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Regular Generalized Closed Sets in Biminimal Structure Spaces

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Abstract

The purpose of the present paper are to introduce the concept of (i, j)-regular generalized closed sets in biminimal structure spaces and study some of their properties. We introduce the concept of (i, j)-regular generalized continuous function on biminimal structure spaces and investigate some of their characterizations.

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1 Introduction

N. Palaniappan and K. Chandrasekhara Rao [6] introduced the concept of regular generalized closed (r-g-closed) sets and study the properties of r-g-closed sets relative to union, intersection and subspaces. V. Popa and T. Noiri [5] introduced the concept of minimal structure. Also they introduced the notion of m_X-open set and m_X-closed set and characterize those sets using m_X-closure and m_X-interior operators respectively. Further they introduced M-continuous functions and studied some of its basic properties. C. Boonpok [1] introduced the concept of biminmal structure spaces and studied some fundamental properties of $m_X^1m_X^2$-closed sets and $m_X^1m_X^2$-open sets in biminimal structure spaces. Moreover, C. Boonpok [2] introduced the notion of M-continuous functions on biminmal structure spaces and studied some characterizations and several properties of such functions.

In this paper, we introduce the concept of (i,j)-regular generalized closed sets in biminimal structure spaces and study some of their properties. We introduce the concept of (i,j)-regular generalized continuous function on biminimal structure spaces and investigate some of their characterizations.

2 Preliminaries

Definition 2.1. [5] Let X be a nonempty set and $\mathcal{P}(X)$ be the power set of X. A subfamily m_X of $\mathcal{P}(X)$ is called a *minimal structure* (briefly *m-structure*) if $\emptyset \in m_X$ and $X \in m_X$.

By (X, m_X), we denote a nonempty set X with a m-structure on X and it is called a *m-space*. Each member of m_X is said to be a *m_X-open* set and the complement of a m_X-open set is said to be *m_X-closed*.

Definition 2.2. [5] Let X be a nonempty set and m_X be m-structure on X. For a subset A of X the *m_X-closure* of A and the *m_X-interior* of A are defined as follows :

(1) $m_X\text{-}Cl(A) = \cap\{F|\ A \subseteq F, X - F \in m_X\}$;

(2) $m_X\text{-}Int(A) = \cup\{U|\ U \subseteq A, U \in m_X\}$.

Lemma 2.3. [3] *Let $X \neq \emptyset$ and m_X is a m-structure on X. For $A, B \subseteq X$ the following properties hold :*

(1) $m_X\text{-}Cl(X - A) = X - (m_X\text{-}Int(A))$ and $m_X\text{-}Int(X - A) = X - (m_X\text{-}Cl(A))$.

(2) If $(X - A) \in m_X$, then $m_X\text{-}Cl(A) = A$ and if $A \in m_X$, then $m_X\text{-}Int(A) = A$.

(3) $m_X\text{-}Cl(\emptyset) = \emptyset$, $m_X\text{-}Cl(X) = X$ $m_X\text{-}Int(\emptyset) = \emptyset$, and $m_X\text{-}Int(X) = X$.

(4) If $A \subseteq B$, then $m_X\text{-}Cl(A) \subseteq m_X\text{-}Cl(B)$ and $m_X\text{-}Int(A) \subseteq m_X\text{-}Int(B)$.

(5) $A \subseteq m_X\text{-}Cl(A)$ and $m_X\text{-}Int(A) \subseteq A$.

(6) $m_X\text{-}Cl(m_X\text{-}Cl(A)) = m_X\text{-}Cl(A)$ and $m_X\text{-}Int(m_X\text{-}Int(A)) = m_X\text{-}Int(A)$.

Lemma 2.4. [3] *Let $X \neq \emptyset$ and m_X is a m-structure on X. For $A \subseteq X$ then $x \in m_X\text{-}Cl(A)$ if and only if $U \cap A \neq \emptyset$ for every $U \in m_X$ containing x.*

Definition 2.5. [3] A m-structure m_X on a nonempty set X is said to have *property $\mathcal{B}$* if the union of any family of subsets belonging to m_X belongs to m_X.

Lemma 2.6. [5] *Let X be a nonempty set and m_X is a m-structure on X sastisfying property $\mathcal{B}$. For $A \subseteq X$ the following properties hold :*

(1) $A \in m_X$ if and only if m_X-$Int(A) = A$;

(2) A is m_X-closed if and only if m_X-$Cl(A) = A$;

(3) m_X-$Int(A) \in m_X$ and m_X-$Cl(A)$ is m_X-closed.

Definition 2.7. [1] Let X be a nonempty set and let m^1_X, m^2_X be minimal structures on X. A triple (X, m^1_X, m^2_X) is called a *biminimal structure space* (briefly *bim-space*).

Throughout the present paper, (X, m^1_X, m^2_X) denote a biminimal structure space and A is a subset of X. The m_X-closure and m_X-interior of A with respect to m^i_X are denoted by m^i_X-$Cl(A)$ and m^i_X-$Int(A)$, respectively, for $i = 1, 2$.

Definition 2.8. [2] Let (X, m^1_X, m^2_X) and (Y, m^1_Y, m^2_Y) be biminimal structure spaces. A function $f : (X, m^1_X, m^2_X) \to (Y, m^1_Y, m^2_Y)$ is said to be (i, j)-M-continuous at a point $x \in X$ if for each $V \in m^i_Y$ containing $f(x)$, there exists $U \in m^i_X$ containing x such that $f(U) \subseteq V$, where $i, j = 1, 2$ and $i \neq j$.

A function $f : (X, m^1_X, m^2_X) \to (Y, m^1_Y, m^2_Y)$ is said to be (i, j)-M-continuous if it has this property at each point $x \in X$.

Theorem 2.9. [2] *For a function $f : (X, m^1_X, m^2_X) \to (Y, m^1_Y, m^2_Y)$, the following properties are equivalent :*

(1) f is (i, j)-M-continuous at a point $x \in X$;

(2) $x \in m^j_X$-$Int((f^{-1}(V))$ for every $V \in m^i_Y$ containing $f(x)$;

(3) $x \in f^{-1}(m^i_Y$-$Cl(f(A)))$ for every subset A of X with $x \in m^j_X$-$Cl(A)$;

(4) $x \in f^{-1}(m^i_Y$-$Cl(B))$ for every subset B of Y with $x \in m^j_X$-$Cl(f^{-1}(B))$;

(5) $x \in m^j_X$-$Int(f^{-1}(B))$ for every subset B of Y with $x \in f^{-1}(m^i_Y$-$Int(B))$;

(6) $x \in f^{-1}(F)$ for every m^i_Y-closed set F of Y such that $x \in m^j_X$-$Cl(f^{-1}(F))$.

Theorem 2.10. [2] *For a function $f : (X, m^1_X, m^2_X) \to (Y, m^1_Y, m^2_Y)$, the following properties are equivalent :*

(1) f is (i, j)-M-continuous;

(2) $f^{-1}(V) = m^j_X$-$Int(f^{-1}(V)$ for every $V \in m^i_Y$;

(3) $f(m^j_X$-$Cl(A)) \subseteq m^i_Y$-$Cl(f(A)))$ for every subset A of X;

(4) m^j_X-$Cl(f^{-1}(B)) \subseteq f^{-1}(m^i_Y$-$Cl(B))$ *for every subset* B *of* Y*;*

(5) $f^{-1}(m^i_Y$-$Int(B)) \subseteq m^j_X$-$Int(f^{-1}(B))$ *for every subset* B *of* Y*;*

(6) m^j_X-$Cl(f^{-1}(F)) = f^{-1}(F)$ *for every* m^i_Y*-closed set* F *of* Y*.*

Definition 2.11. [2] A subset A of biminimal structure space (X, m^1_X, m^2_X) is said to be:

(1) (i,j)-m_X*-regular open* if $A = m^i_X$-$Int(m^j_X$-$Cl(A))$, where $i, j = 1, 2$ and $i \neq j$;

(2) (i,j)-m_X*-semi-open* if $A \subseteq m^i_X$-$Cl(m^j_X$-$Int(A))$, where $i, j = 1, 2$ and $i \neq j$;

(3) (i,j)-m_X*-preopen* if $A \subseteq m^i_X$-$Int(m^j_X$-$Cl(A))$, where $i, j = 1, 2$ and $i \neq j$;

(4) (i,j)-m_X-α*-open* if $A \subseteq m^i_X$-$Int(m^j_X$-$Cl(m^i_X$-$Int(A)))$, where $i, j = 1, 2$ and $i \neq j$;

(5) (i,j)-m_X-β*-open* if $A \subseteq m^i_X$-$Cl(m^j_X$-$Int(m^i_X$-$Cl(A)))$, where $i, j = 1, 2$ and $i \neq j$.

The complement of a (i,j)-m_X-regular open (resp. (i,j)-m_X-semi open, (i,j)-m_X-preopen, (i,j)-m_X-α-open, (i,j)-m_X-β-open) set is called a (i,j)-m_X*-regular closed* (resp. (i,j)-m_X*-semi-closed*, (i,j)-m_X*-preclosed*, (i,j)-m_X-α*-closed*, (i,j)-m_X-β*-closed*) set.

Definition 2.12. [2] A subset A of a biminimal structure space (X, m^1_X, m^2_X) is said to be:

(1) m^i_X*-regular open* if $A = m^i_X$-$Int(m^i_X$-$Cl(A))$, for $i = 1, 2$;

(2) m^i_X*-semi-open* if $A \subseteq m^i_X$-$Cl(m^i_X$-$Int(A))$, for $i = 1, 2$;

(3) m^i_X*-preopen* if $A \subseteq m^i_X$-$Int(m^i_X$-$Cl(A))$, for $i = 1, 2$;

(4) m^i_X-α*-open* if $A \subseteq m^i_X$-$Int(m^i_X$-$Cl(m^i_X$-$Int(A)))$, for $i = 1, 2$;

(5) m^i_X-β*-open* if $A \subseteq m^i_X$-$Cl(m^i_X$-$Int(m^i_X$-$Cl(A)))$, for $i = 1, 2$.

The complement of a m^i_X-regular open (resp. m^i_X-semi open, m^i_X-preopen, m^i_X-α-open,m^i_X-β-open) set is called a m^i_X*-regular closed* (resp. m^i_X*-semi-closed*, m^i_X*-preclosed*, m^i_X-α*-closed*, m^i_X-β*-closed*) set.

Definition 2.13. [7] A subset A of biminimal structure space (X, m^1_X, m^2_X) is said to be $m^{(i,j)}_X$*-closed* if m^i_X-$Cl(m^j_X$-$Cl(A)) = A$, where $i, j = 1, 2$ and $i \neq j$. The complement of a $m^{(i,j)}_X$-closed set is said to be $m^{(i,j)}_X$*-open*.

Proposition 2.14. [7] *Let A be a subset of a biminimal structure space (X, m_X^1, m_X^2), where m_X^1, m_X^2 have property $\mathcal{B}$. Then A is $m_X^{(i,j)}$-closed if and only if A is both m_X^i-closed and m_X^j-closed, where $i, j = 1, 2$ and $i \neq j$.*

Definition 2.15. [7] A subset A of a biminimal structure space (X, m_X^1, m_X^2) is said to be (i, j)-*generalized m-closed* (briefly $gm_X^{(i,j)}$-*closed*) if m_X^j-$Cl(A) \subseteq U$, whenever $A \subseteq U$ and $U \in m_X^i$, where $i, j = 1, 2$ and $i \neq j$. The complement of $gm_X^{(i,j)}$-closed is said to be $gm_X^{(i,j)}$-*open*.

3 Regular generalized closed sets

In this section, we introduce the concept of (i, j)-regular generalized closed sets in biminimal structure spaces and study some of their properties.

Definition 3.1. A subset A of a biminimal structure space (X, m_X^1, m_X^2) is said to be (i, j)-*regular generalized closed* (briefly (i, j)-*rg-closed*) set if m_X^j-$Cl(A) \subseteq U$, whenever $A \subseteq U$ and U is m_X^i-regular open in X, where $i, j = 1, 2$ and $i \neq j$. The complement of (i, j)-rg-closed is said to be (i, j)-*regular generalized open* (briefly (i, j)-*rg-open*). A subset A of a biminimal structure space (X, m_X^1, m_X^2) is called *pairwise* (i, j)-*rg-closed* if A is $(1, 2)$-rg-closed and $(2, 1)$-rg-closed. The complement of pairwise (i, j)-rg-closed is called *pairwise* (i, j)-*rg-open.*

The family of all (i, j)-rg-closed (resp. (i, j)-rg-open) sets of (X, m_X^1, m_X^2) is denote by (i, j)-rg-$C(X)$ (resp. (i, j)-rg-$O(X)$), $i, j = 1, 2$ and $i \neq j$.

Remark 1. *The union of two (i, j)-rg-closed sets is not a (i, j)-rg-closed set in general as can be seen from the following example.*

Example 3.2. *Let $X = \{a, b, c, d\}$. Consider two minimal structures $m_X^1 = \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, X\}$ and $m_X^2 = \{\emptyset, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$. Then $\{c\}$ and $\{d\}$ are $(1, 2)$-rg-closed but $\{c\} \cup \{d\} = \{c, d\}$ is not $(1, 2)$-rg-closed.*

Remark 2. *The intersection of two (i, j)-rg-closed sets is not a (i, j)-rg-closed set in general as can be seen from the following example.*

Example 3.3. *Let $X = \{a, b, c\}$. Consider two minimal structures $m_X^1 = \{\emptyset, \{a\}, \{b\}, X\}$ and $m_X^2 = \{\emptyset, \{a, b\}, X\}$. Then $\{a, b\}$ and $\{b, c\}$ are $(1, 2)$-rg-closed but $\{a, b\} \cap \{b, c\} = \{b\}$ is not $(1, 2)$-rg-closed.*

Remark 3. *The intersection of two (i, j)-rg-open sets is not a (i, j)-rg-open set in general as can be seen from the following example.*

Example 3.4. *Let $X = \{a, b, c, d\}$. Consider two minimal structures $m^1_X = \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, X\}$ and $m^2_X = \{\emptyset, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$. Then $\{a, b, d\}$ and $\{a, b, c\}$ are $(1, 2)$-rg-open but $\{a, b, d\} \cap \{a, b, c\} = \{a, b\}$ is not $(1, 2)$-rg-open.*

Remark 4. *The union of two (i, j)-rg-open sets is not a (i, j)-rg-open set in general as can be seen from the following example.*

Example 3.5. *Let $X = \{a, b, c\}$. Consider two minimal structures $m^1_X = \{\emptyset, \{a\}, \{b\}, X\}$ and $m^2_X = \{\emptyset, \{a, b\}, X\}$. Then $\{a\}$ and $\{c\}$ are $(1, 2)$-rg-open but $\{a\} \cup \{c\} = \{a, c\}$ is not $(1, 2)$-rg-open.*

Proposition 3.6. *If A is a (i, j)-rg-closed set of (X, m^1_X, m^2_X) such that $A \subseteq B \subseteq m^j_X$-$Cl(A)$, then B is (i, j)-rg-closed set, where $i, j = 1, 2$ and $i \neq j$.*

Proof. Let A be a (i, j)-rg-closed set and $A \subseteq B \subseteq m^j_X$-$Cl(A)$. Let $B \subseteq U$ and U is m^i_X-regular open. Then $A \subseteq U$. Since A is (i, j)-rg-closed, we have m^j_X-$Cl(A) \subseteq U$. Since $B \subseteq m^j_X$-$Cl(A)$, then m^j_X-$Cl(B) \subseteq m^j_X$-$Cl(A) \subseteq U$. Hence, B is (i, j)-rg-closed. □

Proposition 3.7. *For a subset A of a biminimal structure space (X, m^1_X, m^2_X). If A is both m^i_X-regular open and (i, j)-rg-closed, then A is m^j_X-closed, where $i, j = 1, 2$ and $i \neq j$.*

Proof. Let A be m^i_X-regular open and (i, j)-rg-closed, we have m^j_X-$Cl(A) = A$. Hence, A is m^j_X-closed. □

Remark 5. $(1, 2)$-*rg*-$C(X)$ *is generally not equal to* $(2, 1)$-*rg*-$C(X)$ *as can be seen from the following example.*

Example 3.8. *Let $X = \{a, b, c\}$. Consider two minimal structure $m^1_X = \{\emptyset, \{a\}, \{b\}, X\}$ and $m^2_X = \{\emptyset, \{a, b\}, X\}$. Then $(1, 2)$-rg-$C(X) = \{\emptyset, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$ and$(2, 1)$-rg-$C(X) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$. Thus $(1, 2)$-rg-$C(X) \neq (2, 1)$-rg-$C(X)$*

Proposition 3.9. *For each element x of a biminimal structure space (X, m^1_X, m^2_X), $\{x\}$ is m^i_X-regular closed or $X - \{x\}$ is (i, j)-rg-closed, where $i, j = 1, 2$ and $i \neq j$.*

Proof. Let $x \in X$ and the singleton $\{x\}$ be not m^i_X-regular closed. Then $X - \{x\}$ is not m^i_X-regular open, and so X is only m^i_X-regular open set which contains $X - \{x\}$. Hence $X - \{x\}$ is (i, j)-rg-closed. □

Proposition 3.10. *Let A be a subset of a biminimal structure space (X, m^1_X, m^2_X). If A is (i, j)-rg-closed, then m^j_X-$Cl(A) - A$ contains no nonempty m^i_X-regular closed set, where $i, j = 1, 2$ and $i \neq j$.*

Proof. Let A be a (i,j)-rg-closed set and $F \neq \emptyset$ is m_X^i-regular closed set such that $F \subseteq m_X^j$-$Cl(A) - A$. Since $A \in (i,j)$-rg-$C(X)$, we have m_X^j-$Cl(A) \subseteq X - F$. Thus $F \subseteq m_X^j$-$Cl(A) \cap (X - m_X^j$-$Cl(A)) = \emptyset$, this is a contradiction. Then m_X^j-$Cl(A) - A$ contains no nonempty m_X^i-regular closed set. □

Corollary 3.11. *Let m_X^1 and m_X^2 be minimal structure on X satisfying property $\mathcal{B}$. If A is (i,j)-rg-closed in (X, m_X^1, m_X^2), then A is m_X^j-closed if and only if m_X^j-$Cl(A) - A$ is m_X^i-regular closed , where $i, j = 1, 2$ and $i \neq j$.*

Proof. If A is m_X^j-closed, then m_X^j-$Cl(A) = A$. i.e. m_X^j-$Cl(A) - A = \emptyset$ and hence m_X^j-$Cl(A) - A$ is m_X^i-regular closed.

Conversely, if m_X^j-$Cl(A) - A$ is m_X^i-regular closed, then by Proposition 3.10 m_X^j-$Cl(A) - A = \emptyset$, since A is (i,j)-rg-closed. Therefore, A is m_X^j-closed. □

Proposition 3.12. *For a biminimal structure space (X, m_X^1, m_X^2) satisfying property $\mathcal{B}$. If every subset of X is (i,j)-rg-closed set, then m_X^i-$RO(X) \subseteq m_X^j$-$C(X)$ whenever m_X^i-$RO(X)$ is a family of all m_X^i-regular open and m_X^j-$C(X)$ is a family of all m_X^j-closed, where $i, j = 1, 2$ and $i \neq j$.*

Proof. suppose that every subset of X is (i,j)-rg-closed . Let $U \in m_X^i$-$RO(X)$. Since U is (i,j)-rg-closed, we have m_X^j-$Cl(U) \subseteq U$. Therefore, $U \in m_X^j$-$C(X)$ and hence m_X^i-$RO(X) \subseteq m_X^j$-$C(X)$. □

Theorem 3.13. *A subset A of a biminimal structure space (X, m_X^1, m_X^2) is (i,j)-rg-open if and only if every subset F of X, $F \subseteq m_X^j$-$Int(A)$ whenever F is m_X^i-regular closed and $F \subseteq A$, where $i, j = 1, 2$ and $i \neq j$.*

Proof. Suppose that A is (i,j)-rg-open. We shall show that $F \subseteq m_X^j$-$Int(A)$ whenever F is m_X^i-regular closed and $F \subseteq A$. Let $F \subseteq A$ and F is m_X^i-regular closed. Then $X - A \subseteq X - F$ and $X - F$ is m_X^i-regular open, we have $X - A$ is (i,j)-rg-closed, then m_X^j-$Cl(X - A) \subseteq X - F$. Thus $X - (m_X^j$-$Int(A)) \subseteq X - F$ and hence $F \subseteq m_X^j$-$Int(A)$.

Conversely, suppose that $F \subseteq m_X^j$-$Int(A)$ whenever F is m_X^i-regular closed and $F \subseteq A$. Let $X - A \subseteq U$ and U is m_X^i-regular open. Then $X - U \subseteq A$ and $X - U$ is m_X^i-regular closed. By assumption, we have $X - U \subseteq m_X^j$-$Int(A)$, then $X - (m_X^j$-$Int(A)) \subseteq U$. Therefore, m_X^j-$Cl(X - A) \subseteq U$. Thus $X - A$ is (i,j)-rg-closed. Hence, A is (i,j)-rg-open. □

Proposition 3.14. *Let A and B be subsets of a biminimal structure space (X, m_X^1, m_X^2) such that m_X^j-$Int(A) \subseteq B \subseteq A$. If A is (i,j)-rg-open, then B is (i,j)-rg-open, where $i, j = 1, 2$ and $i \neq j$.*

Proof. Suppose that m_X^j-$Int(A) \subseteq B \subseteq A$. Let F be m_X^i-regular closed such that $F \subseteq B$. Since A is (i,j)-rg-open, $F \subseteq m_X^j$-$Int(A)$. Since m_X^j-$Int(A) \subseteq B$, we have m_X^j-$Int(m_X^j$-$Int(A)) \subseteq m_X^j$-$Int(B)$. Consequently, m_X^j-$Int(A) \subseteq m_X^j$-$Int(B)$. Hence, $F \subseteq m_X^j$-$Int(B)$. Therefore, B is (i,j)-rg-open. □

Proposition 3.15. *Let A be a subset of a biminimal structure space (X, m_X^1, m_X^2). If A is (i,j)-rg-closed, then m_X^j-$Cl(A) - A$ is (i,j)-rg-open, where $i,j = 1,2$ and $i \neq j$.*

Proof. Suppose that A is (i,j)-rg-closed. We shall show that m_X^j-$Cl(A) - A$ is (i,j)-rg-open. Let $F \subseteq m_X^j$-$Cl(A) - A$ and F is m_X^i-regular closed. Since A is (i,j)-rg-closed, we have m_X^j-$Cl(A) - A$ does not contain nonempty m_X^i-regular closed by Proposition 3.10. Consequently, $F = \emptyset$, Therefore, $\emptyset \subseteq m_X^j$-$Cl(A) - A$. Hence, m_X^j-$Cl(A) - A$ is (i,j)-rg-open. □

4 Regular generalized continuous function

In this section, we introduce the concept of (i,j)-regular generalized continuous function on biminimal structure spaces and investigate some of their characterizations.

Definition 4.1. Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be biminimal structure space. A function $f : (X, m_X^1, m_X^2) \to (Y, m_Y^1, m_Y^2)$ is said to be (i,j)-*regular generalized continuous function* (briefly (i,j)-*rg-continuous*) if $f^{-1}(F)$ is (i,j)-rg-closed in X for every $m_Y^{(i,j)}$-closed F of Y, where $i,j = 1,2$ and $i \neq j$.

A function $f : (X, m_X^1, m_X^2) \to (Y, m_Y^1, m_Y^2)$ is (i,j)-regular generalized continuous if and only if $f^{-1}(U)$ is (i,j)-rg-open in X for every $m_Y^{(i,j)}$-open U of Y, where $i,j = 1,2$ and $i \neq j$.

Definition 4.2. A biminimal structure space (X, m_X^1, m_X^2) is said to be $m^{(i,j)}$-$T_{\frac{1}{2}}$ space if for every (i,j)-rg-closed set is $m_X^{(i,j)}$-closed set, where $i,j = 1,2$ and $i \neq j$.

Theorem 4.3. *Let (X, m_X^1, m_X^2) be a $m^{(i,j)}$-$T_{\frac{1}{2}}$ space and let (Y, m_Y^1, m_Y^2) be a biminimal structure space, where m_Y^1, m_Y^2 have property $\mathcal{B}$. For an injective function $f : (X, m_X^1, m_X^2) \to (Y, m_Y^1, m_Y^2)$, the following properties are equivalent:*

(1) f is (i,j)-rg-continuous.

(2) For each $x \in X$ and for every $m_Y^{(i,j)}$-open set V containing $f(x)$, there exists a (i,j)-rg-open set U containing x such that $f(U) \subseteq V$.

(3) $f(m_X^j\text{-}Cl(A)) \subseteq m_Y^j\text{-}Cl(f(A))$ for every subset A of X.

(4) $m_X^j\text{-}Cl(f^{-1}(B)) \subseteq f^{-1}(m_Y^j\text{-}Cl(B))$ for every subset B of Y.

Proof. (1) ⇒ (2): Let $x \in X$ and V be a $m_Y^{(i,j)}$-open subset of Y containing $f(x)$. Then by (1), $f^{-1}(V)$ is (i,j)-rg-open of X containing x. If $U = f^{-1}(V)$, then $f(U) \subseteq V$.

(2) ⇒ (3): Let A be a subset of X and $f(x) \notin m_Y^j\text{-}Cl(f(A))$. Then, there exists a $m_Y^{(i,j)}$-open subset V of Y containing $f(x)$ such that $V \cap f(A) = \emptyset$. Then by (2), there exists a (i,j)-rg-open set U such that $f(x) \in f(U) \subseteq V$. Hence, $f(U) \cap f(A) = \emptyset$ implies $U \cap A = \emptyset$. Consequently, $x \notin m_Y^j\text{-}Cl(A)$ and $f(x) \notin f(m_X^j\text{-}Cl(A))$.

(3) ⇒ (4): Let B be a subset of Y. By (3), we obtain $f(m_X^j\text{-}Cl(f^{-1}(B))) \subseteq m_Y^j\text{-}Cl(f(f^{-1}(B)))$. Thus $m_X^j\text{-}Cl(f^{-1}(B)) \subseteq f^{-1}(m_Y^j\text{-}Cl(B))$.

(4) ⇒ (1): Let F be a $m_Y^{(i,j)}$-closed subset of Y. Let U be a m_X^i-regular open subset of X such that $f^{-1}(F) \subseteq U$. Since $m_Y^j\text{-}Cl(F) = F$ and by (4), $m_X^j\text{-}Cl(f^{-1}(F)) \subseteq U$. Hence, f is (i,j)-rg-continuous. □

Definition 4.4. Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be biminimal structure spaces. A function $f : (X, m_X^1, m_X^2) \to (Y, m_Y^1, m_Y^2)$ is said to be *i-regular generalized irresolute* (briefly *i-rg-irresolute*) if $f^{-1}(F)$ is m_X^i-regular closed in X for every m_X^i-regular closed F of Y, for $i = 1, 2$.

Definition 4.5. Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be biminimal structure spaces. A function $f : (X, m_X^1, m_X^2) \to (Y, m_Y^1, m_Y^2)$ is said to be *i-r-closed* (resp. *i-r-open*) if $f(F)$ is m_Y^i-regular closed (resp. m_Y^i-regular open) of (Y, m_Y^i) for every m_X^i-regular closed (resp. m_X^i-regular open) F of (X, m_X^i), for $i = 1, 2$.

Definition 4.6. Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be biminimal structure spaces. A function $f : (X, m_X^1, m_X^2) \to (Y, m_Y^1, m_Y^2)$ is said to be *i-closed* (resp. *i-open*) if $f(F)$ is m_Y^i-closed (resp. m_Y^i-open) of (Y, m_Y^i) for every m_X^i-closed (resp. m_X^i-open) F of (X, m_X^i), for $i = 1, 2$.

Proposition 4.7. *Let (Y, m_Y^1, m_Y^2) be a biminimal structure space, where m_Y^1, m_Y^2 satisfying property $\mathcal{B}$. If $f : (X, m_X^1, m_X^2) \to (Y, m_Y^1, m_Y^2)$ is i-rg-irresolute and i-closed, then $f(A)$ is (i,j)-rg-closed subset of Y for every (i,j)-rg-closed subset A of X, where $i, j = 1, 2$ and $i \neq j$.*

Proof. Let A be a (i,j)-rg-closed subset of X. Suppose that $f(A) \subseteq U$, where U is m_Y^i-regular open. Then $A \subseteq f^{-1}(U)$ and $f^{-1}(U)$ is m_X^i-regular open subset of X, since f is i-rg-irresolute. Since A is (i,j)-rg-closed, we have $m_X^j\text{-}Cl(A) \subseteq f^{-1}(U)$ and hence $f(m_X^j\text{-}Cl(A)) \subseteq U$. Since f is i-closed, we have $m_Y^j\text{-}Cl(f(A)) \subseteq m_Y^j\text{-}Cl(f(m_X^j\text{-}Cl(A))) = f(m_X^j\text{-}Cl(A)) \subseteq U$. Therefore, $f(A)$ is (i,j)-rg-closed subset of Y. □

Lemma 4.8. *If $f : (X, m_X^1, m_X^2) \to (Y, m_Y^1, m_Y^2)$ is i-r-closed, then for each subset S of Y and each m_X^i-regular open subset U of X containing $f^{-1}(S)$, there exists a m_Y^i-regular open subset V of Y such that $f^{-1}(V) \subseteq U$.*

Proposition 4.9. *Let (X, m_X^1, m_X^2) be a $m^{(i,j)}$-$T_{\frac{1}{2}}$ space and let (Y, m_Y^1, m_Y^2) be a biminimal structure space, where m_Y^1, m_Y^2 have property $\mathcal{B}$. If $f : (X, m_X^1, m_X^2) \to (Y, m_Y^1, m_Y^2)$ is i-r-closed and (i,j)-rg-continuous, then $f^{-1}(B)$ is (i,j)-rg-closed subset of X for every (i,j)-rg-closed subset B of Y, where $i, j = 1, 2$ and $i \neq j$.*

Proof. Let B be a (i,j)-rg-closed subset of Y. Let U be a m_X^i-regular open subset of X such that $f^{-1}(B) \subseteq U$. Since f is i-r-closed and by Lemma 4.8, there exists a m_Y^i-regular open subset V of Y such that $B \subseteq V$ and $f^{-1}(V) \subseteq U$. Since B is (i,j)-rg-closed set and $B \subseteq V$, then m_Y^j-$Cl(B) \subseteq V$. Consequently, $f^{-1}(m_Y^j\text{-}Cl(B)) \subseteq f^{-1}(V) \subseteq U$. By Theorem 4.3, m_X^j-$Cl(f^{-1}(B)) \subseteq U$ and hence $f^{-1}(B)$ is (i,j)-rg-closed subset of X. □

Proposition 4.10. *Let (Y, m_Y^1, m_Y^2) be a $m^{(i,j)}$-$T_{\frac{1}{2}}$ space and let $f : (X, m_X^1, m_X^2) \to (Y, m_Y^1, m_Y^2)$ and $g : (Y, m_Y^1, m_Y^2) \to (Z, m_Z^1, m_Z^2)$ be functions. If f and g are (i,j)-rg-continuous, then $g \circ f$ is (i,j)-rg-continuous.*

Proof. Let F be a $m_Z^{(i,j)}$-closed subset of Z. Since g is (i,j)-rg-continuous, then $g^{-1}(F)$ is (i,j)-rg-closed subset of Y. Since (Y, m_Y^1, m_Y^2) be a $m^{(i,j)}$-$T_{\frac{1}{2}}$ space, then $g^{-1}(F)$ is $m_Y^{(i,j)}$-closed subset of Y. Since f is (i,j)-rg-continuous, then $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is (i,j)-rg-closed subset of X. Hence, $g \circ f$ is (i,j)-rg-continuous. □

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