**Int. Journal of Math. Analysis, Vol. 4, 2010, no. 11, 537 - 545**

# Fixed Point Theorems for Coincidence Maps in Intuitionistic Fuzzy Metric Space

**Sanjay Kumar*, S. K. Garg* and Ramesh Kumar Vats****

* Deen Bandhu Chhotu Ram University of Science and Techonology, Murthal, Sonepat -131039, Haryana (sanjaymudgal2004@yahoo.com)

**National Institute of Technology, Hamirpur (INDIA)-177005
ramesh_vats@rediffmail.com

**Abstract.** This paper is two folds. First, we prove a common fixed point theorem for weakly compatible map in intuitionistic fuzzy metric spaces which generalizes the result of Turkoglu, Alaca, Cho and Yildiz [22] and secondly, we discuss some results related to variants of R-weakly commuting mappings.

**Mathematics Subject Classifications:** 47H10, 54H25

**Keywords:** Intuitionistic fuzzy metric space, Weakly compatible mappings, variants of R-weakly commuting mappings(coincidence maps)

## 1. Introduction

It proved a turning point in the development of mathematics when the notion of fuzzy set was introduced by Zadeh [24] which laid the foundation of fuzzy mathematics. Consequently the last three decades were very productive for fuzzy mathematics and the recent literature has observed the fuzzification in almost every direction of mathematics such as arithmetic, topology, graph theory, probability theory, logic etc. The fuzzy set theory has applications in neural network theory, stability theory, mathematical programming, modeling theory, engineering sciences, medical sciences, image processing, control theory, communication etc. There are many view points of the notion of the metric space in fuzzy topology.

Atanassov [4] introduced and studied the concept of intuitionistic fuzzy sets. Intuitionistic fuzzy set deals with both degree of nearness and non-nearness. Coker [6] introduced the concept of intuitionistic fuzzy topological spaces and proved the well-known fixed point theorems of Banach [5] in the setting of intuitionistic fuzzy metric spaces. Turkoglu et al. [22] proved Jungck's [11] common fixed point theorem in the setting of intuitionistic fuzzy metric space. Turkoglu et al. [22] further formulated the notions of weakly commuting and R-weakly commuting mappings in intuitionistic fuzzy metric spaces and proved the intuitionistic fuzzy version of Pant's theorem [16]. Recently, Kumar and Vats [14] introduced the notion of variants of R-weakly commuting mappings.

## 2. Preliminaries

The concepts of triangular norms (t-norm) and triangular conorms (t-conorm) are originally introduced by Schweizer and Sklar [21] in study of statistical metric spaces.

**Definition 2.1[21]** A binary operation *: $[0,1]\times[0,1] \to [0,1]$ is continuous t-norm if * is satisfying the following conditions:

(i) * is commutative and associative;
(ii) * is continuous;
(iii) $a * 1 = a$ for all $a\in[0, 1]$;
(iv) $a * b \le c * d$ whenever $a \le c$ and $b \le d$ for all $a, b, c, d\in[0, 1]$.

**Definition 2.2[21]** A binary operation $\Diamond$ : $[0,1]\times[0,1] \to [0,1]$ is continuous t-conorm if $\Diamond$ is satisfying the following conditions:

(i) $\Diamond$ is commutative and associative;
(ii) $\Diamond$ is continuous;
(iii) $a \Diamond 0 = a$ for all $a\in[0, 1]$;
(iv) $a \Diamond b \le c \Diamond d$ whenever $a \le c$ and $b \le d$ for all $a, b, c, d\in[0, 1]$.

Alaca et al. [2] defined the notion of intuitionistic fuzzy metric space as follows:

**Definition 2.3[2]** A 5-tuple $(X, M, N, *, \Diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, * is a continuous t-norm, $\Diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2\times(0, \infty)$ satisfying the following conditions:

(i) $M(x, y, t) + N(x, y, t) \le 1$ for all $x, y\in X$ and $t > 0$;
(ii) $M(x, y, 0) = 0$ for all $x, y\in X$;
(iii) $M(x, y, t) = 1$ for all $x, y\in X$ and $t > 0$ if and only if $x = y$;
(iv) $M(x, y, t) = M(y, x, t)$ for all $x, y\in X$ and $t > 0$;
(v) $M(x, y, t) * M(y, z, s) \le M(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;
(vi) for all $x, y\in X$, $M(x, y, .) : [0, \infty) \to [0, 1]$ is continuous;
(vii) $\lim_{t\to\infty}M(x, y, t) = 1$ for all $x, y\in X$ and $t > 0$;
(viii) $N(x, y, 0) = 1$ for all $x, y\in X$;
(ix) $N(x, y, t) = 0$ for all $x, y\in X$ and $t > 0$ if and only if $x = y$;
(x) $N(x, y, t) = N(y, x, t)$ for all $x, y\in X$ and $t > 0$;
(xi) $N(x, y, t) \Diamond N(y, z, s) \ge N(x, z, t + s)$ for all $x, y, z\in X$ and $s, t > 0$;
(xii) for all $x, y\in X$, $N(x, y, .) : [0, \infty)\to[0, 1]$ is continuous;
(xiii) $\lim_{t\to\infty}N(x, y, t) = 0$ for all x, y in X.

(M, N) is called an intuitionistic fuzzy metric on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively. We know that **e**very fuzzy metric space (X, M, *) is an intuitionistic fuzzy metric space of the form $(X, M, 1\text{-}M, *, \Diamond)$ such that t-norm * and t-conorm $\Diamond$ are associated i.e., $x \Diamond y = 1-((1-x) * (1-y))$ for all $x, y\in X$. Further, the intuitionistic fuzzy metric spaces with continuous t-norm * and continuous t-conorm $\Diamond$ defined by $a*a \ge a$ and $(1-a) \Diamond (1-a) \le (1-a)$ for all $a\in[0, 1]$. Then for all $x, y\in X$, $M(x, y, *)$ is non-decreasing and $N(x, y, \Diamond)$ is non-increasing. Alaca, Turkoglu and Yildiz[2] introduced the following notions:

**Definition 2.4.** Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space. Then

(a) a sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all $t > 0$ and $p > 0$, $\lim_{n\to\infty}M(x_{n+p}, x_n, t) = 1$, $\lim_{n\to\infty}N(x_{n+p}, x_n, t) = 0$.
(b) a sequence $\{x_n\}$ in X is said to be convergent to a point $x\in X$ if, for all $t > 0$, $\lim_{n\to\infty}M(x_n, x, t) = 1$, $\lim_{n\to\infty}N(x_n, x, t) = 0$.

Since * and $\Diamond$ are continuous, the limit is uniquely determined from (v) and (xi) of Definition 2.3, respectively.

**Definition 2.5.** An intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ is said to be complete if and only if every Cauchy sequence in X is convergent.

Turkoglu, Alaca and Yildiz [23] introduced the notions of compatible mappings akin to the concept of compatible mappings introduced by Jungck [12] in metric spaces as follows:

**Definition 2.6.** A pair of self-mappings (f, g) of a intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be

(i) compatible if $\lim_{n\to\infty} M(fgx_n, gfx_n, t) = 1$ and $\lim_{n\to\infty} N(fgx_n, gfx_n, t) = 0$ for every $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$ for some $z \in X$.

(ii) non-compatible if $\lim_{n\to\infty} M(fgx_n, gfx_n, t) \neq 1$ or non-existent and $\lim_{n\to\infty} N(fgx_n, gfx_n, t) \neq 0$ or non-existent for every $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$ for some $z \in X$.

In 1996, Jungck and Rhoades [13] introduced the concept of weakly compatible maps as follows:

**Definition 2.7.** Two self maps f and g are said to be weakly compatible if they commute at coincidence points.

One can refer for the example that weakly compatible mappings need not be compatible, see[14].

Turkoglu et al. [22] first formulate the definition of weakly commuting and R-weakly commuting mappings in intuitionistic fuzzy metric spaces and proved the intuitionistic fuzzy version of Pant's theorem [16].

**Definition 2.8.** A pair of self-mappings (f, g) of a intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be

(i) weakly commuting if $M(fgx, gfx, t) \geq M(fx, gx, t)$ and $N(fgx, gfx, t) \leq N(fx, gx, t)$ for all $x \in X$ and $t > 0$.

(ii) R-weakly commuting if there exists some $R > 0$ such that $M(fgx, gfx, t) \geq M(fx, gx, t/R)$ and $N(fgx, gfx, t) \leq N(fx, gx, t/R)$ for all $x \in X$ and $t > 0$.

Recently, Kumar and Vats [14] introduced the notions of variants of R-weakly commuting mappings.

**Definition 2.9.** A pair of self-mappings (f, g) of an intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be

(i) R-weakly commuting mappings of type (Ag) if there exists some $R > 0$ such that

$$M(gfx, ffx, t) \geq M(fx, gx, t/R) \text{ and}$$
$$N(gfx, ffx, t) \leq N(fx, gx, t/R) \text{ for all } x \in X \text{ and } t > 0.$$

(ii) R-weakly commuting mappings of type $(A_f)$ if there exists some $R > 0$ such that

$$M(fgx, ggx, t) \geq M(fx, gx, t/R) \text{ and}$$
$$N(fgx, ggx, t) \leq N(fx, gx, t/R) \text{ for all } x \in X \text{ and } t > 0.$$

(iii) R-weakly commuting mappings of type (P) if there exists some $R > 0$ such that

$$M(ffx, ggx, t) \geq M(fx, gx, t/R) \text{ and}$$
$$N(ffx, ggx, t) \leq N(fx, gx, t/R) \text{ for all } x \in X \text{ and } t > 0.$$

Independency of R-weakly commuting of different kind is illustrating by the following example.

**Example 2.1[14]** Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space with X = [0, 1], t-norm * and t-conorm ◊ defined by a*b = min{a, b} and a◊b = max{a, b}, $a, b \in [0, 1]$, respectively. Let (M, N) is the intuitionistic fuzzy set on $X^2 \times (0,\infty)$, defined by

$$M(x, y, t) = \begin{cases} \left(\exp\left(\dfrac{|x\text{-}y|}{t}\right)\right)^{-1} & \text{for all } x,y \in X \text{ and } t > 0 \\ 0 & \text{for all } x,y \in X \text{ and } t = 0. \end{cases}$$

and

$$N(x, y, t) = \begin{cases} \left[\exp\left(\dfrac{|x\text{-}y|}{t}\right)\text{-}1\right]\left[\exp\left(\dfrac{|x\text{-}y|}{t}\right)\right]^{-1} & \text{for all } x,y \in X \text{ and } t > 0 \\ 1 & \text{for all } x,y \in X \text{ and } t = 0 \end{cases}$$

Then it is well known that (X, M, N, *, ◊) is an intuitionistic fuzzy metric space. Define $fx = 2x-1$ and $gx = x^2$. Then by a straightforward calculation, one can show that

$$M(fgx, gfx, t) = \left(\exp\left(\frac{2|x\text{-}1|^2}{t}\right)\right)^{-1} = M(fx, gx, t/2)$$

$$N(fgx, gfx, t) = \left[\exp\left(\frac{2|x\text{-}1|^2}{t}\right)\text{-}1\right]\left[\exp\left(\frac{2|x\text{-}1|^2}{t}\right)\right]^{-1} = N(fx, gx, t/2), \text{ which}$$

shows that the pair (f, g) is R-weakly commuting for R = 2. Note that the pair (f,g) is not weakly commuting due to a strict increasing property of the exponential function.

However, various kinds of above mentioned ‘R-weak commutativity’ notions are independent of one another and none implies the other. The earlier example can be utilized to demonstrate this inter-independence.

To demonstrate the independence of ‘R-weak commutativity’ with ‘R-weak commutativity of type ($A_f$)’ notice that

$$M(fgx, ggx, t) = \left(\exp\left(\frac{|x^4\text{-}2x^2\text{+}1|}{t}\right)\right)^{-1} = \left(\exp\left(\frac{R(x\text{-}1)^2}{t}\frac{(x\text{+}1)^2}{R}\right)\right)^{-1}$$

$$< \left(\exp\left(\frac{R|x\text{-}1|^2}{t}\right)\right)^{-1} = M(fx, gx, t/R) \text{ when } x > 1$$

$$N(fgx, ggx, t) = \left[\exp\left(\frac{|x^4\text{-}2x^2\text{+}1|^2}{t}\right)\text{-}1\right]\left[\exp\left(\frac{|x^4\text{-}2x^2\text{+}1|^2}{t}\right)\right]^{-1}$$

$$= \left[\exp\left(\frac{R(x\text{-}1)^2}{t}\frac{(x+1)^2}{R}\right)\text{-}1\right]\left[\exp\left(\frac{R(x\text{-}1)^2}{t}\frac{(x+1)^2}{R}\right)\right]^{-1}$$

$$> \left[\exp\left(\frac{R|x\text{-}1|^2}{t}\right)\text{-}1\right]\left[\exp\left(\frac{R|x\text{-}1|^2}{t}\right)\right]^{-1}$$

= N(fx, gx, t/R), which shows that ‘R-weak commutativity’ does not imply ‘R-weak commutativity of type ($A_f$)’.

Secondly, in order to demonstrate the independence of ‘R-weak commutativity’ with ‘R-weak commutativity of type (P)’ note that

$$M(ffx, ggx, t) = \left(\exp\left(\frac{|x^4 - 4x + 3|}{t}\right)\right)^{-1} = \left(\exp\left(\frac{R(x\text{-}1)^2}{t}\frac{(x^2 + 2x + 3)}{R}\right)\right)^{-1}$$

$$< \left( \exp\left( \frac{R|x-1|^2}{t} \right) \right)^{-1} = M(fx, gx, t/R)$$

and

$$N(ffx, ggx, t) = \left[ \exp\left( \frac{|x^4-4x+3|}{t} \right) - 1 \right] \left[ \exp\left( \frac{|x^4-4x+3|}{t} \right) \right]^{-1}$$

$$= \left[ \exp\left( \frac{R(x-1)^2}{t} \frac{(x^2+2x+3)}{R} \right) - 1 \right] \left[ \exp\left( \frac{R(x-1)^2}{t} \frac{(x^2+2x+3)}{R} \right) \right]^{-1}$$

$$> \left[ \exp\left( \frac{R|x-1|^2}{t} \right) - 1 \right] \left[ \exp\left( \frac{R|x-1|^2}{t} \right) \right]^{-1}$$

$= N(fx, gx, t/R)$ when $x > 1$, which implies R-weak commutativity does not imply R-weak commutativity of type (P).
Finally, for a change the pair (f, g) is R-weakly commuting of type ($A_g$) as

$$M(gfx, ffx, t) = \left( \exp\left( \frac{|(2x-1)^2 - 4x + 3|}{t} \right) \right)^{-1} = \left( \exp\left( \frac{4|x-1|^2}{t} \right) \right)^{-1}$$

$$= M(fx, gx, t/4)$$

$$N(gfx, ffx, t) = \left[ \exp\left( \frac{|(2x-1)^2-4x+3|}{t} \right) - 1 \right] \left[ \exp\left( \frac{|(2x-1)^2-4x+3|}{t} \right) \right]^{-1}$$

$$= \left[ \exp\left( \frac{4|x-1|^2}{t} \right) - 1 \right] \left[ \exp\left( \frac{4|x-1|^2}{t} \right) \right]^{-1}$$

$= N(fx, gx, t/4)$, which shows that (f, g) is R-weakly commuting of type ($A_g$) for $R = 4$. This situation may also be utilized to interpret that an R-weakly commuting pair of type ($A_g$) need not be R-weakly commuting pair of type ($A_f$) or type (P). It is not difficult to find examples to establish the independence of one of these definitions from the others which shows that there exist situations to suit a definition but not the others.

## 3. Main Results

**Lemma 3.1[1]**. Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space and for all $x, y \in X$, $t > 0$ and if for a number $k \in (0, 1)$,

(3.1) $M(x, y, kt) \geq M(x, y, t)$ and $N(x, y, kt) \leq N(x, y, t)$ then $x = y$.

**Lemma 3.2[1].** Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space and $\{y_n\}$ be a sequence in X. If there exists a number $k \in (0, 1)$ such that

(3.2) $M(y_{n+2}, y_{n+1}, kt) \geq M(y_{n+1}, y_n, t)$, $N(y_{n+2}, y_{n+1}, kt) \leq N(y_{n+1}, y_n, t)$ for all $t > 0$ and $n = 1, 2, \ldots$, then $\{y_n\}$ is a Cauchy sequence in X.

Turkoglu et al. [22] proved the following theorem:

**Theorem 3.1.** Let (X, M, N, *, ◊) be a complete intuitionistic fuzzy metric spaces. Let f, g: X→X be self mappings satisfying the following conditions:

(3.3) $g(X) \subset f(X)$

(3.4) f is continuous

(3.5) There exists a number $k \in (0,1)$ such that

$$M(gx, gy, kt) \geq M(fx, fy, t)$$

$$N(gx, gy, kt) \le N(fx, fy, t)$$

for all x, $y \in X$ and $t > 0$. Then f and g have a unique common fixed point in X provided f and g commute.

Now we prove the following results:

**Theorem 3.2.** Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric spaces. Let f, g: $X \to X$ be weakly compatible mappings satisfying (3.3), (3.5) and the following conditions:

(3.6) one of the subspace g(X) or f(X) is complete

Then f and g have a unique common fixed point in X.

**Proof.** By (3.3), since $g(X) \subset f(X)$, for any $x_0 \in X$, there exists a point $x_1 \in X$ such that $gx_0 = fx_1$. In general, chose $x_{n+1}$ such that $y_n = fx_{n+1} = gx_n$. From Theorem 2 of Turkoglu et al. [22], we conclude that $\{y_n\}$ is a Cauchy sequence in X. Since either f(X) or g(X) is complete, for definiteness assumes that f(X) is complete. Since f(X) is complete, so there exists a point $p \in X$ such that fp = z. Now using (3.5), we have

$$M(gp, gx_n, kt) \ge M(fp, fx_n, t) \text{ and } N(gp, gx_n, kt) \le N(fp, fx_n, t)$$

Taking limit as $n \to \infty$, we obtain

$M(gp, gx_n, kt) \ge M(fp, fx_n, t)$ and $N(gp, gx_n, kt) \le N(fp, fx_n, t)$, which implies gp = z.

Therefore, we have fp = gp =z.

Since f and g are weakly compatible, therefore fgp = gfp, i.e., fz = gz. Now we show that z is a common fixed point of f and g.

From (3.5), we get

$$M(gz, gx_n, kt) \ge M(fz, fx_n, t) \text{ and } N(gz, gx_n, kt) \le N(fz, fx_n, t)$$

Proceeding limit as $n \to \infty$, we obtain gz = z. Hence z is a common fixed point of f and g. Uniqueness follows easily from (3.5), see [22].

**Example 3.1.** Let X = $\{\frac{1}{n} : n \in N\} \cup \{0\}$ with * continuous t-norm and ◊ continuous t-conorm defined by a * b = ab and a ◊ b = min {1, a+ b} respectively, for all $a,b \in [0,1]$. For each $t \in [0, \infty)$ and x, $y \in X$, define (M, N) by

$$M(x, y, t) = \begin{cases} \frac{t}{t+|x-y|}, & t > 0, \\ 0 & t=0 \end{cases} \quad \text{and } N(x, y, t) = \begin{cases} \frac{|x-y|}{t+|x-y|}, & t > 0, \\ 1 & t=0 \end{cases}$$

Clearly, (X, M, N, *, ◊) is an intuitionistic fuzzy metric space.

Define gx $=\frac{x}{6}$ and fx $=\frac{x}{2}$ on X. It is clear that $g(X) \subseteq f(X)$.

$$\text{Now } M(gx, gy, \frac{t}{3}) = \begin{cases} \frac{\frac{t}{2}}{\frac{t}{2}+\frac{|x-y|}{6}} = \frac{3t}{3t+|x-y|} \ge \frac{t}{t+\frac{|x-y|}{3}} = \frac{3t}{3t+|x-y|} = M(fx, fy, t), \end{cases}$$

$$\text{Now } N(gx, gy, \frac{t}{3}) = \begin{cases} \frac{\frac{|x-y|}{6}}{\frac{t}{2}+\frac{|x-y|}{6}} = \frac{|x-y|}{3t+|x-y|} \le \frac{\frac{|x-y|}{3}}{t+\frac{|x-y|}{3}} = \frac{|x-y|}{3t+|x-y|} = N(fx, fy, t), \end{cases}$$

Thus all the conditions of Theorem 3.2 are satisfied and so f and g have a unique common fixed point 0.

**Theorem 3.3.** Theorem 3.2 remains true if a weakly compatible pair (coincidentally commuting) property is replaced by any one (retaining the rest of the hypotheses) of the following:

(i) R-weakly commuting property,
(ii) R-weakly commuting property of type $(A_f)$,
(iii) R-weakly commuting property of type (Ag),
(iv) R-weakly commuting property of type (P),
(v) weakly commuting property.

**Proof.** Since all the conditions of Theorem 3.2 are satisfied, then the existence of coincidence points for both the pairs is insured. Let x be an arbitrary point of coincidence for the pair (f, g), then using R-weak commutativity one gets

$M(fgx, gfx, t) \geq M(fx, gx, t/R) = 1$

$N(fgx, gfx, t) \leq N(fx, gx, t/R) = 0$, which amounts to say that fgx = gfx. Thus the pair (f, g) is weakly compatible pair. Now applying Theorem 3.2, one concludes that f and g have a unique common fixed point.

In case (f, g) is an R-weakly commuting pair of type $(A_f)$, then

$M(fgx, g^2x, t) \geq M(fx, gx, t/R) = 1$, $N(fgx, g^2x, t) \leq N(fx, gx, t/R) = 0$

which amounts to say that $fgx = g^2x$.

Now $M(fgx, gfx, t) \geq M(fgx, g^2x, \frac{t}{2}) * M(g^2x, gfx, \frac{t}{2}) = 1 * 1 = 1$

$N(fgx, gfx, t) \leq N(fgx, g^2x, \frac{t}{2}) \Diamond N(g^2x, gfx, \frac{t}{2}) = 0 \Diamond 0 = 0$, yielding

thereby fgx = gfx. Similarly, if pair is R-weakly commuting mappings of type (Ag) or type (P) or weakly commuting, then (A, S) also commutes at their points of coincidence. Now in view of Theorem 3.2, f and g have a unique common fixed point. This completes the proof.

As an application of Theorem 3.2, we prove a common fixed point theorem for four finite families of mappings which runs as follows:

**Theorem 3.4.** Let $\{f_1, f_2, \ldots, f_m\}$ and $\{g_1, g_2, \ldots, g_n\}$ be two finite families of self-mappings of a intuitionistic fuzzy metric spaces with continuous t-norm * and continuous t-conorm ◊ defined by $a*a \geq a$ and $(1-a) \Diamond (1-a) \leq (1-a)$ for all $a \in [0, 1]$ such that $f = f_1f_2 \ldots f_m$, $g = g_1g_2 \ldots g_n$, satisfy condition (3.3) ,(3.5) and (3.6). Then f and g have a point of coincidence. Moreover, if $f_if_j = f_jf_i$, $g_kg_l = g_lg_k$ for all $i, j \in I_1 = \{1, 2, \ldots, m\}$, $k, l \in I_2 = \{1, 2, \ldots, n\}$, then (for all $i \in I_1$, $k \in I_2$) $f_i$ and $g_k$ have a common fixed point.

**Proof.** The conclusions is immediate i.e., f and g have a point of coincidence as f, and g satisfy all the conditions of Theorem 3.2. Now appealing to component wise commutativity of various pairs, one can immediately prove that fg = gf, hence, obviously pairs (f, g) is coincidentally commuting. Note that all the conditions of Theorem 3.2 are satisfied which ensured the existence of a unique common fixed point, say z. Now one need to show that z remains the fixed point of all the component maps.

For this consider

$f(f_iz) = ((f_1f_2 \ldots f_m)f_i)z = (f_1f_2 \ldots f_{m-1})((f_mf_i)z) = (f_1 \ldots f_{m-1})(f_if_mz) = (f_1 \ldots f_{m-2})$
$(f_{m-1}f_i(f_mz) = (f_1 \ldots f_{m-2})(f_if_{m-1}(f_mz)) = \ldots f_1f_i(f_2f_3f_4 \ldots f_mz) = f_if_1(f_2f_3 \ldots f_mz)$
$= f_i(fz) = f_iz$.

Similarly, one can show that

$f(g_kz) = g_k(fz) = g_kz$, $g(g_kz) = g_k(gz) = g_kz$ and $g(f_iz) = f_i(gz) = f_iz$,

which show that (for all I and k) $f_iz$ and $g_kz$ are other fixed points of the pair (f,g). Now appealing to the uniqueness of common fixed points of both pairs separately, we get

$z = f_iz = g_k z$, which shows that z is a common fixed point of $f_i$, $g_k$ for all i and k.

By setting $f = f_1 = f_2 = \ldots = f_m$, $g = g_1 = g_2 = \ldots = g_n$ ,in Theorem 2.4 we deduces the following:

**Corollary 3.1.** Let Let f and g be two self-mappings of a intuitionistic fuzzy metric spaces with continuous t-norm * and continuous t-conorm ◊ defined by $a*a \geq a$ and $(1\text{-}a) \lozenge (1\text{-}a) \leq (1\text{-}a)$ for all $a \in [0, 1]$ such that that $f_m$ and $g_n$ satisfy the conditions satisfy condition (3.3), (3.5) and (3.6). If one of $f_m(X)$ or $g_n(X)$ is a complete subspace of X, then f, and g have a unique common fixed point provided (f, g) commute.

## References


[1] C. Alaca, I. Altun and D. Turkoglu: On compatible mappings of type (I) and type (II) in intuitionistic fuzzy metric spaces, *Commun. Korean Math. Soc.*, **23**(2008), No. 3, 427-446.

[2] C. Alaca, D. Turkoglu and C. Yildiz: Fixed points in intuitionistic fuzzy metric spaces, *Chaos, Solitons & Fractals*, **29(5)**(2006), 1073–1078.

[3] C. Alaca, D. Turkoglu and C. Yildiz: Common fixed points of compatible maps in intuitionistic fuzzy metric spaces, *Southeast Asian Bulletin of Mathematics*, **32**(2008), 21–33.

[4] K. Atanassov: Intuitionistic fuzzy sets, *Fuzzy Sets and System*, **20**(1986), 87-96.

[5] S. Banach: Theorie les operations lineaires, *Manograie Mathematyezne Warsaw Poland*, 1932.

[6] D. Coker: An introduction to intuitionistic fuzzy topological spaces, *Fuzzy Sets and System*, **88**(1997), 81–89.

[7] M. Edelstein: On fixed and periodic points under contractive mappings, *J. London Math. Soc.*, **37**(1962), 74–79.

[8] M. Grabiec: Fixed points in fuzzy metric spaces, *Fuzzy Sets and Systems*, **27**(1988), 385–389.

[9] V. Gregori, S. Romaguera and P. Veeramani: A note on intuitionistic fuzzy metric spaces, *Chaos, Solitons & Fractals*, **28**(2006), 902–905.

[10] M. Imdad and Javid Ali: Some common fixed point theorems in fuzzy metric spaces, *Mathematical Communication*, **11**(2006), 153–163.

[11] G. Jungck: Commuting mappings and fixed points, *Amer. Math. Monthly*, **83**(1976), 261–263.

[12] G. Jungck: Compatible mappings and common fixed points, *Internat. J. Math. Math. Sci.*, **9**(1986), 771–779.

[13] G. Jungck and B.E. Rhoades: Fixed point for set valued functions without continuity, *Indian J. Pure and Appl. Math.*, **29(3)**(1998), 227-238.

[14] S. Kumar and R.K. Vats: Common fixed points for weakly compatible maps in intuitionistic fuzzy metric spaces, *Advances in Fuzzy Mathematics*, ISSN 0973-533X, **4(1)**(2009), 9–22.

[15] K. Menger: Statistical metrices, *Proc. Nat. Acad. Sci.*, (USA), **28**(1942).

[16] R.P. Pant: Common fixed points of noncommuting mappings, *J. Math. Anal. Appl.*, **188**(1994), 436–440.

[17] J.H. Park: Intuitionistic fuzzy metric spaces, *Chaos, Solitons & Fractals*, **22**(2004), 1039–1046.

[18] J.S. Park, Y.C. Kwun, and J.H. Park: A fixed point theorem in the intuitionistic fuzzy metric spaces, *Far East J. Math. Sci.*, **16**(2005), 137–149.

[19] H.K. Pathak, Y.J. Cho, S.M. Kang: Remarks on R-Weakly commuting mappings and common fixed point theorems, *Bull. Korean Math. Soc.*, **34(2)**(1997), 247–257.

[20] R. Saadati and J.H. Park: On the intuitionistic fuzzy topological spaces, *Chaos, Solitons & Fractals*, **27**(2006), 331–344.

[21] B. Schweizer and A. Sklar: Statistical metric spaces, *Pacific J. Math.*, **10**(1960), 314–334.

[22] D. Turkoglu, C. Alaca, Y. J. Cho and C. Yildiz: Common fixed point theorems in intuitionistic fuzzy metric spaces, *J. Appl. Math. & Computing*, **22**(2006), 411–424.

[23] D. Turkoglu, C. Alaca and C. Yildiz: Compatible maps and compatible maps of types(α) and (β) in intuitionistic fuzzy metric spaces, *Demonstration Math.*, **39**(2006), 671–684.

[24] L. A. Zadeh: Fuzzy sets, *Inform. and Control*, **8**(1965), 338–353.

Received: May, 2009