian metric with the resulting structure and called it a Lorentzian almost paracontact manifold. In a Lorentzian almost paracontact manifold given by Matsumoto, the semi-Riemannian metric has only index 1 and the structure vector field ξ is always lite like. Motivated by these circumstances M.M. Tripathi et. al. [9] associated a semi-Riemannian metric, not necessarily Lorentzian, with an almost paracontact structure, and they called this indefinite almost paracontact metric structure an ϵ-almost paracontact structure, where the structure vector field ξ is space like or time like according as $\epsilon = 1$ or $\epsilon = -1$. In [9] the authors studied ϵ- almost para contact manifolds, and in particular ϵ-para Sasakian manifold. They gave basic definitions and some examples of ϵ- almost paracontact manifolds and introduced the notion of an ϵ-para Sasakian structure. The basic properties, some typical, identities for curvature tensor and Ricci tensor of ϵ-para Sasakian manifold were also studied in [9]. The authors proved that if a semi-Riemannian manifold is one of flat proper recurrent or proper Ricci-Recurrent than it can not admit an ϵ- para Sasakian structure. Also, they showed that, for an ϵ- para Sasakian manifold, the conditions of being symmetric, semi-symmetric or of constant sectional curvature are all identical.

In subsequent paper, M.M. Tripathi et.al studied 3-dimensional ϵ-para Sasakian manifolds. They obtained a necessary and sufficient condition for an ϵ-para Sasakian 3-manifolds to be an indefinite space form.

The present paper is the continuation of previous studies. In section 2, we sketch the notion of ϵ- almost paracontact metric manifold, ϵ-para Sasakian manifold and its properties. In section-3, we study some properties of fundamental 2-form of ϵ-almost paracontact manifold. Section-4, deals with affine connections satisfying various properties.

2 Preliminaries

Let M be an n-dimensional almost paracontact manifold [4] equipped with an almost paracontact structure (φ, ξ, η) consisting of a tensor field of φ of type (1, 1), a vector field ξ and a 1-form η satisfying

$$\varphi^2 = I - \eta \otimes \xi, \tag{1}$$

$$\eta(\xi) = 1, \tag{2}$$

$$\varphi(\xi) = 0, \tag{3}$$

and

$$\eta \circ \varphi = 0. \tag{4}$$

Throughout this paper, we assume that $X, Y, Z, U, V, W \in \chi(M)$, where $\chi(M)$ is the Lie algebra of vector fields in M, unless specifically stated otherwise. By a semi-Riemannian metric [2] on a manifold M, we understand a non-degenerate symmetric tensor field g of type (0, 2). In particular, if its index is 1, it becomes a Lorentzian metric [5]. Let g be a semi-Riemannian metric with index $(g) = v$ in an n-dimensional almost paracontact manifold M such that

$$g(\varphi X, \varphi Y) = g(X, Y) - \epsilon\eta(X)\eta(Y), \tag{5}$$

where $\epsilon = \pm 1$. Then M is called an- almost paracontact metric manifold equipped with an ϵ-almost paracontact metric structure $(\varphi, \xi, \eta, g, \epsilon)$ [9]. In particular, if index $(g) = 1$, then an ϵ-almost paracontact metric manifold will be called a Lorentzian almost paracontact manifold. In particular, if the metric is positive definite, then an ϵ-almost paracontact metric manifold is the usual almost paracontact metric manifold [4].
The condition (5) is equivalent to

$$g(X, \varphi Y) = g(\varphi X, Y) \tag{6}$$

along with

$$g(X, \xi) = \epsilon\eta(X). \tag{7}$$

From (7), it follows that

$$g(\xi, \xi) = \epsilon. \tag{8}$$

that is, structure vector field ξ is never light like. Defining

$$\phi(X, Y) = g(X, \varphi Y), \tag{9}$$

we note that

$$\phi(X, \xi) = 0. \tag{10}$$

From (9), we also have

$$(\nabla_X \phi)(Y, Z) = g((\nabla_X \varphi)Y, Z) = (\nabla_X \phi)(Z, Y). \tag{11}$$

If on an ϵ-almost paracontact metric manifold M

$$2\phi(X, Y) = (\nabla_X \eta)(Y) + (\nabla_Y \eta)(X) \tag{12}$$

for all $X, Y \in \chi(M)$, then M is called asϵ-paracontact metric manifold [9]. An ϵ-almost paracontact metric structure $(\varphi, \xi, \eta, g, \epsilon)$ is called an ϵ-S-Paracontact metric structure if

$$\nabla\xi = \epsilon\varphi. \tag{13}$$

A manifold equipped with an ϵ-S-paracontact metric structure is called an ϵ-S-paracontact metric manifold. Equation (13) is equivalent to

$$\phi(X, Y) = g(\varphi X, Y) = \epsilon g(\nabla_X \xi, Y) = (\nabla_X \eta)(Y) \tag{14}$$

for all $X, Y \in \chi(M)$. An ϵ-almost paracontact metric structure is called an ϵ-para Sasakian structure if [9]

$$(\nabla_X \varphi)(Y) = g(\varphi X, \varphi Y)\xi - \epsilon\eta(Y)\varphi^2 X, \tag{15}$$

where $\nabla$is the Levi-Civita connection with respect to g. A manifold endowed with an ϵ-para Sasakian structure is called an ϵ-para Sasakian manifold. For $\epsilon = 1$ and g Riemannian, M is the usual para-Sasakian manifold [4]. For, $\epsilon = -1$, g Lorentzian and ξ replaced by $-\xi$, M becomes a Lorentzian Para Sasakian manifold [8].

It is known that on an ϵ-para Sasakian manifold [9]

$$R(X, Y, \xi) = \eta(X)Y - \eta(Y)X, \tag{16}$$

$$R(X, Y, Z, \xi) = -\eta(X)g(Y, Z) + \eta(Y)g(X, Z), \tag{17}$$

$$\eta(R(X, Y, Z)) = -\epsilon\eta(X)g(Y, Z) + \epsilon\eta(Y)g(Y, Z), \tag{18}$$

$$R(\xi, X, Y) = -\epsilon g(X, Y)\xi + \eta(Y)X, \tag{19}$$

and

$$S(X, \xi) = -(n-1)\eta(X), \tag{20}$$

where S denotes the Ricci tensor of the manifold.

3 Fundamental 2-form

In this section, we have studied various properties of fundamental 2-form $\phi(Y, Z)$ of indefinite almost paracontact metric manifold

Theorem 3.1. *On an indefinite almost paracontact metric manifold M, we have*

1. $(\nabla_X\eta)(\bar{Y}) = -\eta((\nabla_X\varphi)(Y))$,
2. $(\nabla_X\phi)(Y,\xi) = -\phi(Y,\nabla_X\xi)$,
3. $(\nabla_X\phi)(Y,\xi) = -\varepsilon(\nabla_X\eta)(\bar{Y})$,
4. $\phi(Y,\nabla_X\xi) = \varepsilon(\nabla_X\eta)(\bar{Y})$.

Proof: (i) From (4), we have

$$\eta(\varphi(Y)) = 0.$$

Differentiating the above equation covariantly, with respect to the Levi-Civita connection ∇, we get

$$(\nabla_X\eta)(\bar{Y}) + \eta((\nabla_X\varphi)(Y)) + \eta(\overline{\nabla_X Y}) = 0,$$

where, we have put

$$\varphi(X) = \bar{X}.$$

Using (3), (7) and (8), in above equation, we obtain

$$(\nabla_X\eta)(\bar{Y}) = -\eta((\nabla_X\varphi)(Y)). \tag{21}$$

(ii) Differentiating (10), covariantly, we obtain

$$(\nabla_X\phi)(Y,\xi) = -\phi(Y,\nabla_X\xi). \tag{22}$$

(iii) Putting $Z = X$ in (11), we get

$$(\nabla_X\phi)(Y,\xi) = g((\nabla_X\varphi)(Y),\xi),$$

which in view of (7) and (21), reduces to

$$(\nabla_X\phi)(Y,\xi) = -\varepsilon(\nabla_X\eta)(\bar{Y}). \tag{23}$$

(iv) This result follows from (ii) and (iii), i.e.,

$$\phi(Y,\nabla_X\xi) = \varepsilon(\nabla_X\eta)(\bar{Y}). \tag{24}$$

Theorem 3.2. *On an indefinite almost paracontact manifold, we have*

$$(\nabla_X\phi)(\bar{Y},Z) + (\nabla_X\phi)(Y,\bar{Z}) = -\varepsilon[(\nabla_X\eta)(Y)\eta(Z) - (\nabla_X\eta)\,(Z)\,\eta(Y)].$$

Proof : Barring Y in (9) and using (1), we obtain

$$\phi(\bar{Y}, Z) = g(\bar{Y}, \bar{Z}) = g(Y, Z) - \varepsilon\eta(Y)\eta(Z) \tag{25}$$

and also

$$\phi(Y, \bar{Z}) = \phi(\bar{Y}, Z) = g(Y, Z) - \varepsilon\eta(Y)\eta(Z). \tag{26}$$

Now, differentiating (25), covariantly, we get

$$(\nabla_X\phi)(\bar{Y}, Z) + \phi((\nabla_X\varphi)(Z), Y) = -\varepsilon[(\nabla_X\eta)(Y)\eta(Z) + \eta(Y)(\nabla_X\eta(Z)]. \tag{27}$$

Again, Differentiating (25), we get

$$(\nabla_X\phi)(Y, \bar{Z}) + \phi((\nabla_X\varphi)(Z), Y) = -\varepsilon[(\nabla_X\eta)(Y)\eta(Z\eta(Y)(\nabla_X\eta)(Z)]. \tag{28}$$

From (5), we have

$$g(\bar{Y}, \bar{Z}) = g(Y, Z) - \varepsilon\eta(Y)\eta(Z).$$

Differentiation of this equation yields

$$\begin{aligned} g((\nabla_X\varphi)(Y), \bar{Z}) + g(\varphi(\nabla_X Y), \bar{Z}) + g(\bar{Y}(\nabla_X\varphi)(Z)) + g(\bar{Y}, \varphi(\nabla_X Z)) \\ = g(\nabla_X Y, Z) + g(Y, \nabla_X Z) - \varepsilon(\nabla_X\eta)(Y)\eta(Z) - \varepsilon\eta(\nabla_X Y)\eta(Z) \\ - \varepsilon\eta(Y)(\nabla_X\eta)(Z) - \varepsilon\eta(Y)\eta(\nabla_X Z), \end{aligned}$$

which, after simplification, gives

$$g((\nabla_X\varphi)(Y), \bar{Z}) + g(\bar{Y}, (\nabla_X\varphi)(Z)) = -\varepsilon[(\nabla_X\eta)(Y)\eta(Z) + \eta(Y)(\nabla_X\eta)(Z)]. \tag{29}$$

Now, adding (27) and (28), we obtain

$$\begin{aligned} (\nabla_X\phi)(\bar{Y}, Z) + (\nabla_X\phi)(Y, \bar{Z}) + \phi((\nabla_X\varphi)(Y), Z) + \phi(Y, (\nabla_X\varphi)(Z)) \\ = -2\varepsilon[(\nabla_X\eta)(Y)\eta(Z) + (\nabla_X\eta)(Z)\eta(Y)], \end{aligned}$$

which, on using (29) and (30), gives

$$(\nabla_X\phi)(\bar{Y}, Z) + (\nabla_X\phi)(Y, \bar{Z}) = -\varepsilon[(\nabla_X\eta)(Y)\eta(Z) + (\nabla_X\eta)\,(Z)\,\eta(Y)]. \tag{30}$$

which proves the result.

Corollary 3.3. *On an indefinite almost paracontact manifold, we have*

$$(\nabla_X\phi)(\bar{Y}, \bar{Z}) + (\nabla_X\phi)(Y, Z) = -\varepsilon[\eta(Z)(\nabla_X\eta)\bar{Y} + \eta(Y)(\nabla_X\eta)(\bar{Z})].$$

Proof: Barring Z in (30) and using (1), we have

$$(\nabla_X\phi)(\bar{Y},\bar{Z}) + (\nabla_X\phi)(Y, Z - \eta(Z)\xi) = -\varepsilon[\eta(\bar{Z})(\nabla_X\eta)(\bar{Y}) + \eta(Y)(\nabla_X\eta)(\bar{Z})],$$

which after simplification, produces

$$(\nabla_X\phi)(\bar{Y},\bar{Z}) + (\nabla_X\phi)(Y, Z) = -\varepsilon[\eta(Z)(\nabla_X\eta)(\bar{Y}) + \eta(Y)(\nabla_X\eta)(\bar{Z})]. \quad (31)$$

Theorem 3.4. *On an ϵ-S-paracontact manifold, we have*

$$(\nabla_X\phi)(Y, Z) - (\nabla_Y\phi)(X, Z) = -\eta(K(X, Y, Z).$$

where K represents curvature tensor of the manifold.

Proof : On an ϵ-S-paracontact manifold, we have

$$\nabla_X\xi = \varphi X, \quad (32)$$

which produces

$$\phi(X, Y) = (\nabla_X\eta)(Y) = X(\eta(Y)) - \eta(\nabla_X Y). \quad (33)$$

Now, we have

$$\begin{aligned}(\nabla_X\phi)(Y, Z) + \phi(\nabla_X Y, Z) + \phi(Y, \nabla_X Z)\\ = X(Y(\eta(Z))) - (\nabla_X\eta)(\nabla_Y Z) - \eta(\nabla_X\nabla_Y Z).\end{aligned} \quad (34)$$

Interchanging X and Y in above, we get

$$\begin{aligned}(\nabla_Y\phi)(X, Z) + \phi(\nabla_Y X, Z) + \phi(X, \nabla_Y Z)\\ = Y(X(\eta(Z))) - (\nabla_Y\eta)(\nabla_X Z) - \eta(\nabla_Y\nabla_X Z).\end{aligned} \quad (35)$$

Subtracting (35) from (34), we get

$$\begin{aligned}(\nabla_X\phi)(Y, Z) - (\nabla_X\phi)(X, Z) + \phi(\nabla_X Y, Z) + \phi(Y, \nabla_X Z) - \phi(\nabla_Y X, Z) - \phi(X, \nabla_Y Z)\\ = X(Y(\eta(Z))) - (\nabla_X\eta)(\nabla_Y Z) - \eta(\nabla_X\nabla_Y Z)\\ - Y(X(\eta(Z))) + (\nabla_Y\eta)(\nabla_X Z) + \eta(\nabla_Y\nabla_X Z),\end{aligned} \quad (36)$$

which after simplification gives

$$(\nabla_X\phi)(Y, Z) - (\nabla_Y\phi)(X, Z) + \phi([X, Y], Z) = [X, Y](\eta(Z)) - \eta(\nabla_X\nabla_Y Z - \nabla_Y\nabla_X Z).$$

Using (33) in above equation, we get

$$(\nabla_X\phi)(Y, Z) - (\nabla_Y\phi)(X, Z) = -\eta(K(X, Y, Z)), \quad (37)$$

which proves the result.

4 Affine Connection

In this section, we study properties of some affine connections. Let us put

$$B_X Y = D_X Y + H(X, Y). \tag{38}$$

where D is Levi-Civita connection of the indefinite almost para contact manifold M^n. The torsion tensor $S(X, Y)$ of the affine connection B is given by

$$S(X, Y) = H(X, Y) - H(Y, X) \tag{39}$$

and

$$\grave{S}(X, Y, Z) = g(S(X, Y, Z)) = \grave{H}(X, Y, Z) - \grave{H}(X, Y, Z), \tag{40}$$

where $\grave{H}(X, Y, Z) = g((H, X, Y), Z)$.

Theorem 4.1. *If the affine connection B is a metric connection,i.e.,*

$$(B_X g)(Y, Z) = 0, \tag{41}$$

then
(i) $\grave{H}\,(X, Y, Z) + \grave{H}\,(X, Y, Z) = 0$,
(ii) $2g\,(B_Y Z, X) = 2g\,(D_Y Z, X) + \grave{S}\,(X, Y, Z) + \grave{S}\,(Y, Z, X) - \grave{S}\,(Z, X, Y)$.

Proof: (i) We have

$$(B_X g)\,(Y, Z) = X\,(g\,(Y, Z)) - g\,(B_X Y, Z) - g\,(Y, B_X Z)\,,$$

which, due to (38), reduces to

$$(B_X Y)\,(Y, Z) = (D_X g)\,(Y, Z) - g\,(H\,(X, Y)\,, Z) - g\,(Y, H\,(X, Z))\,.$$

using (41) and (40), we obtain

$$\grave{H}\,(X, Y, Z) + \grave{H}\,(X, Y, Z) = 0. \tag{42}$$

(ii) We have

$$\begin{aligned}\grave{S}\,(X, Y, Z) + \grave{S}\,(Y, Z, X) - \grave{S}\,(Z, X, Y) = {} & \grave{H}\,(X, Y, Z) \\ & - \grave{H}\,(Y, X, Z) + \grave{H}\,(Y, Z, X) - \grave{H}\,(Z, Y, X) \\ & + \grave{H}\,(Z, X, Y) - \grave{H}\,(X, Z, Y)\,.\end{aligned}$$

Now, using (42), we get

$$\grave{S}\,(X, Y, Z) + \grave{S}\,(Y, Z, X) - \grave{S}\,(Z, X, Y) = 2\grave{H}\,(Y, Z, X)\,.$$

This, in view of (40) and (38), reduces to

$$\grave{S}\,(X, Y, Z) + \grave{S}\,(Y, Z, X) - \grave{S}\,(Z, X, Y) = 2g\,(B_Y Z - D_Y Z, X)\,.$$

This gives

$$2g\,(B_Y Z, X) = 2g\,(D_Y Z, X) + \grave{S}\,(X, Y, Z) + \grave{S}\,(Y, Z, X) - \grave{S}\,(Z, X, Y)\,. \tag{43}$$

Theorem 4.2. *Let the connection B satisfies*
(a) $(B_X\phi)(Y,Z) = 0$ *and*
(b) $\grave{H}\left(X,Z,\bar{Y}\right) - \grave{H}\left(Y,Z,\bar{X}\right) = 0,$
then
(i) $\sigma_{(X,Y,Z)}(D_X\phi)(Y,Z) = 2H\left(X,Y,\bar{Z}\right) + 2\grave{H}\left(X,Z,\bar{Y}\right) + 2\grave{H}\left(Z,X,\bar{Y}\right),$
(ii) $(D_X\phi)(Y,Z) - (D_X\phi)(Z,X) = S(X,Y,\bar{Z}).$

Proof: (i) We have

$$(B_X\phi)(Y,Z) = X(\phi(Y,Z)) - \phi(B_XY,Z) - \phi(Y,B_XZ).$$

Using (a) and (38), in above, we get

$$(D_X\phi)(Y,Z) = \grave{H}\left(X,Y,\bar{Z}\right) + \grave{H}\left(X,Z,\bar{Y}\right). \tag{44}$$

Now writing two more equation by cyclic permutations of X, Y and Z, we have

$$(D_Y\phi)(Z,X) = \grave{H}\left(Y,Z,\bar{X}\right) + \grave{H}\left(Y,X,\bar{Z}\right) \tag{45}$$

and

$$(D_Z\phi)(X,Y) = \grave{H}\left(Z,X,\bar{Y}\right) + \grave{H}\left(Z,Y,\bar{X}\right). \tag{46}$$

Adding (44), (45) and (46) and then using (b), we get desired result.

(ii) Now subtracting (4.9) from (4.8), we get

$$\begin{aligned}(D_X\phi)(Y,Z) - (D_Y\phi)(Z,X) = {} & \grave{H}\left(X,Y\bar{Z}\right) + \grave{H}\left(X,Z,\bar{Y}\right) \\ & - \grave{H}\left(Y,Z,\bar{X}\right) - \grave{H}\left(Y,X,\bar{Z}\right),\end{aligned} \tag{47}$$

which, on using condition (b), gives

$$(D_X\phi)(Y,Z) - (D_Y\phi)(Z,X) = \grave{H}\left(X,Y,\bar{Z}\right) - \grave{H}\left(Y,X\bar{Z}\right).$$

Using (40), in above, we get

$$(D_X\phi)(Y,Z) - (D_Y\phi)(Z,X) = S(X,Y,\bar{Z}). \tag{48}$$

which is the desired result.

Theorem 4.3. *If on an ϵ-S-paracontact manifold the connection B*
(a) $(B_X\phi)(Y,Z) = 0$
and (b) $\grave{H}\left(X,Z,\bar{Y}\right) - \grave{H}\left(Y,Z,\bar{X}\right) = 0,$
then

$$(D_X\phi)(Y,Z) - (D_Y\phi)(Z,X) = S\left(X,Y\bar{Z}\right) = -\eta(K(X,Y,Z)). \tag{49}$$

Proof: Proof of this theorem follows from the theorem (3.3) and (ii) result of theorem (4.2).

Corollary 4.4. *If on an ϵ-S–paracontact manifold the connection B satisfies properties (a) and $(b)$of the theorem (4.3), then*

$$K\left(X,Y,\xi\right)=\varepsilon\overline{S\left(X,Y\right)}. \tag{50}$$

Proof: From (49), we have

$$g\left(S\left(X,Y\right),\bar{Z}\right)=-\varepsilon g\left(K\left(X,Y,Z\right),\xi\right).$$

Using (6) and skew-symmetry property of K, we have

$$g\left(S\left(\bar{X},Y\right),Z\right)=\varepsilon g\left(K\left(X,Y,\xi\right),Z\right),$$

which gives

$$K\left(X,Y,\xi\right)=\varepsilon\overline{S\left(X,Y\right)},$$

which proves result.

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