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$(1,2)^* - \pi g\alpha$-Closed Maps in Bitopological Spaces

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Abstract

In this paper,we introduce $(1,2)^* - \pi g\alpha$-closed maps from a bitopological space X to a bitopological space Y as the image of every $\tau_{1,2}$-closed set is $(1,2)^* - \pi g\alpha$-closed. Also we discuss about Almost $(1,2)^* - \pi g\alpha$-closed mappings.Further we obtain several characterizations of these classes,study their bitopological properties and investigate their relation.

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1.Preliminaries

Generalized closed mappings were introduced and studied by Malghan[9].wg-closed maps and rwg-closed maps were introduced and studied by Nagavani[10]. Regular closed maps, gpr-closed maps and rg-closed maps have been introduced and studied by Long[6], Gnanambal[4] and Arockiarani[1] respectively.A. Vadivel and K. Vairamanickam[13]were discussed rg-Closed and rg-Open Maps in Topological Spaces. Further Lellis Thivagar and Ravi.O[8] initiated the study of the notion of a (1,2)*-g-closed map, (1,2)*-sg-closed map and(1,2)*-gs-closed map in bitopological spaces.In this paper,a new class of maps called

$(1,2)^* - \pi g\alpha$-closed maps,Almost $(1,2)^* - \pi g\alpha$-closed maps have been introduced and stuided their various results.We prove that the composition of two $(1,2)^* - \pi g\alpha$-closed maps need not be $(1,2)^* - \pi g\alpha$-closed map.Also we obtain some important results in bitopological settings.

Throughout the present paper$(X, \tau_1, \tau_2), (Y, \sigma_1, \sigma_2), (Z, \eta_1, \eta_2)$briefly X,Y,Z be
bitopological spaces.

Definition 1.1:[7] A subset S of a bitopological space(X, τ_1, τ_2) is said to be $\tau_{1,2}$-open if $S = A\cup$ B where $A \in \tau_1$and $B \in \tau_2$. A subset S of X is said to be

[1] $\tau_{1,2}$-closed if the complement of S is $\tau_{1,2}$-open.

[2] $\tau_{1,2}$-clopen if S is both$\tau_{1,2}$-open and $\tau_{1,2}$-closed.

Definition 1.2:[7]Let S be a subset of the bitopological space (X, τ_1, τ_2).Then

[1] The $\tau_{1,2}$-interior of S,denoted by $\tau_{1,2}$-int(S) is defined by $\cup$ G:G $\subseteq$ S and G is $\tau_{1,2}$-open

[2] The $\tau_{1,2}$-closure of S,denoted by $\tau_{1,2}$-cl(S) is defined by $\cap$ F:S $\subseteq$ F and F is $\tau_{1,2}$-closed

Remark 1.3:[7]$\tau_{1,2}$-open sets need not form a topology.

Definition 1.4:[7]A subset A of a bitoplogical space (X, τ_1, τ_2) is called

[1] $(1,2)^*$-regular open if $A = \tau_{1,2}$-int$(\tau_{1,2} - cl(A))$.

[2] $(1,2)^* - \alpha$-open if $A \subseteq \tau_{1,2}$-int$(\tau_{1,2} - cl(\tau_{1,2} - int(A)))$.

[3] $(1,2)^*$-generalized closed (briefly $(1,2)^*$-g-closed) if $\tau_{1,2} - cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$-open in X.
The complement of the sets mentioned from (i) and (ii) are called their respective closed sets and the complements of the sets mentioned above (iii) is called the respective open set.

Definition 1.5:[2]The finite union of $(1,2)^*$-regular open sets is said to be $\tau_{1,2} - \pi$- open.The complement of $\tau_{1,2} - \pi$- open is said to be $\tau_{1,2} - \pi$- closed.

Definition 1.6:[5]A map $f : (X, \tau) \to (Y, \sigma)$ is said to be $\pi g\alpha$-closed (resp.Almost $\pi g\alpha$-closed) if for every closed set (resp.regualr closed) F of X,f(F) is $\pi g\alpha$-closed in Y.

Definition 1.7:[5]A function $f : (X, \tau) \to (Y, \sigma)$ is called π-continuous if $f^{-1}(V)$ is π-closed in X for every closed set V in Y.

Definition 1.8:[3]A space (X, τ_1, τ_2) is called $(1,2)^* - \pi g\alpha$ -$T_{1/2}$-space if every $(1,2)^* - \pi g\alpha$ -closed set is $(1,2)^* - \alpha$ -closed.

Definition 1.9:A map $f : X \to Y$ is called

[1] $(1,2)^*$-g- closed [11] if f(U) is $(1,2)^*$-g-closed set in Y for every $\tau_{1,2}$-closed set U in X.

[2] $(1,2)^*$-g-open [11] if f(U) is$(1,2)^*$-g-open set in Y for every $\tau_{1,2}$-open set U in X.

[3] $(1,2)^*$-continuous [11] if $f^{-1}(V)$ is $\tau_{1,2}$-closed in X for every $\sigma_{1,2}$-closed set V in Y.

[4] $(1,2)^* - \pi g\alpha$ -irresolute [3] if $f^{-1}(V)$is $(1,2)^* - \pi g\alpha$-closed in X,for every $(1,2)^* - \pi g\alpha$-closed set V of Y.

[5] $(1,2)^* - \pi g\alpha$ -continuous [3]if $f^{-1}(V)$ is $(1,2)^* - \pi g\alpha$-closed in X for every $\sigma_{1,2}$-closed.

Definition 1.10:[3]A function $f : X \to Y$ is said to be $M-(1,2)^*-\pi g\alpha$-closed map if the image f(A) is $(1,2)^*-\pi g\alpha$ -closed in Y for every $(1,2)^*-\pi g\alpha$-closed set A in X.
The complement of the $M-(1,2)^*-\pi g\alpha$-closed map is said to be $M-(1,2)^*-\pi g\alpha$-open map.
Definition 1.11:[2]A space$(X,\tau_1,\tau_2)$is called a $(1,2)^*$-quasi -α-normal,if for every pair of disjoint $\tau_{1,2}-\pi$-closed subsets H,K ,there exist disjoint $(1,2)^*-\alpha$-open sets U ,V of X ,H$\subseteq$ U and K$\subseteq$ V.
Definition 1.12:[5]A space (X,τ) is called a α-normal, if for every pair of disjoint closed subsets H,K there exist disjoint α-open sets U,V suchthat H$\subset$ αintU,K$\subset$ αintV and αintU $\cap$ αintV=ϕ.
Definition 1.13:[12] A function $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is said to be almost continuous if $f^{-1}(V)$ is closed in X for every regular closed in Y.
Definition 1.14:[5] A function $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is said to be almost $\pi g\alpha$-continuous if $f^{-1}(V)$ is $\pi g\alpha$-closed in X for every regular closed in Y.

2.$(1,2)^* - \pi g\alpha$-CLOSED MAPS

Definition 2.1: A map $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is said to be $(1,2)^* - \pi g\alpha$-closed if for every $\tau_{1,2}$ -closed F of X,f(F) is $(1,2)^* - \pi g\alpha$-closed in Y.

Theorem 2.2: Every $\tau_{1,2}$-closed(resp.$(1,2)^*$-g-closed)map is $(1,2)^* - \pi g\alpha$ closed .
Proof:Since every $\tau_{1,2}$-closed set (resp.$(1,2)^*$-g-closed) is $(1,2)^* - \pi g\alpha$ closed. Proof straight forward.
However the converses need not be true.
Example 2.3: Let X={a,b,c},τ_1={ϕ,X,{a},{b},{a,b}}, τ_2={ϕ,X,{b},{a,b}},

σ_1={ϕ,X,{a},{a,c}}, σ_2={ϕ,X,{a,b}}.$f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ be the identity map then f is $(1,2)^*-\pi g\alpha$ closed but not $\tau_{1,2}$-closed map, not $(1,2)^*$-g-closed map.

Proposition 2.4: If a mapping $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is $(1,2)^*-\pi g\alpha$ closed then for every subset A of X,$(1,2)^*-\pi g\alpha-cl(f(A))\subset f(\sigma_{1,2}-cl(A))$.
Proof:Let $A\subset X$.Since f is $(1,2)^*-\pi g\alpha$ closed,$f(\sigma_{1,2}-cl(A))$ is $(1,2)^*-\pi g\alpha$ closed in Y.Now $f(A)\subset f(\sigma_{1,2}-cl(A))$.Also $f(A)\subset(1,2)^*-\pi g\alpha-cl(f(A))$.By definition,we have $(1,2)^*-\pi g\alpha-cl(f(A))\subset f(\sigma_{1,2}-cl(A))$.
Converse need not be true as seen in the following example.
Example 2.5: Let X={a,b,c,d},τ_1={ϕ,X,{b,c,d}}, τ_2={ϕ,X},σ_1={ϕ,X,{b}}, σ_2={ϕ,X,{a},{a,b}}, $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ be the identity map.For every subset A of X ,$(1,2)^*-\pi g\alpha-cl(f(A))\subset f(\sigma_{1,2}-cl(A))$,but f is not $(1,2)^*-\pi g\alpha$-closed map .

Proposition 2.6: If for every subset A of X,$\tau_{1,2}-cl(\tau_{1,2}-int(\tau_{1,2}-cl(A)))\subset f(\sigma_{1,2}-cl(A))$ then a mapping $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is $(1,2)^*-\pi g\alpha$ closed.
Proof:Let A be closed in X.Since $\tau_{1,2}-cl(\tau_{1,2}-int(\tau_{1,2}-cl(A)))\subset f(\sigma_{1,2}-cl(A))\subset f(A)$.f(A)is $(1,2)^*-\alpha$ closed and hence $(1,2)^*-\pi g\alpha$ closed .
Converse of the above need not be true as seen in the following example.
Example 2.7: Let X={a,b,c},τ_1={ϕ,X,{a}},τ_2={ϕ,X,{a,c}}, σ_1={ϕ,X,{b}}, σ_2={ϕ,X }, f:$(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ be the identity map then f is $(1,2)^*-\pi g\alpha$ closed map , but $\tau_{1,2}-cl(\tau_{1,2}-int(\tau_{1,2}-cl(A)))\subset f(\sigma_{1,2}-cl(A))$ is not true.

Proposition 2.8: If a mapping $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is $(1,2)^*-\pi g\alpha$ closed and Y is $(1,2)^*-\pi g\alpha-T_{1/2}$ space then $\tau_{1,2}-cl(\tau_{1,2}-int(\tau_{1,2}-cl(A)))\subset f(\sigma_{1,2}-cl(A))$.
Proof:Let $A\subset X$.Then $\tau_{1,2}-cl(A)$ is closed in X.Since f is $(1,2)^*-\pi g\alpha$ closed,$f(\sigma_{1,2}-cl(A))$ is $(1,2)^*-\pi g\alpha$ closed in Y and so $(1,2)^*-\alpha cl(f(\tau_{1,2}-cl(A)))\subset f(\sigma_{1,2}-cl(A))$.Hence $\tau_{1,2}-cl(\tau_{1,2}-int(\tau_{1,2}-cl(A)))\subset\tau_{1,2}-cl(\tau_{1,2}-int(\tau_{1,2}-cl(f(\tau_{1,2}-cl(A)))))\subset f(\sigma_{1,2}-cl(A))$.

Theorem 2.9: A surjection $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is $(1,2)^*-\pi g\alpha$ closed iff for each subset S of Y and each $\tau_{1,2}$-open set U containing $f^{-1}(S)$ thereexist a $(1,2)^*-\pi g\alpha$ open set V of Y suchthat $S\subset V$ and $f^{-1}(V)\subset U$.
Proof:Necessity:Suppose that f is $(1,2)^*-\pi g\alpha$ closed.Let S be a subset of Y and Y be an $\tau_{1,2}$-open subset of X containing $f^{-1}(S)$.If V=Y-f(X-U),then V is a $(1,2)^*-\pi g\alpha$ open set of Y,suchthat $S\subset V$ and $f^{-1}(V)\subset U$.
Sufficiency:Let F be any $\tau_{1,2}$-closed set of X.Then $f^{-1}(Y-f(F))\subset X-F$ and X-F is $\tau_{1,2}$-open in X.Thereexists $(1,2)^*-\pi g\alpha$ open set V of Y suchthat $Y-f(F)\subset V$ and $f^{-1}(V)\subset X-F$.Therefore, $Y-V\subset f(F)\subset f(X-$

$f^{-1}(V)) \subset Y - V$.Hence we obtain $f(F) = Y - V$ and f(F) is $(1,2)^* - \pi g\alpha$ closed in Y which shows that f is $(1,2)^* - \pi g\alpha$ closed.

Theorem 2.10: If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $(1,2)^* - \pi g\alpha$ closed and A is $\tau_{1,2}$-closed subset of X then $f|A : (A) \to (Y)$ is $(1,2)^* - \pi g\alpha$ closed.
Proof:Let $B \subset A$ be $\tau_{1,2}$-closed in A.Then B is $\tau_{1,2}$-closed in X.Since f is $(1,2)^* - \pi g\alpha$ closed,f(B) is $(1,2)^* - \pi g\alpha$ closed in Y.But $f(B) = (f|A)(B)$.So $f|A$ is $(1,2)^* - \pi g\alpha$ closed.

Remark 2.11: Composition of two $(1,2)^* - \pi g\alpha$ closed maps need not be $(1,2)^* - \pi g\alpha$ closed map.

Example 2.12: Let X=Y=Z={a,b,c},τ_1={ϕ,X,{b,c}},τ_2={ϕ,X,{a}}, σ_1={ϕ,X,{a}}, σ_2={ϕ,X}, η_1={ϕ,X,{b}},η_2={ϕ,X,{a},{a,b}}, $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ be the identity maps then f and g are $(1,2)^* - \pi g\alpha$ closed maps, but $gof : (X, \tau_1, \tau_2) \to (Z, \eta_1, \eta_2)$ is not $(1,2)^* - \pi g\alpha$ closed map.

Proposition 2.13: If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $\tau_{1,2}$-closed and
$g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ be the $(1,2)^* - \pi g\alpha$ closed then $gof : (X, \tau_1, \tau_2) \to (Z, \eta_1, \eta_2)$ is $(1,2)^* - \pi g\alpha$ closed.
Proof : Let A be $\tau_{1,2}$-closed in X then $f(A)$ is $\sigma_{1,2}$-closed in Y.Since g is $(1,2)^* - \pi g\alpha$-closed,$g(f(A))$ is$(1,2)^* - \pi g\alpha$-closed in Z.Hence gof is $(1,2)^* - \pi g\alpha$-closed.

Theorem 2.14: Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ be two mappings and let $gof : (X, \tau_1, \tau_2) \to (Z, \eta_1, \eta_2)$ be $(1,2)^* - \pi g\alpha$-closed.Then

[1] If f is $(1,2)^*$-continuous and surjection then g is $(1,2)^* - \pi g\alpha$-closed.

[2] If g is $(1,2)^* - \pi g\alpha$-irresolute and injective then f is $(1,2)^* - \pi g\alpha$-closed.

Proof : (i) Let A be $\sigma_{1,2}$-closed in Y.Since f is $(1,2)^*$-continuous,$f^{-1}(A)$ is $\tau_{1,2}$-closed in X. Since gof is $(1,2)^* - \pi g\alpha$-closed.$gof(f^{-1}(A)) = g(f(f^{-1}(A))) = g(A)$ is $(1,2)^* - \pi g\alpha$-closed.
ii) Let A be $\tau_{1,2}$-closed in X.Since gof is $(1,2)^* - \pi g\alpha$-closed,(gof)(A) is $(1,2)^* - \pi g\alpha$-closed in Z.Since g is $(1,2)^* - \pi g\alpha$-continuous,$g^{-1}((gof)(A)) = f(A)$ is $(1,2)^* - \pi g\alpha$-closed in Y.Hence f is $(1,2)^* - \pi g\alpha$-closed.

Proposition 2.15: For any bijection $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ the following statements are equivalent.

[1] f is a $(1,2)^* - \pi g\alpha$-open map.

[2] f is a $(1,2)^* - \pi g\alpha$-closed map.

[3] $f^{-1} : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $(1,2)^* - \pi g\alpha$-continuous.

Proof : (i)⇒(ii)Let f be a $(1,2)^* - \pi g\alpha$-open map.Let U be $\tau_{1,2}$-closed in X.Then X-U is $\tau_{1,2}$-open in X.By assumption,$f(X-U)$ is a $(1,2)^* - \pi g\alpha$-open map and it implies Y-f(U) is $(1,2)^* - \pi g\alpha$-open map and hence f(U) is $(1,2)^* - \pi g\alpha$-closed.
(ii)⇒(iii) Let V be $\tau_{1,2}$-closed in X.By (ii)$f(V) = (f^{-1})^{-1}(V)$ is $(1,2)^* - \pi g\alpha$-closed in Y.
(iii)⇒(i) Let V be $\tau_{1,2}$-open in X.By (iii)$(f^{-1})^{-1}(V) = f(V)$ is $(1,2)^* - \pi g\alpha$-open in Y.

Definition 2.16:A map $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be $M-(1,2)^* - \pi g\alpha$-closed map if the image f(A) is $(1,2)^* - \pi g\alpha$-closed in Y for every $(1,2)^* - \pi g\alpha$-closed set A in X.

Remark 2.17: Every $M - (1,2)^* - \pi g\alpha$-closed map is $(1,2)^* - \pi g\alpha$-closed map,but the converse need not be true as seen in the following example.

Example 2.18: Let X={a,b,c},τ_1={ϕ,X,{a}}, τ_2={ϕ,X},σ_1={ϕ,X,{b}}, σ_2={ϕ,X,{a},{a,b}}. $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be the identity map then f is $(1,2)^* - \pi g\alpha$-closed map,but not $M - (1,2)^* - \pi g\alpha$-closed map

Definition 2.19: A map $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called $(1,2)^* - \pi$-continuous if $f^{-1}(V)$ is $\tau_{1,2} - \pi$-closed in X for every $\sigma_{1,2}$-closed set V in Y.

Theorem 2.20: If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a $(1,2)^* - \pi$-continuous and $M - (1,2)^* - \pi g\alpha$-closed map in X then f(A) is $(1,2)^* - \pi g\alpha$-closed in Y for every $(1,2)^* - \pi g\alpha$-closed set A of X.
Proof : Let A be any $(1,2)^* - \pi g\alpha$-closed set of X and V be any $\sigma_{1,2} - \pi$-open set of Y containing f(A).Since f is $(1,2)^* - \pi$-continuous,$f^{-1}(V)$ is $\tau_{1,2} - \pi$-open in X and $A \subset f^{-1}(V)$.Therefore $(1,2)^* - \alpha cl(A) \subset f^{-1}(V)$ and hence $f((1,2)^* - \alpha cl(A)) \subset V$.Since f is $M - (1,2)^* - \pi g\alpha$-closed,$f((1,2)^* - \alpha cl(A))$ is $(1,2)^* - \pi g\alpha$-closed in Y and hence we obtain $(1,2)^* - \alpha cl(f(A)) \subset (1,2)^* - \alpha cl(f((1,2)^* - \alpha cl(A))) \subset V$.Hence f(A) is $(1,2)^* - \pi g\alpha$-closed in Y.

Proposition 2.21: For any bijection $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ the following statements are equivalent.

[1] $f^{-1} : (Y, \sigma_1, \sigma_2) \to (X, \tau_1, \tau_2)$ is a $(1,2)^* - \pi g\alpha$-irresolute.

[2] f is a $M - (1,2)^* - \pi g\alpha$-open map.

[3] f is a $M - (1,2)^* - \pi g\alpha$-closed map.

Proof : (i)⇒(ii)Let U be a $(1,2)^* - \pi g\alpha$-open in X.By (i) $(f^{-1})^{-1}(U) = f(U)$ is $(1,2)^* - \pi g\alpha$-open in Y.Hence (ii) holds.
(ii)⇒(iii) Let V be $(1,2)^* - \pi g\alpha$-closed in X.By (ii)$f(X - V) = Y - f(V)$ is $(1,2)^* - \pi g\alpha$-open in Y.That is f(V) is $(1,2)^* - \pi g\alpha$-closed in Y and so f is a $M - (1,2)^* - \pi g\alpha$-closed map.
(iii)⇒(i) Let V be $(1,2)^* - \pi g\alpha$-closed in X.By (iii)$f(V) = (f^{-1})^{-1}(V)$ is $(1,2)^* - \pi g\alpha$-closed in Y. Hence i) holds.

3.ALMOST $(1,2)^* - \pi g\alpha$-CLOSED MAPS

Definition 3.1: A map $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be almost $(1,2)^* - \pi g\alpha$-closed if for every $(1,2)^*$ -regular closed F of X,f(F) is $(1,2)^* - \pi g\alpha$-closed in Y.
Theorem 3.2:

[1] Every almost-$\tau_{1,2}$-closed map is almost $(1,2)^* - \pi g\alpha$ closed .

[2] Every $(1,2)^* - \pi g\alpha$ closed map is almost $(1,2)^* - \pi g\alpha$ closed.

Proof:Proof straight forward.
But the converse is not true. However the converse of the above need not be true as seen in the following example.

Example 3.3: Let X={a,b,c},τ_1={ϕ,X,{a},{b},{a,b}},τ_2={ϕ,X,{a,b}}, σ_1={ϕ,X,{a},{a,b}},σ_2={ϕ,X,{a},{a,c}}, $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be the identity mapping then f is almost $(1,2)^* - \pi g\alpha$ closed set but not almost $\tau_{1,2}$-closed map.

Example 3.4: Let X={a,b,c,d},τ_1={ϕ,X,{a,b,d}}, τ_2={ϕ,X,{c},{b,d},{b,c,d}}, σ_1={ϕ,X,{a},{c,d},{a,c,d}}, σ_2={ϕ,X,{d},{a,d}},
$f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be the identity mapping.Here f is almost $(1,2)^* - \pi g\alpha$ closed set but not $(1,2)^* - \pi g\alpha$ closed.

Theorem 3.5: A surjection $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is almost-$(1,2)^* - \pi g\alpha$ closed iff for each subset S of Y and $U \in (1,2)^*$-regular open of X containing $f^{-1}(S)$ thereexist a $(1,2)^* - \pi g\alpha$ open set V of Y suchthat $S \subset V$ and $f^{-1}(V) \subset U$.
Proof:Necessity:Suppose that f is almost $(1,2)^* - \pi g\alpha$ closed.Let S be a subset of Y and $U \in (1,2)^*$-regular open of X containing $f^{-1}(S)$.If V=Y-f(X-U),then V is a $(1,2)^* - \pi g\alpha$ open set of Y,suchthat $S \subset V$ and $f^{-1}V \subset U$.
Sufficiency:Let F be any $(1,2)^*$-regular closed set of X.Then $f^{-1}(Y - f(F)) \subset$

$X-F$ and X-F is $(1,2)^*$-regular open in X.There exists $(1,2)^*-\pi g\alpha$ open set V of Y suchthat $Y-f(F)\subset V$ and $f^{-1}(V)\subset X-F$.Therefore, $Y-V\subset f(F)\subset f(X-f^{-1}(V))\subset Y-V$.Hence we obtain $f(F)=Y-V$ and f(F) is $(1,2)^*-\pi g\alpha$ closed in Y which shows that f is almost $(1,2)^*-\pi g\alpha$ closed.

Definition 3.6: A space (X,τ_1,τ_2) is called a $(1,2)^*-\alpha$-normal,if for every pair of disjoint $\tau_{1,2}$-closed subsets H,K ,there exist disjoint $\tau_{1,2}-\alpha$-open sets U,V of X, $H\subseteq(1,2)^*-\alpha intU, K\subseteq(1,2)^*-\alpha intV$ and $(1,2)^*-\alpha intU\cap(1,2)^*-\alpha intV=\phi$.

Definition 3.7: A map $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is said to be almost $(1,2)^*-\pi g\alpha$-continuous if $f^{-1}(V)$ is $(1,2)^*-\pi g\alpha$-closed in X for every $(1,2)^*$-regular closed in Y.

Definition 3.8: A map $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is said to be almost $(1,2)^*$-continuous if $f^{-1}(V)$ is $\tau_{1,2}$-closed in X for every $(1,2)^*$-regular closed in Y.

PRESERVATION THEOREMS:

Theorem 3.9: If $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is an almost-$(1,2)^*-\pi g\alpha$ continuous $\tau_{1,2}-\pi$-closed injection and Y is $(1,2)^*$-quasi-α-normal then X is $(1,2)^*$-quasi-α-normal.

Proof:Let A and B be any disjoint $\tau_{1,2}-\pi$-closed sets of X.Since f is a $\tau_{1,2}-\pi$-closed injection f(A) and f(B) are disjoint $\sigma_{1,2}-\pi$-closed sets of Y.Since Y is $(1,2)^*$-quasi-α-normal,thereexist disjoint $(1,2)^*-\alpha$-open sets U and V of Y such that $f(A)\subset U$ and $f(B)\subset V$.Now if $G=\tau_{1,2}-int(\tau_{1,2}-cl(U))$ and $H=\sigma_{1,2}-int(\sigma_{1,2}-cl(V))$.Then G and H are disjoint $(1,2)^*$-regular open sets such that $f(A)\subset G$ and $f(B)\subset H$.Since f is almost $(1,2)^*-\pi g\alpha$ continuous,$f^{-1}(G)$ and $f^{-1}(H)$are disjoint $(1,2)^*-\pi g\alpha$-open sets containing A and B which shows that X is $(1,2)^*$-quasi-α-normal.

Theorem 3.10: If $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is an $(1,2)^*-\pi$ continuous ,almost $\tau_{1,2}-\alpha$-closed surjection and X is $(1,2)^*$-quasi-α-normal then Y is $(1,2)^*$-quasi-α-normal.

Proof:Let A and B be any disjoint $\sigma_{1,2}$-closed sets of Y.Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint $\tau_{1,2}-\pi$-closed sets of X.Since X is $(1,2)^*$-quasi-α-normal,thereexist disjoint $(1,2)^*-\alpha$-open sets U and V of X such that $f^{-1}(A)\subset U$ and $f^{-1}(B)\subset V$.Let $G=\tau_{1,2}-int(\tau_{1,2}-cl(U))$ and $H=\sigma_{1,2}-int(\sigma_{1,2}-cl(V))$.Then G and H are disjoint $(1,2)^*$-regular open sets such that $f^{-1}(A)\subset G$ and $f^{-1}(B)\subset H$.Set $K=Y-f(X-G), L=Y-f(X-H)$.Then K and L are $(1,2)^*-\alpha$ open sets of Y,suchthat $A\subset K$,$B\subset L$,$f^{-1}(K)\subset G$,$f^{-1}(L)\subset H$.Since G and H are disjoint, K and L are disjoint.Since K and L are $(1,2)^*-\alpha$-open and we obtain $A\subset(1,2)^*-\alpha-intK$,$B\subset(1,2)^*-\alpha-intL$ and

$(1,2)^* - \alpha - intK \cap (1,2)^* - \alpha - intL = \phi$.Therefore Y is $(1,2)^*$-quasi-α-normal.

Corollary 3.11: If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is almost $(1,2)^*$-continuous ,almost $\tau_{1,2}$-closed surjection and X is $(1,2)^*$-normal space then Y is $(1,2)^*$-quasi-α-normal.
Proof:Since every almost $(1,2)^*$-closed map is almost $(1,2)^* - \pi g\alpha$-closed by theorem 3.9 Y is $(1,2)^*$-quasi-α-normal.

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